Astrophysical Cosmology

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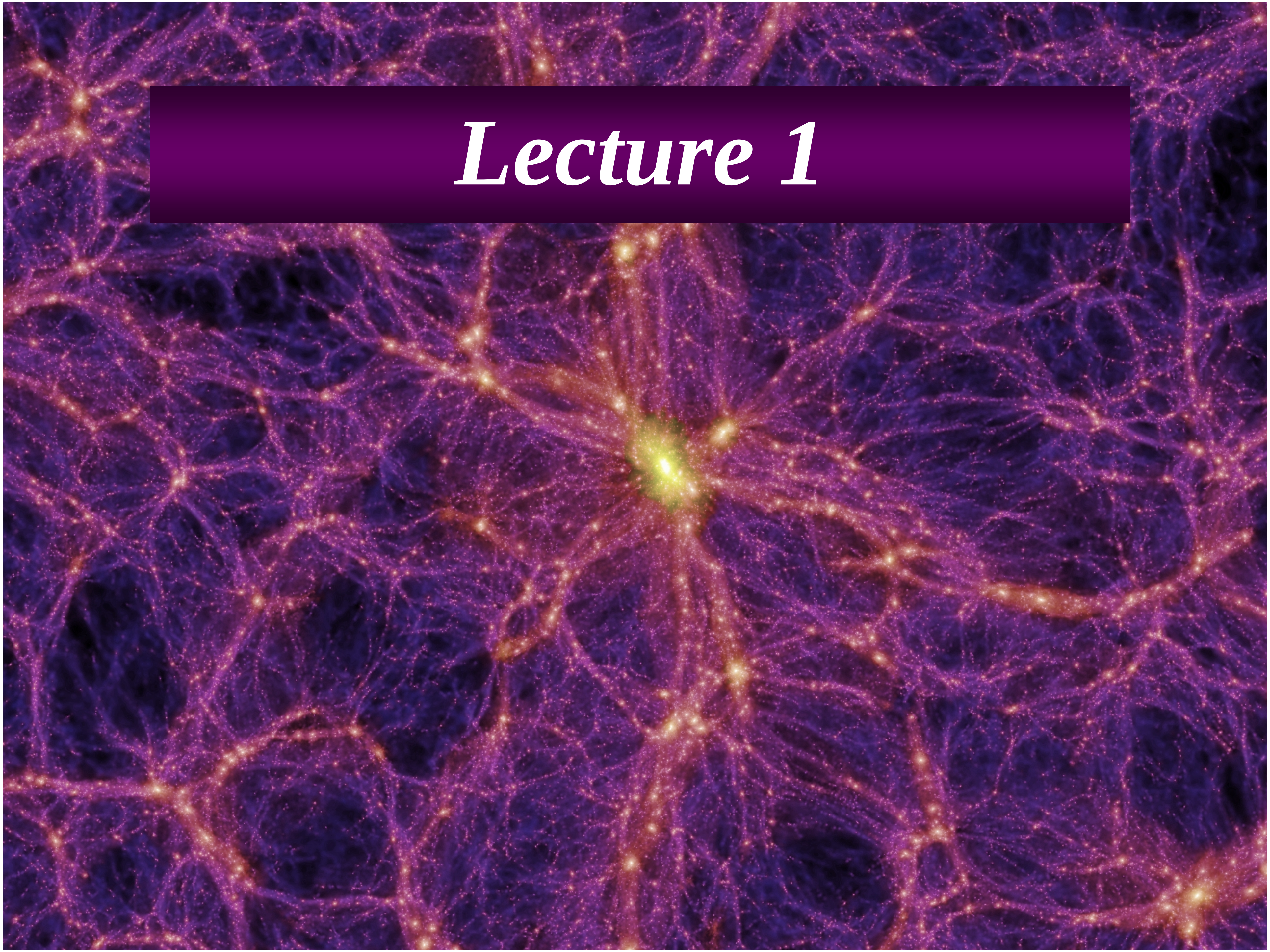
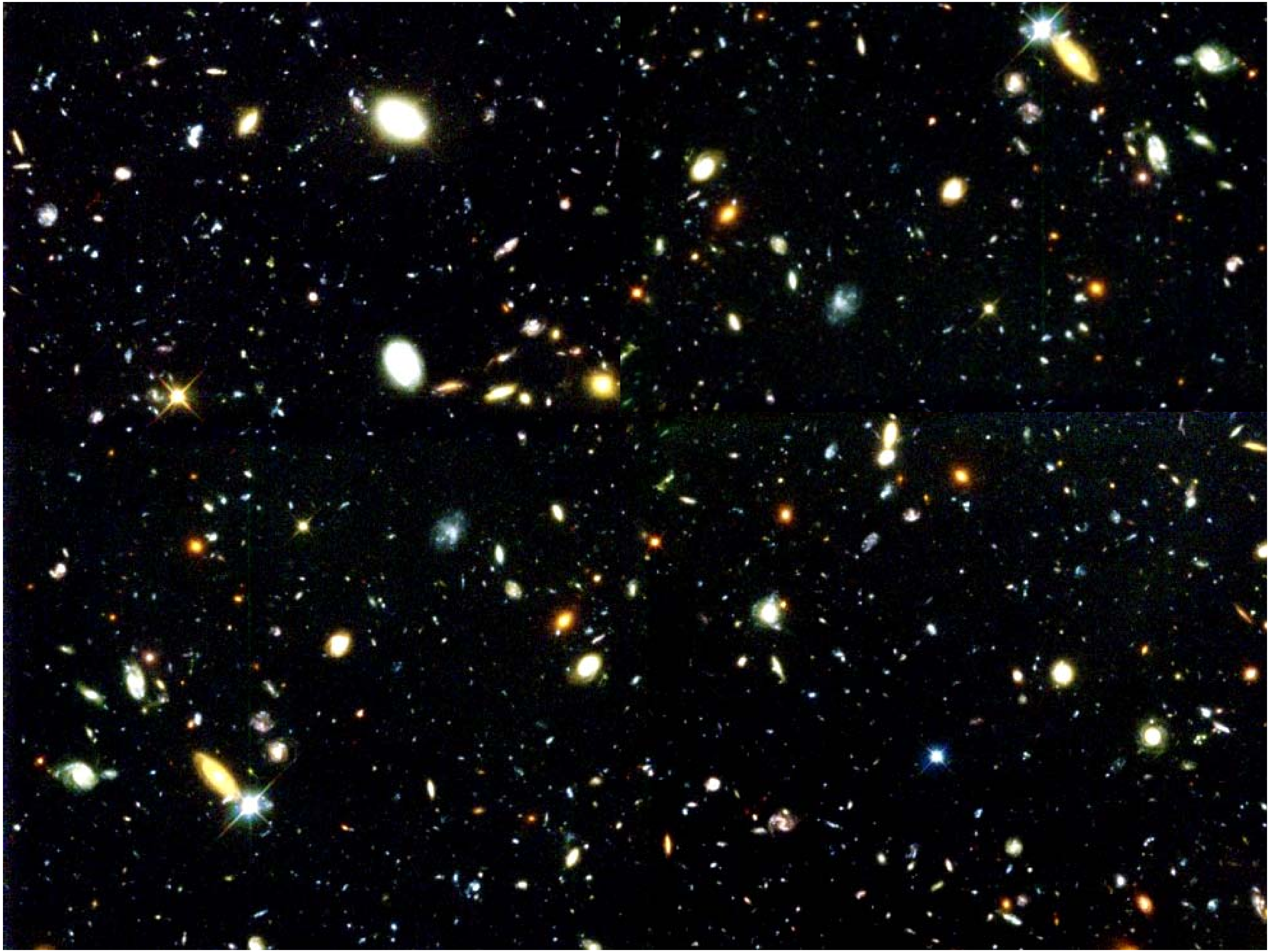
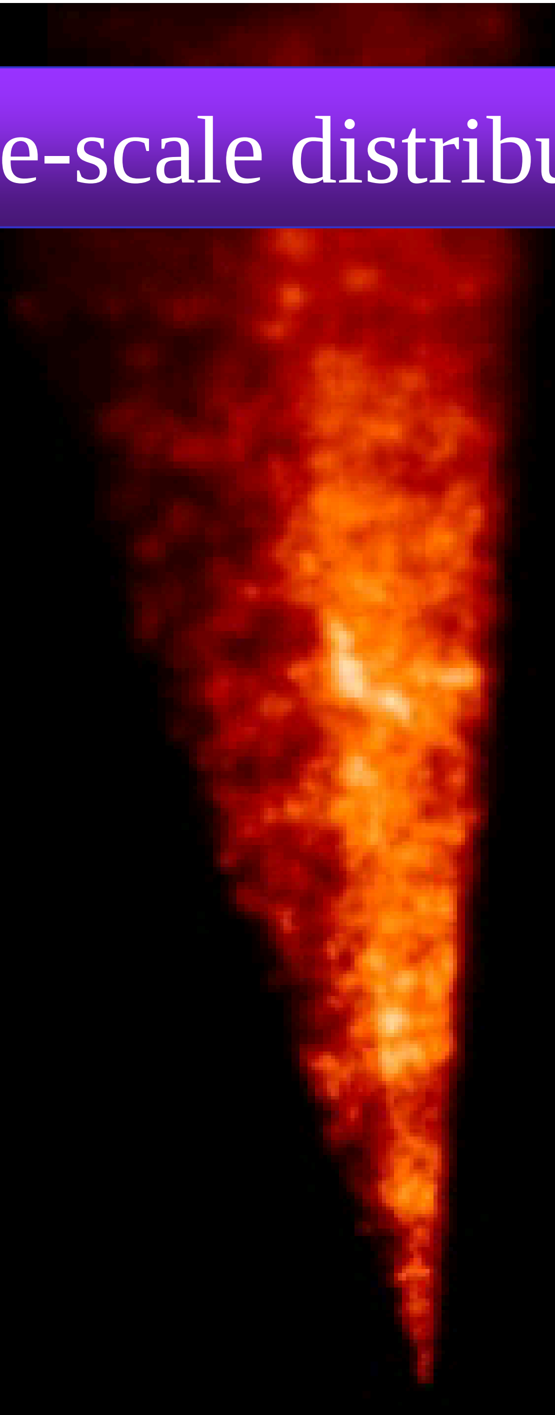
Lecture 1

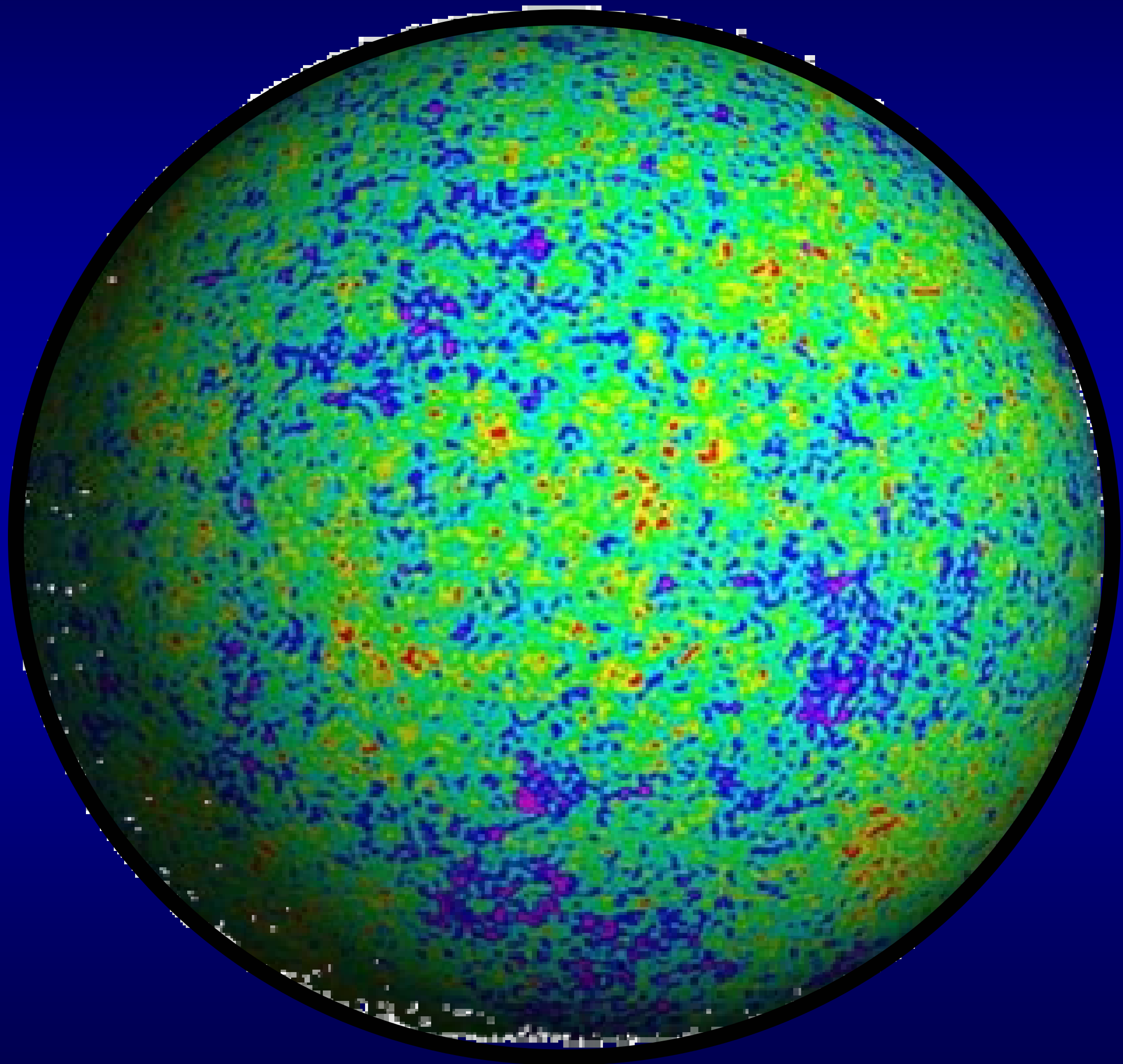




The large-scale distribution of galaxies

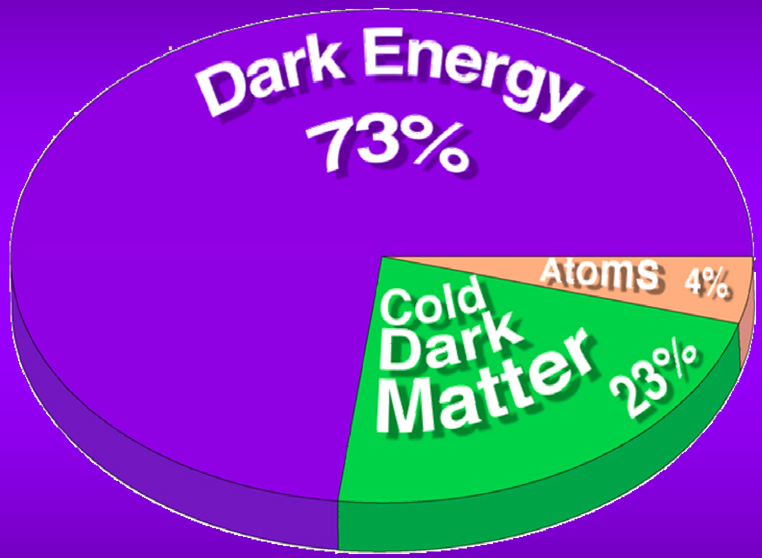


Temperature Variations in the Cosmic Microwave Background



Properties of the Universe

- Universe is expanding.
- Components of the Universe are:

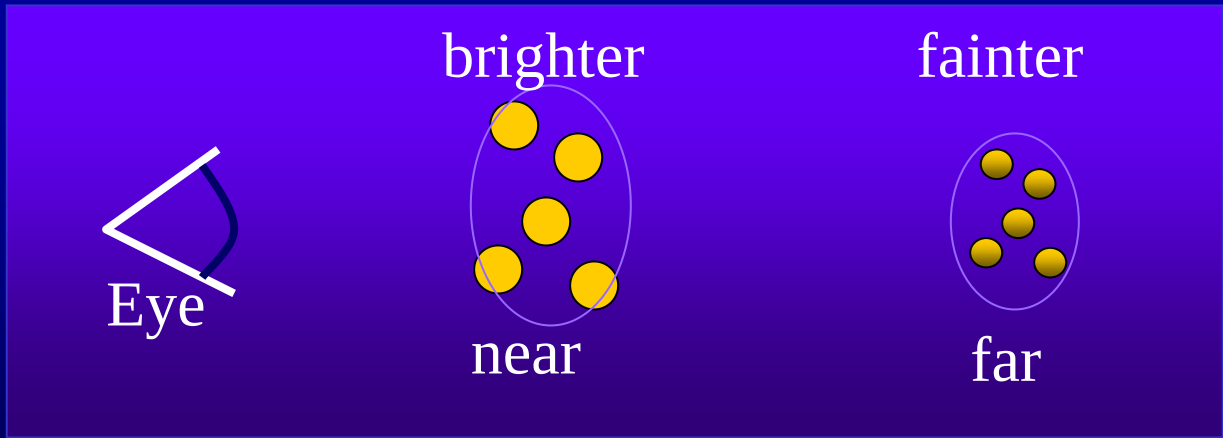


- Universe is 13.7 Billion years old.
- Expansion is currently accelerating.


1920's: The Great Debate
Are these nearby clouds of gas?
Or distant stellar systems (galaxies)

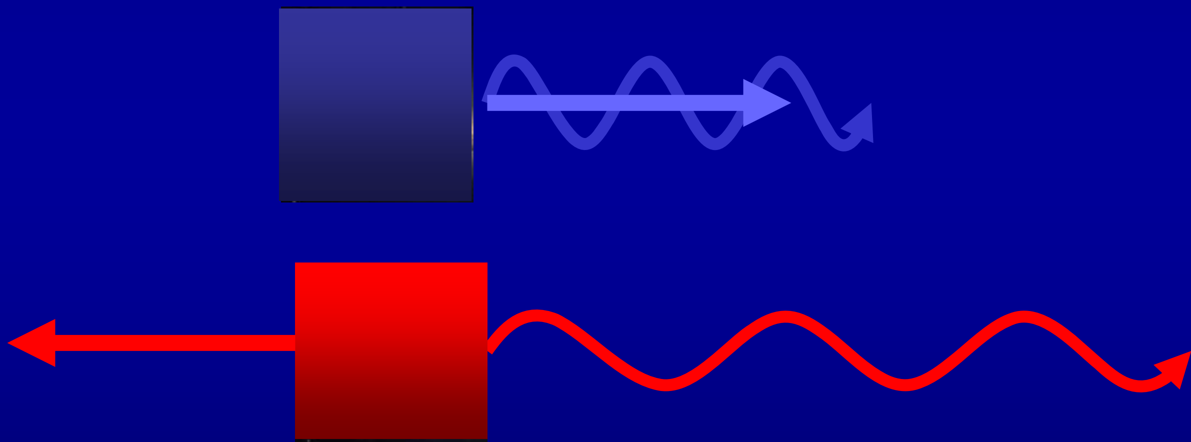
1920's: The Great Debate

- In 1924 Edwin Hubble finds Cepheid Variable stars in M31.
- Cepheid intrinsic brightness correlate with variability (standard candle), so can measure their distance.
- Measured 3 million light years (1Mpc) to M31.



The Expanding Universe

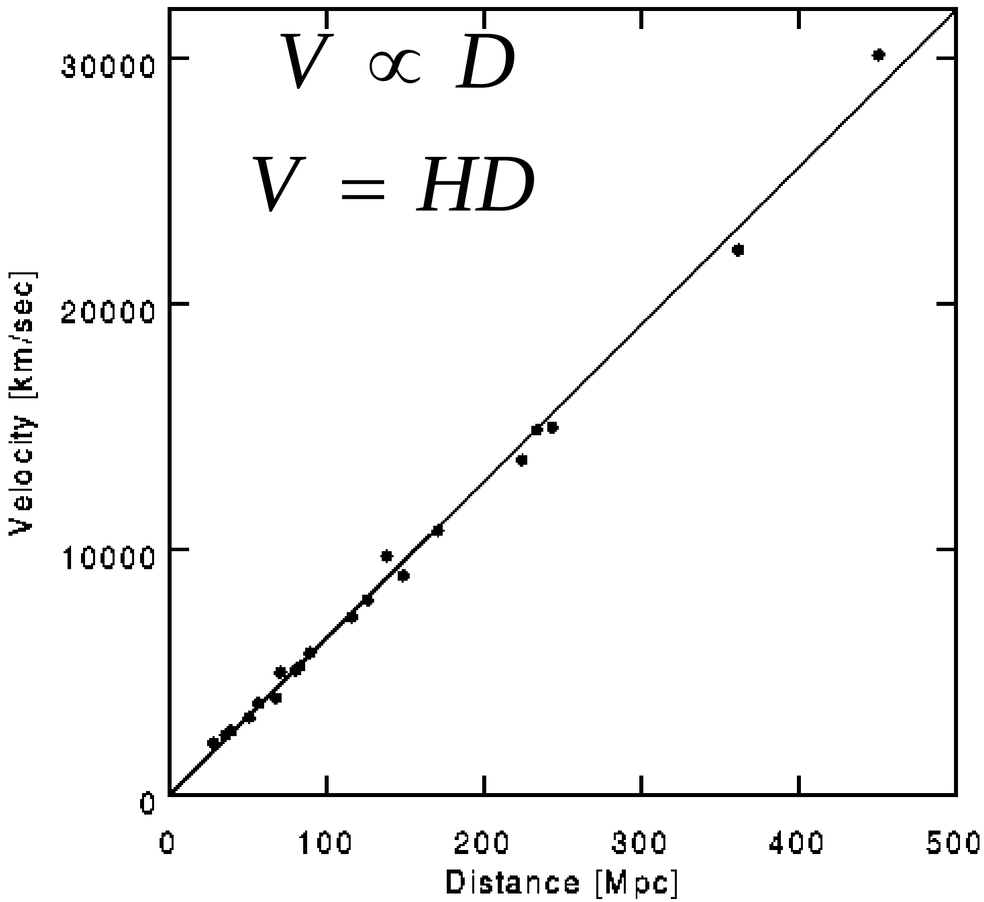
• Between 1912 and 1920 Vesto Slipher finds most galaxy’s spectra are redshifted.



Slipher is first to suggest the Universe is expanding !

Hubble's Law

In 1929 Hubble also finds fainter galaxies are more redshifted.
Infers that recession velocities increase with distance.

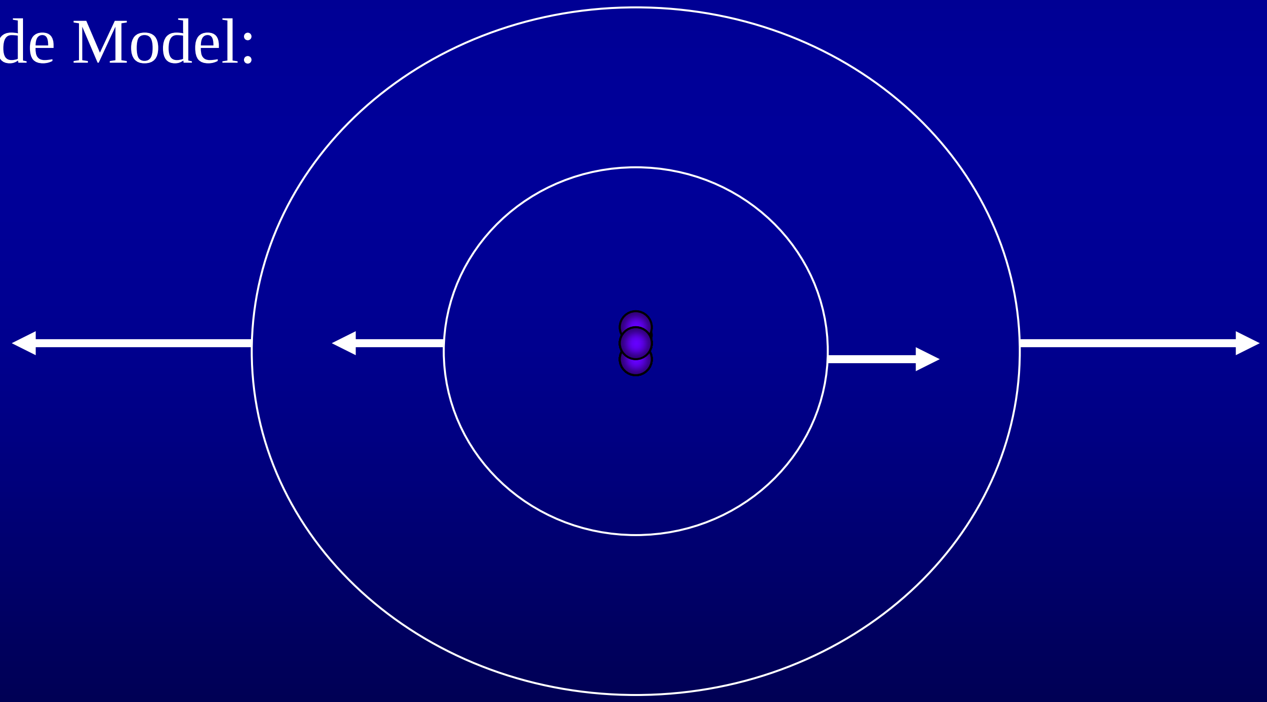


The Expanding Universe

Distance = Velocity x Time

V = HD, H = 1/t

1. Grenade Model:



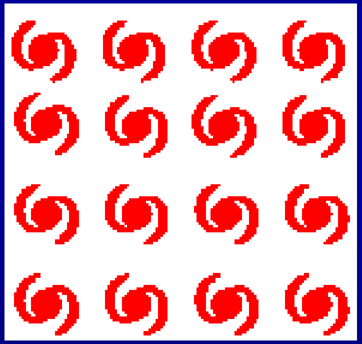
The Expanding Universe

Distance = Velocity x Time

$V = HD, \qquad H = 1/t$

2. Scaling Model:

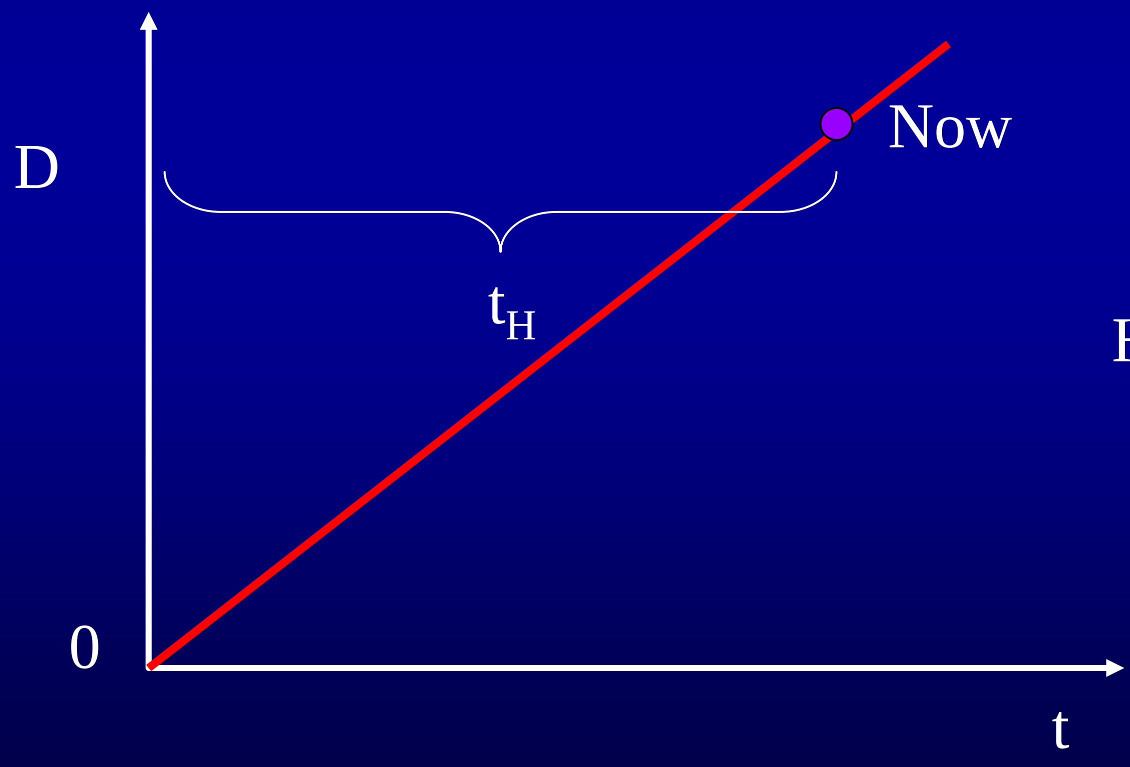
$x(t) = R(t)\, x_0$



The Expanding Universe

Distance = Velocity x Time

$V = HD, \qquad H = 1/t$

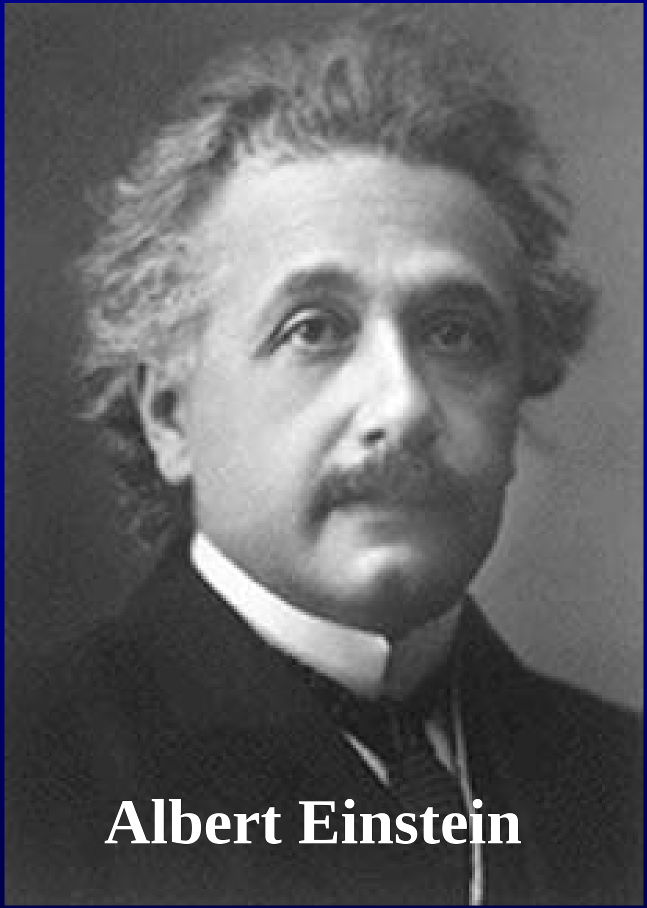


Hubble Time:
$t_H = 1/H$

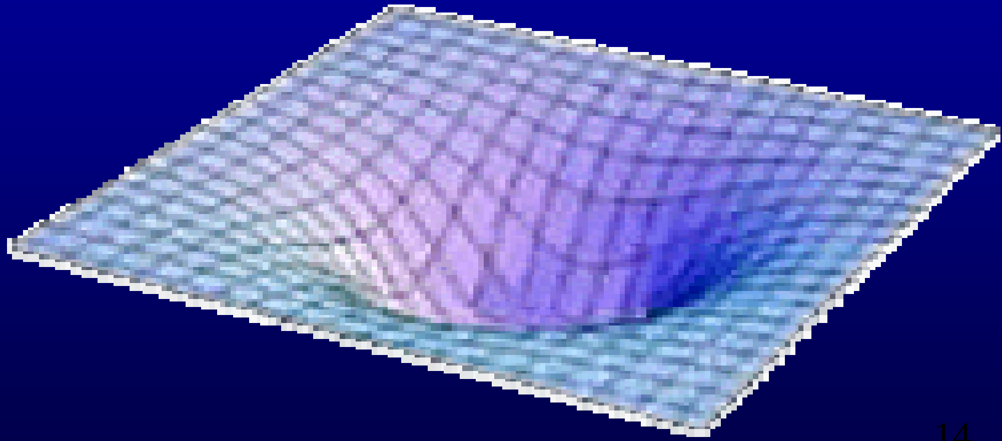
H=70 km/s/Mpc
t_H=14Gyrs

Relativistic Cosmologies

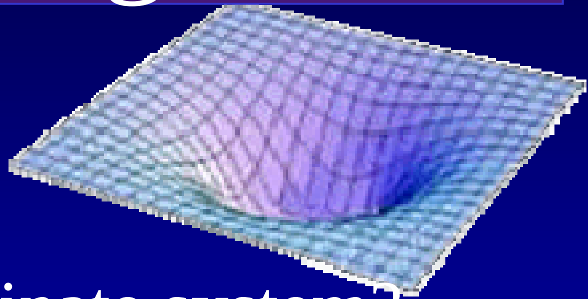
In 1915 Albert Einstein showed that the geometry of spacetime is shaped by the mass-energy distribution.



General Theory of Relativity required to describe the evolution of spacetime.



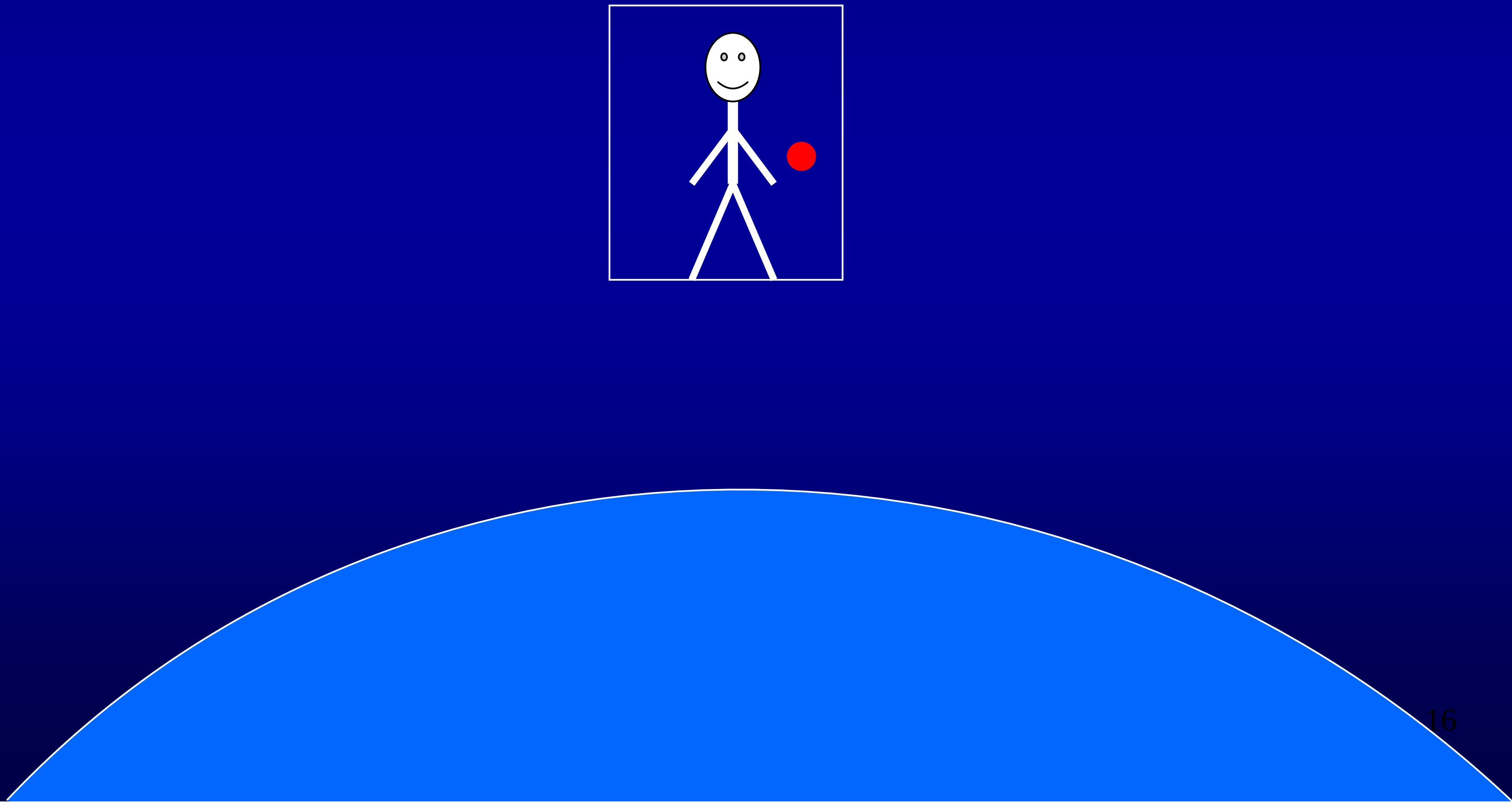
Relativistic Cosmologies



- Cosmological Coordinates (*t* , ***x***):
  - How do we lay down a global coordinate system?
  - In general we cannot.
- Can we lay down a local coordinate system?
  - Yes, can use Special Relativity locally, if we can cancel gravity.
- We can cancel gravity by free-falling (equivalence principle).

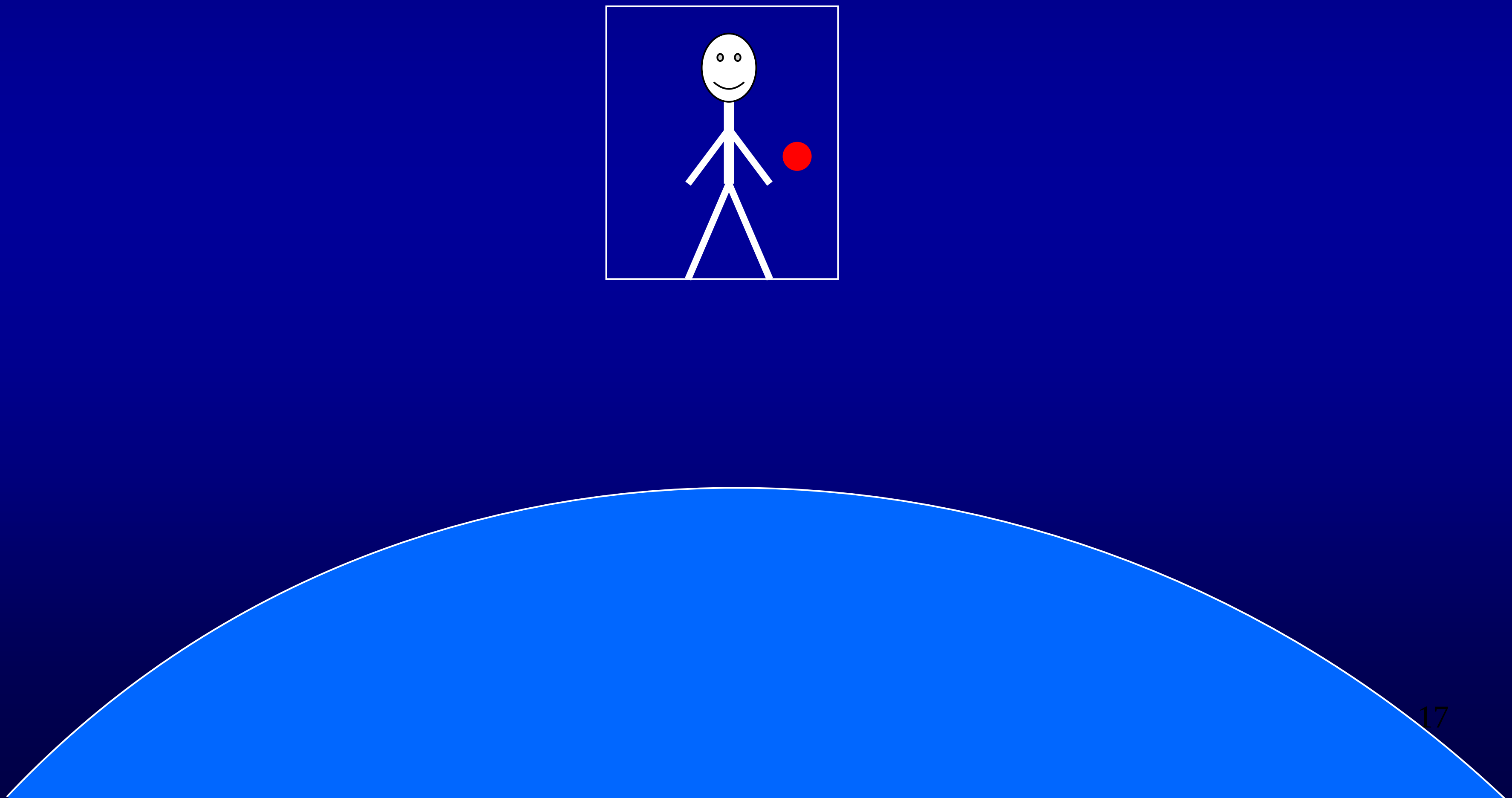
Relativistic Cosmologies

- Equivalence Principle:



Relativistic Cosmologies

- Equivalence Principle:



Relativistic Cosmologies

- In free-fall, a **Fundamental Observer** locally measures the spacetime of Special Relativity.

- Special Relativity Minkowski-space line element:

$$-ds^2 = c^2 d\tau^2 = c^2 dt^2 - dx^2 - dy^2 - dz^2$$

- So all Fundamental Observers will measure time changing at the same rate, dt.

- **Universal cosmological time coordinate**, t.

Relativistic Cosmologies

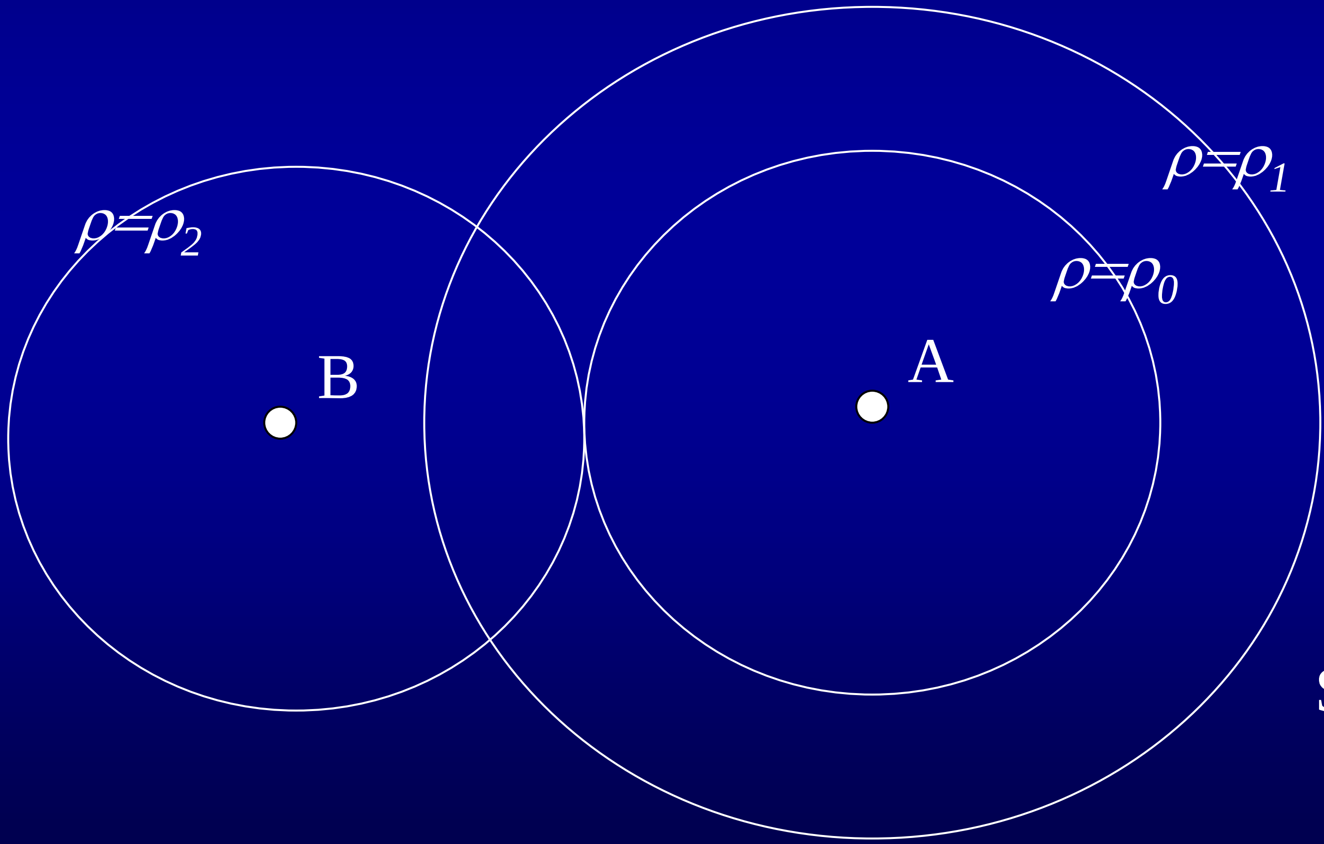
- How can we synchronize this Universal cosmological time coordinate, t, everywhere?

- With a Symmetry Principle:
  - On large-scales Universe seems **isotropic** (same in all directions, eg, Hubble expansion, galaxy distribution, CMB).
  - Combine with **Copernican Principle** (we're not in a special place).

Relativistic Cosmologies

- **Isotropy + Copernican Principle = homogeneity**

(same in all places)

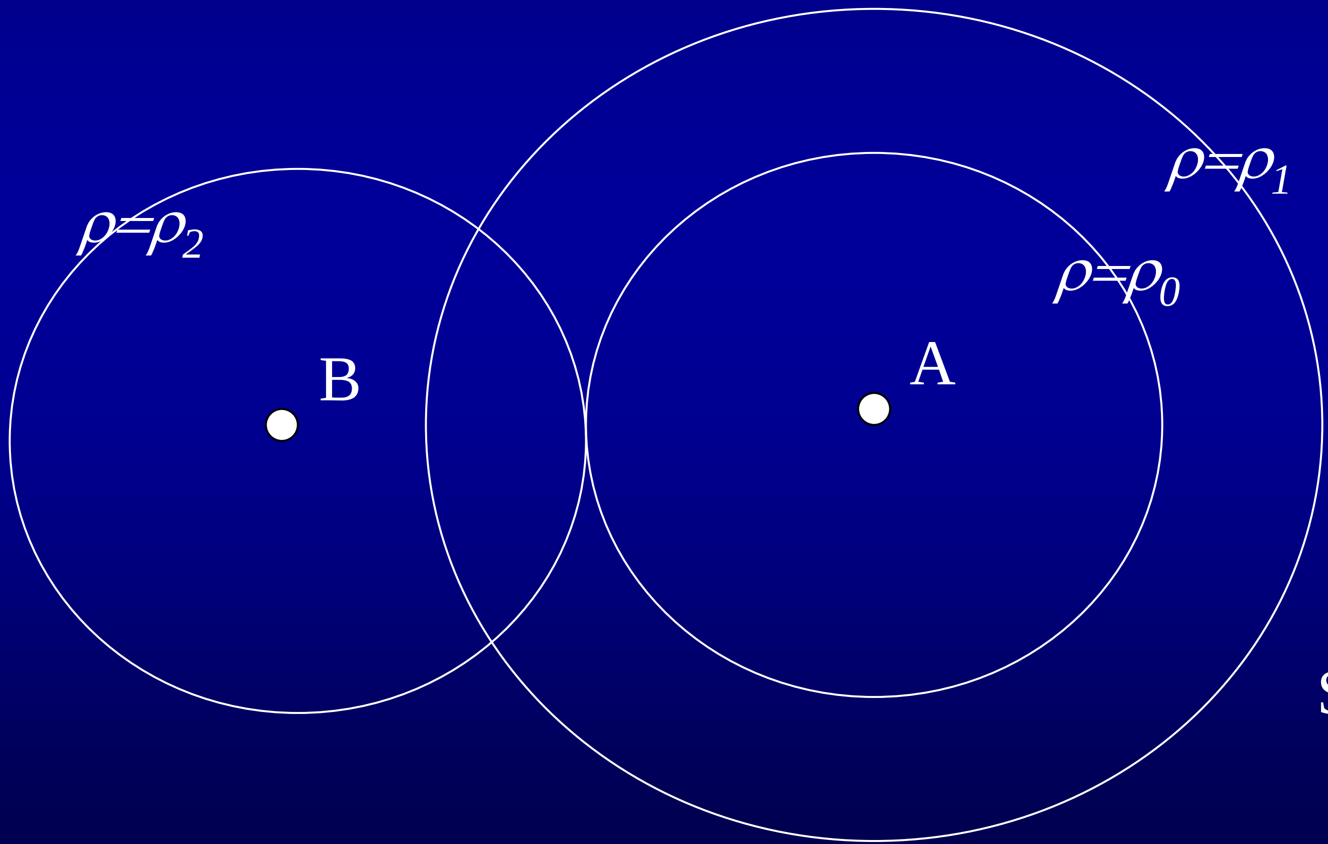


So $\rho_2=\rho_1=\rho_0$.

So uniform density everywhere

Relativistic Cosmologies

- **Isotropy + homogeneity = Cosmological Principle**



So $\rho_2=\rho_1=\rho_0$.

So uniform density everywhere

Relativistic Cosmologies

- With the Cosmological Principle, we have uniform density everywhere.

- Density will decrease with expansion, so $\rho = \rho\,(t)$.

- So can synchronize all Fundamental Observers clocks at pre-set density, ρ_0, and time, t_0:

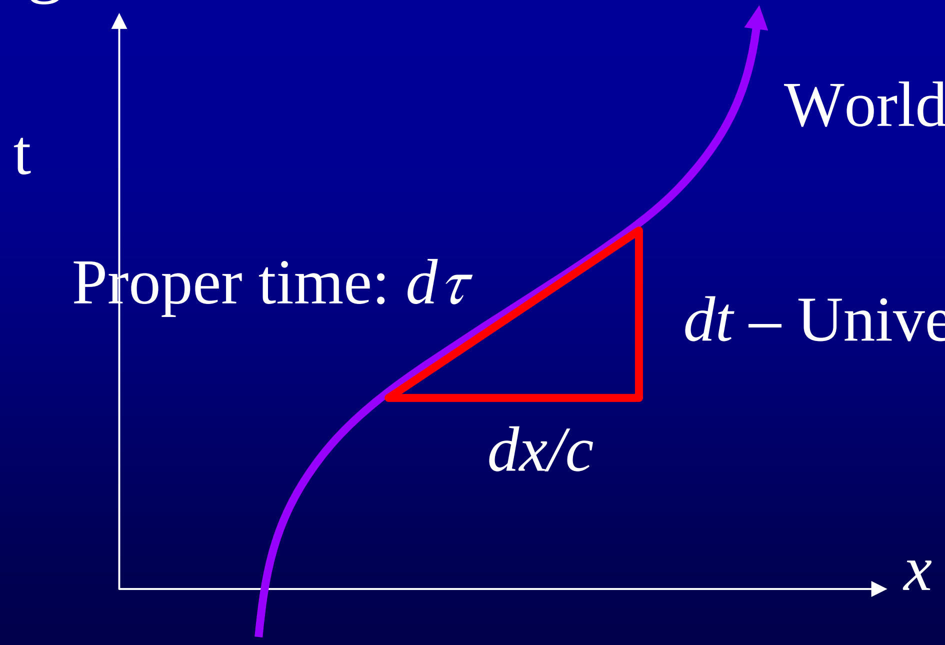
$$t_0 : \qquad \rho(t_0) = \rho_0$$

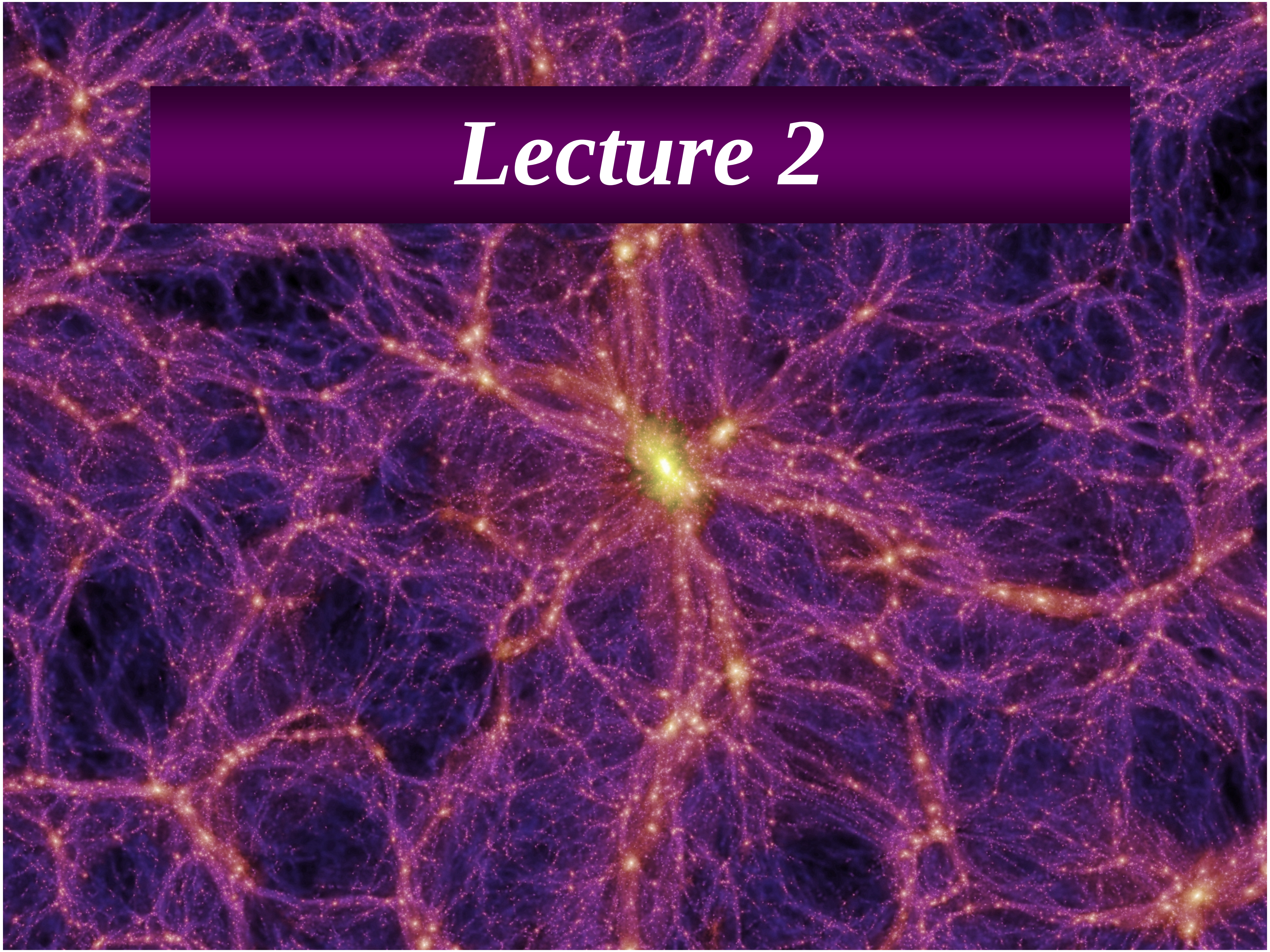
Relativistic Cosmologies

- What is the line element (metric) of a relativistic cosmology?
- Locally Minkowski line element (Special Relativity):

$$-ds^2 = c^2 d\tau^2 = c^2 dt^2 - dx^2 - dy^2 - dz^2$$

- Spacetime Diagram:




Lecture 2

Relativistic Cosmologies

- A general line element (Pythagoras on curved surface):

$$c^2 d\tau^2 = g_{\nu\mu} dx^\nu dx^\mu \qquad x^\mu = (ct, x, y, z)$$

- Minkowski metric tensor:

$$g_{\nu\mu} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix}$$

- We have Universal Cosmic Time of Special Relativity, t, so

$$c^2 d\tau^2 = c^2 dt^2 - d\sigma_3^2$$

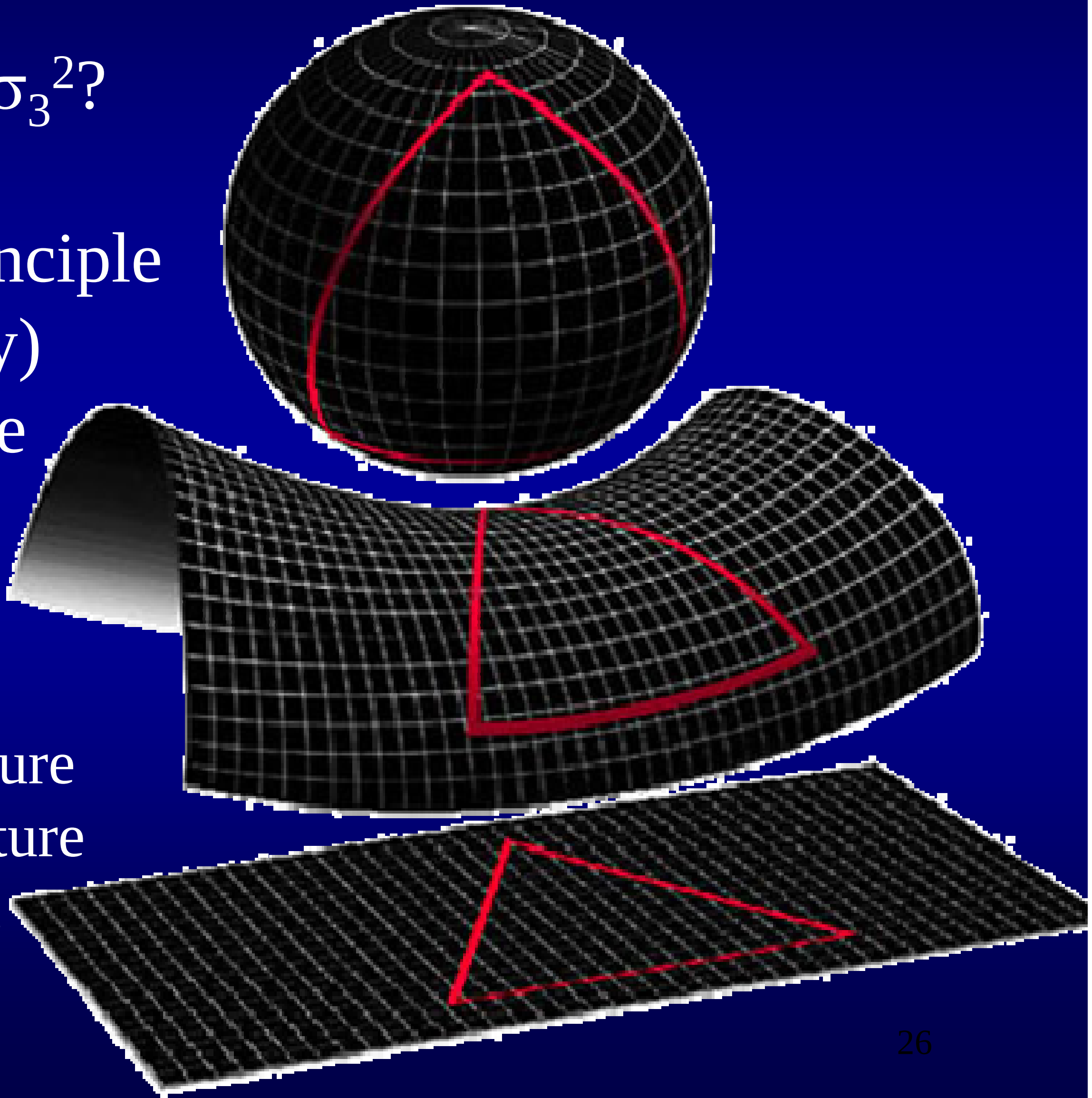
$$\sigma_3^2 - \text{Spatial part of metric}$$

Relativistic Cosmologies

- What is spatial metric, σ_3^2?

- From Cosmological Principle (homogeneity + isotropy) spatial curvature must be constant everywhere.

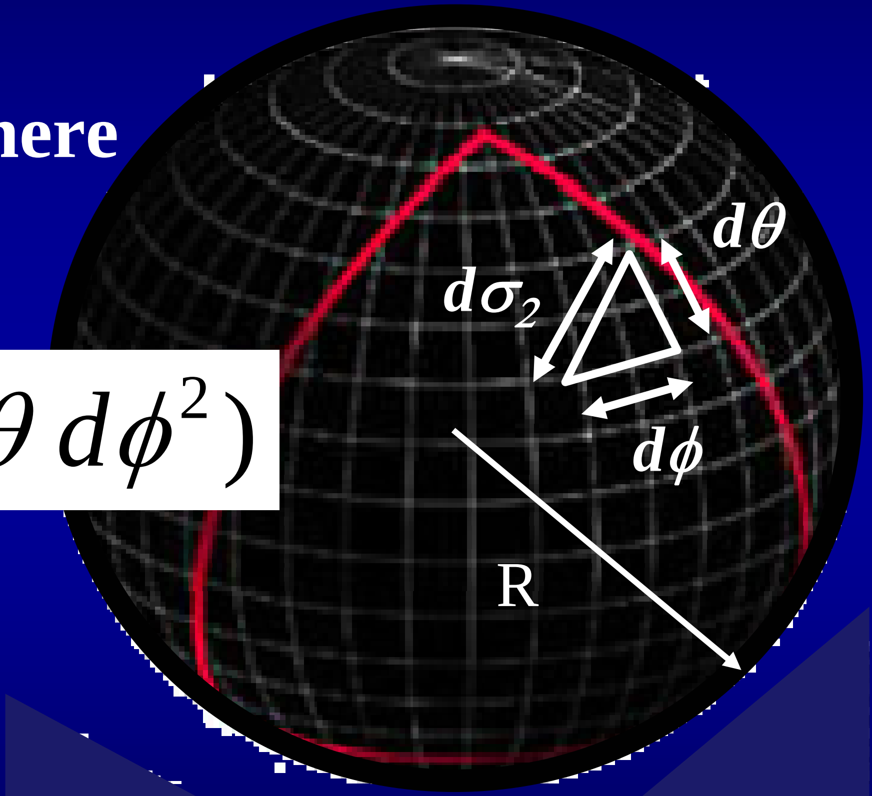
- Only 3 possibilities:
  - Sphere – positive curvature
  - Saddle – negative curvature
  - Flat – zero curvature.



Relativistic Cosmologies

- What is form of σ_3^2?
- Consider the metric on a **2-sphere** of radius R, σ_2^2:

$$d\sigma_2^2 = R^2(d\theta^2 + \sin^2\theta\, d\phi^2)$$



Relativistic Cosmologies

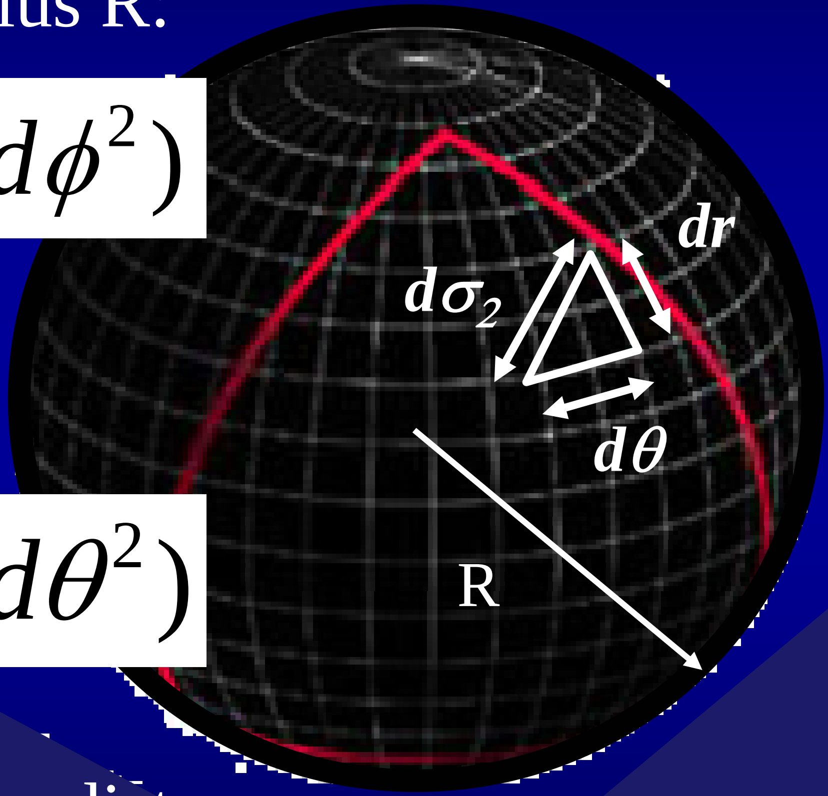
- The metric on a 2-sphere of radius R:

$$d\sigma_2^2 = R^2(d\theta^2 + \sin^2\theta d\phi^2)$$

- Now re-label θ as r and ϕ as θ:

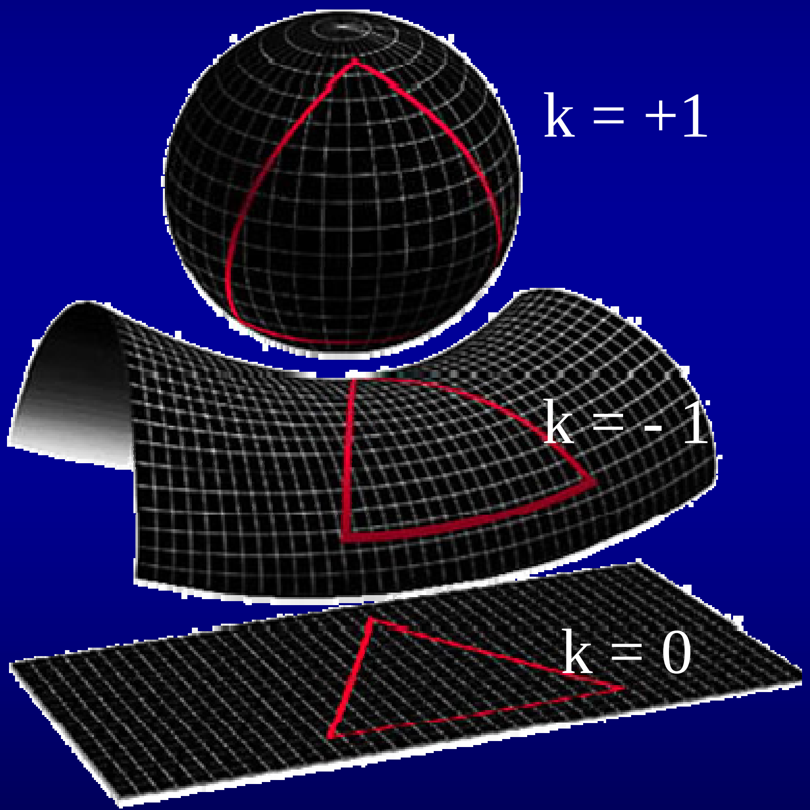
$$d\sigma_2^2 = R^2(dr^2 + \sin^2 r d\theta^2)$$

where $r = (0,\pi)$ is a dimensionless distance.



Relativistic Cosmologies

- Can generate other 2 models from the 2-sphere:



$$d\sigma_2^2 = R^2(dr^2 + \sin^2 r d\theta^2)$$

$$\downarrow \quad r \to ir,\ R \to iR$$

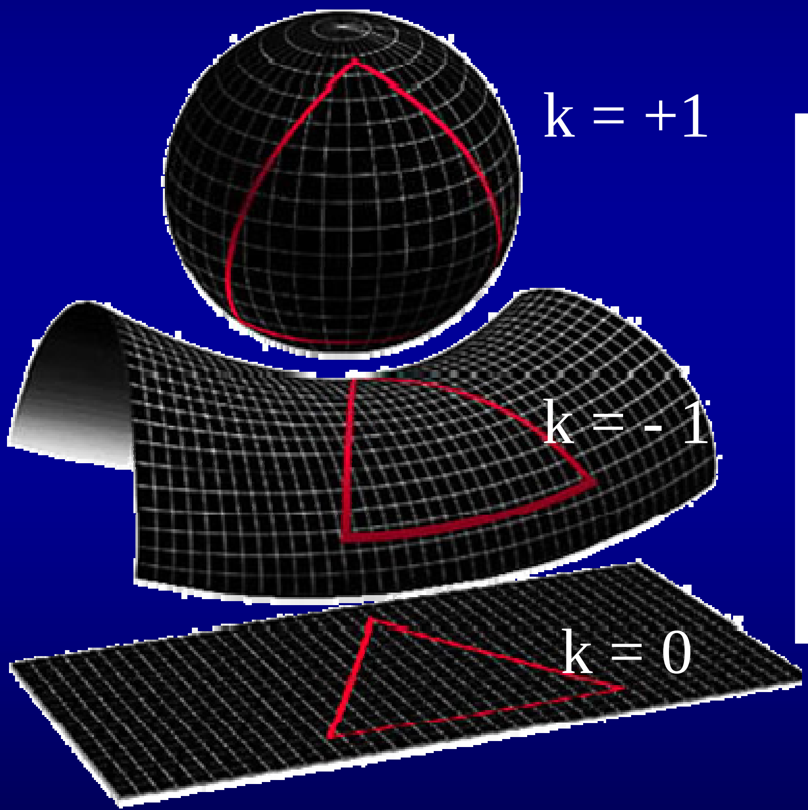
$$d\sigma_2^2 = R^2(dr^2 + \sinh^2 r d\theta^2)$$

$$\downarrow \quad r << R$$

$$d\sigma_2^2 = R^2(dr^2 + r^2 d\theta^2)$$

Relativistic Cosmologies

- General 3-metric for 3 curvatures:

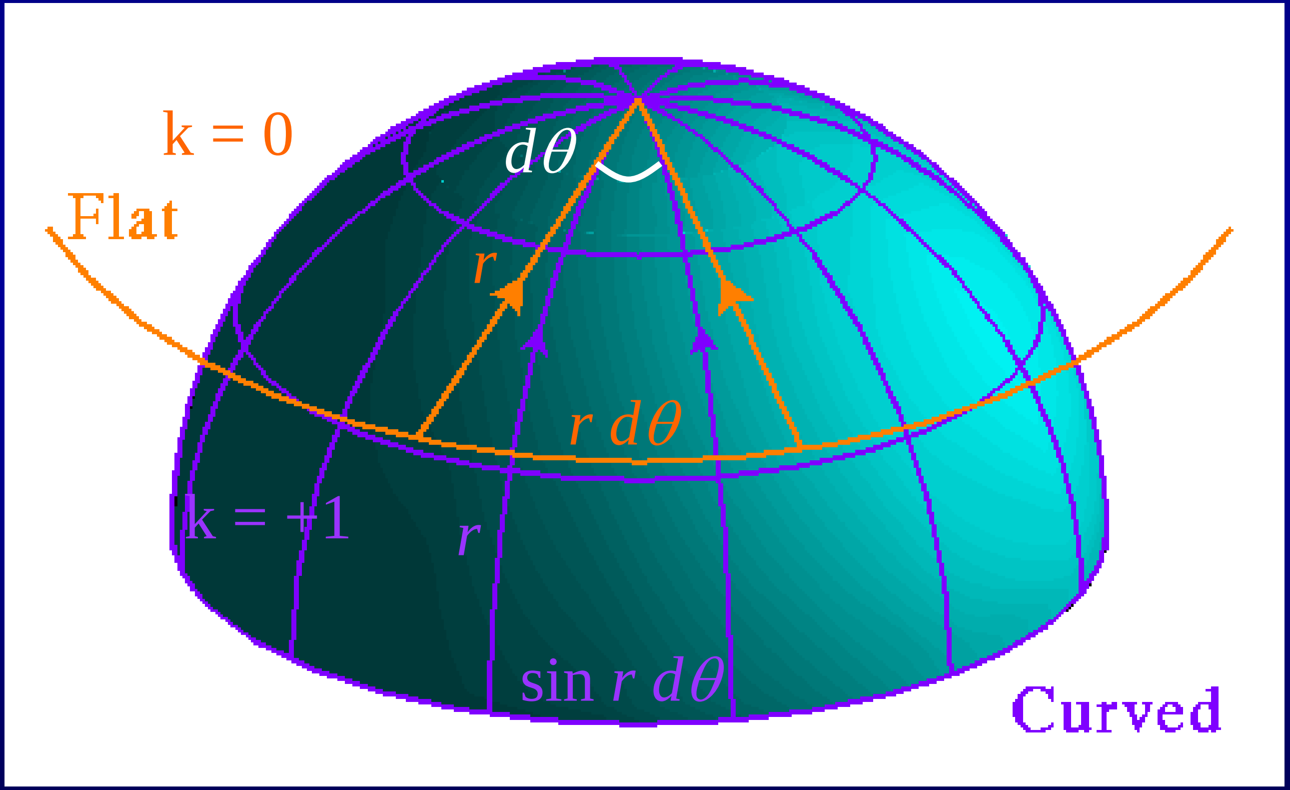


$$d\sigma_2^2 = R^2(dr^2 + S_k^2(r)d\theta^2)$$

$$S_k(r) = \begin{cases} \sin(r), & k = +1 \\ r, & k = 0 \\ \sinh(r), & k = -1 \end{cases}$$

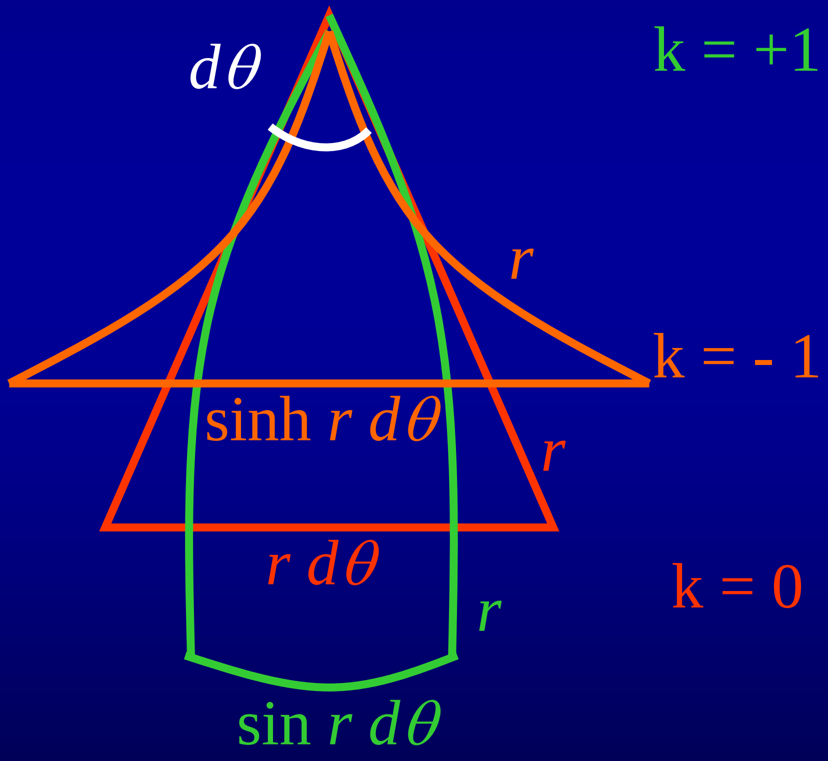
Relativistic Cosmologies

- Different properties of triangles on curved surfaces:



Relativistic Cosmologies

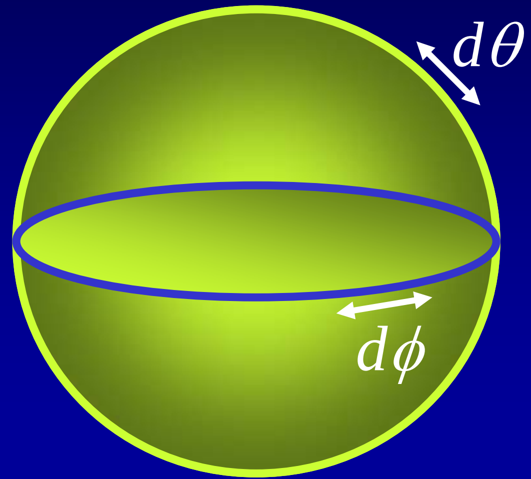
- Different properties of triangles on curved surfaces:



$$d\sigma_2^2 = R^2(dr^2 + S_k^2(r)d\theta^2)$$

$$S_k(r) = \begin{cases} \sin(r), & k = +1 \\ r, & k = 0 \\ \sinh(r), & k = -1 \end{cases}$$

Relativistic Cosmologies



- Finally add extra compact dimension:

$$d\theta^2 \rightarrow d\theta^2 + \sin^2\theta d\phi^2$$

- Promote a 2-sphere to a 3-sphere

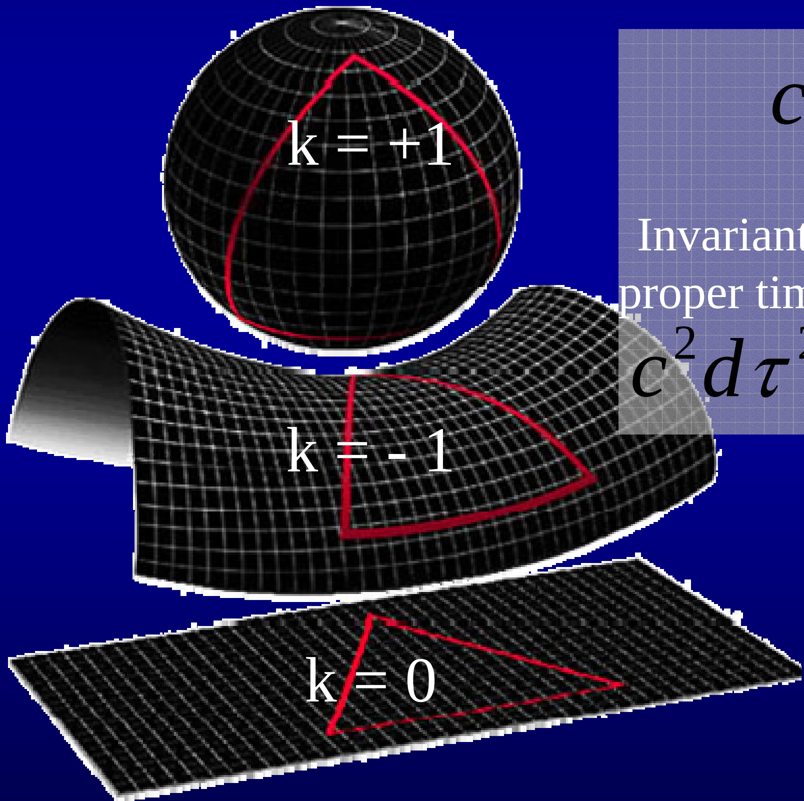
$$dr^2 + S_k^2(r)d\theta^2 \rightarrow dr^2 + S_k^2(r)(d\theta^2 + \sin^2\theta\, d\phi^2)$$

- So metric of 3-sphere is

$$d\sigma_3^2 = R^2(dr^2 + S_k^2(r)d\psi^2),$$
$$d\psi^2 = d\sigma_2^2 = d\theta^2 + \sin^2\theta\, d\phi^2$$

Relativistic Cosmologies

- The **Robertson-Walker metric** generalizes the Minkowski line element for symmetric cosmologies:



$$c^2 d\tau^2 = c^2 dt^2 - dr^2 - r^2 d\psi^2$$

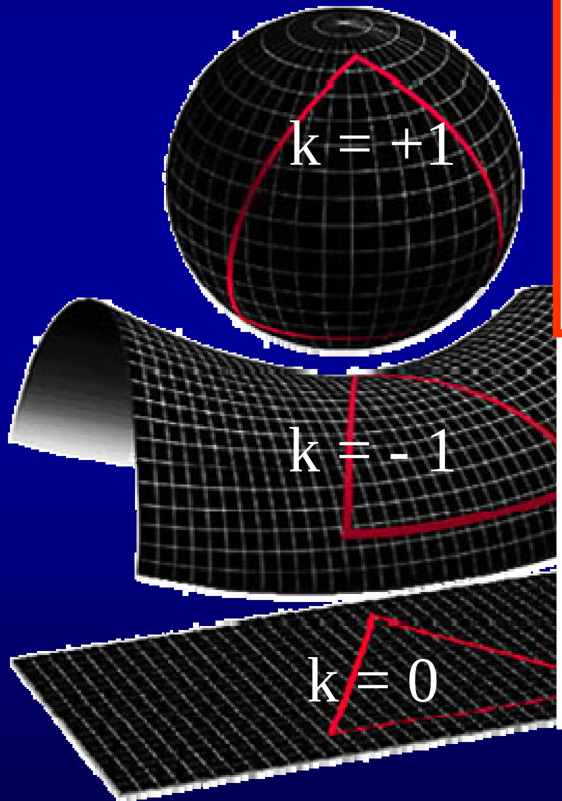
Invariant proper time | Universal Cosmic time | Scale factor ↓ | Co-moving radial distance | Co-moving angular distance

$$c^2 d\tau^2 = c^2 dt^2 - R^2 (dr^2 + S_k^2(r) d\psi^2)$$

- The Robertson-Walker Metric

Relativistic Cosmologies

- Alternative form of the Robertson-Walker metric:



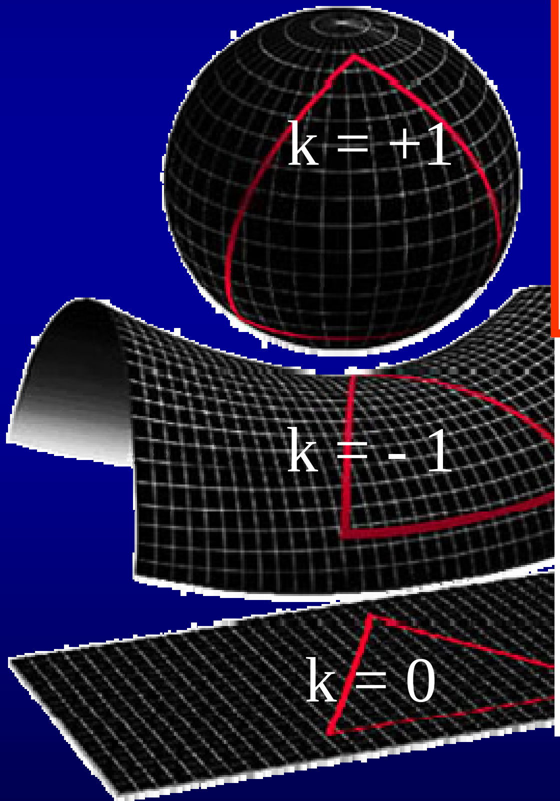
$$c^2 d\tau^2 = c^2 dt^2 - R^2 (dr^2 + \sin^2(r) d\psi^2)$$
$$= c^2 dt^2 - R^2 \left(\frac{dy^2}{1-y^2} + y d\psi^2 \right)$$

$$y = \sin r$$

$$\frac{d \sin^{-1} y}{dy} = \frac{1}{\sqrt{1-y^2}}$$

Relativistic Cosmologies

- Alternative form of the Robertson-Walker metric:



$$c^2 d\tau^2 = c^2 dt^2 - R^2 (dr^2 + S_k^2(r) d\psi^2)$$

$$= c^2 dt^2 - R^2 \left(\frac{dy^2}{1 - ky^2} + y d\psi^2 \right)$$

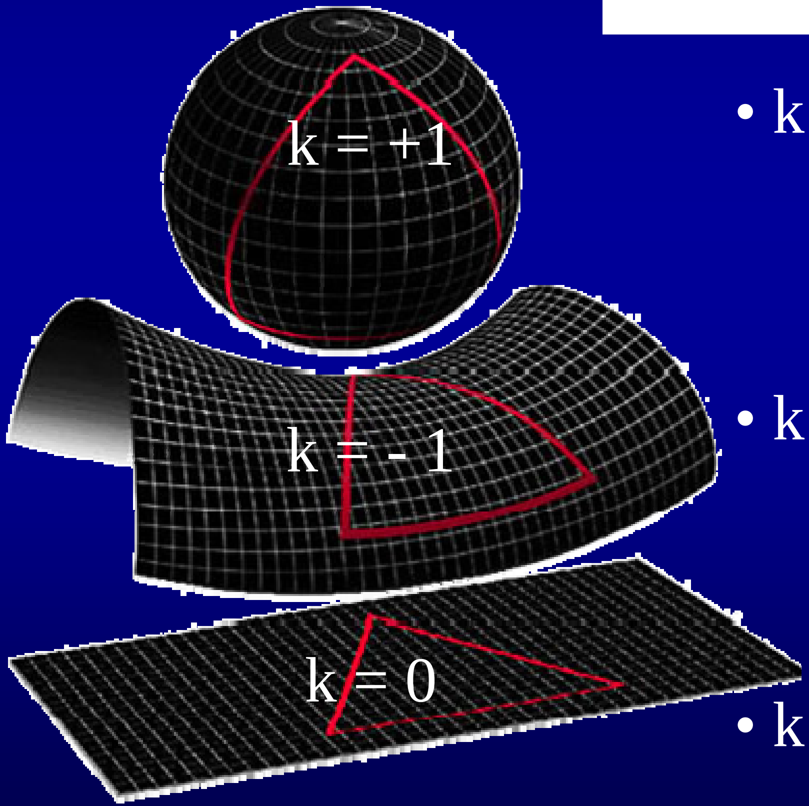
$$y = S_k(r)$$

$$\frac{dS_k^{-1} y}{dy} = \frac{1}{\sqrt{1 - ky^2}}$$

Relativistic Cosmologies

- The Robertson-Walker models.

$$c^2 d\tau^2 = c^2 dt^2 - R^2 (dr^2 + S_k^2(r) d\psi^2)$$



- k = +1: positive curvature everywhere, spatially closed, finite volume, unbounded.
- k = - 1: negative curvature everywhere, spatially open, infinite volume, unbounded.
- k = 0: flat space, spatially open, infinite volume, unbounded.

Relativistic Cosmologies

- The Robertson-Walker models.

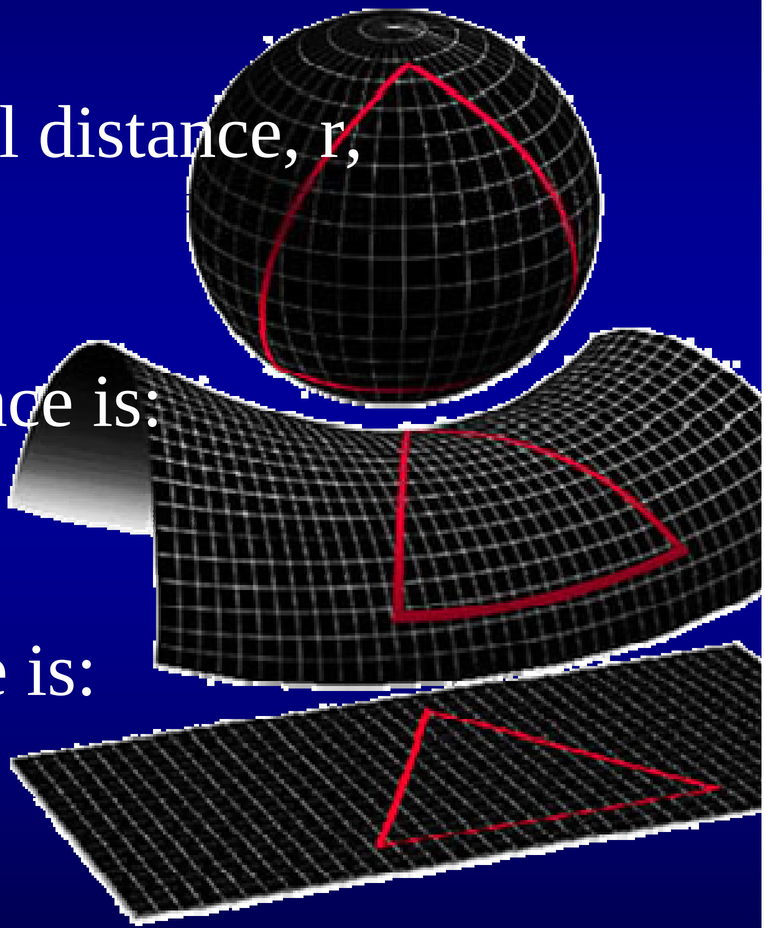
- We have defined the comoving radial distance, r, to be dimensionless.

- The current comoving angular distance is:

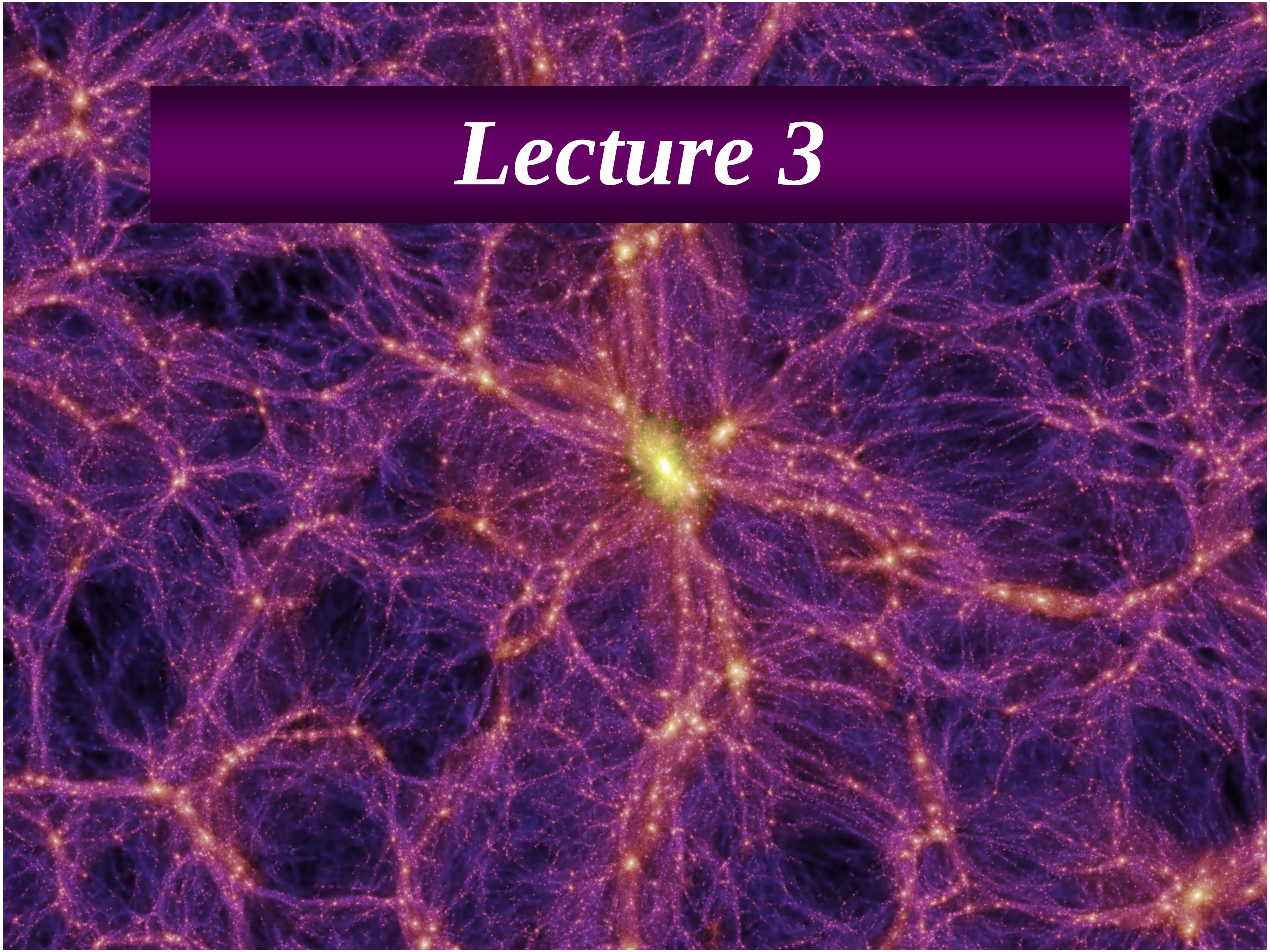
$$d = R_0 S_k(r) \text{ (Mpc)}.$$

- The proper physical angular distance is:

$$d(t) = R(t) S_k(r) \text{ (Mpc)}.$$



Lecture 3



Relativistic Cosmologies

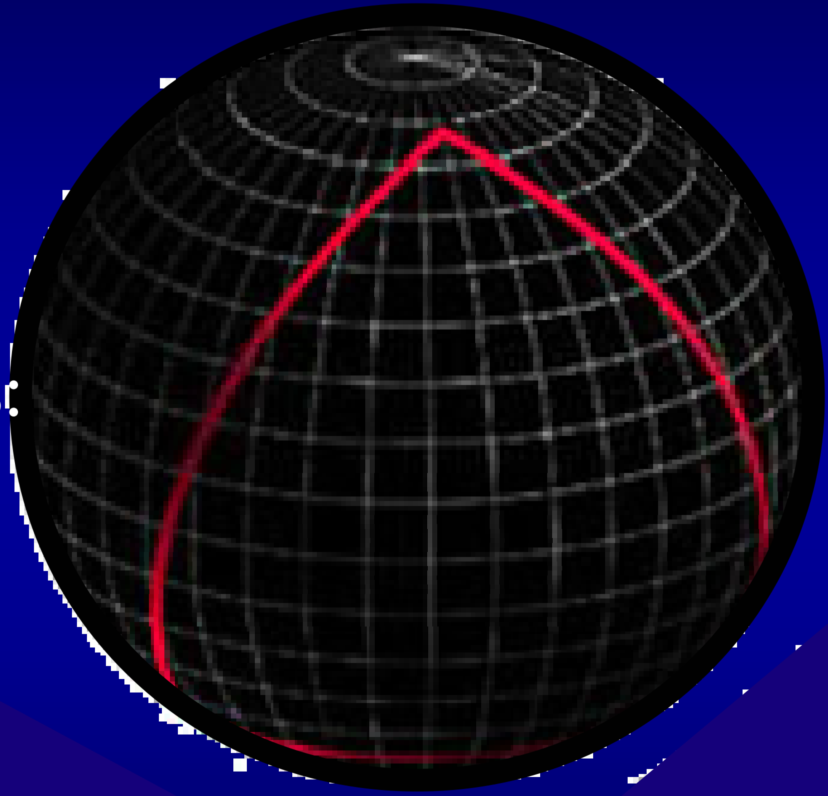
- Superluminal expansion:

The proper radial distance is

$$d(t) = R(t)r$$

The proper recession velocity is:

$$v(t) = \dot{d}(t) = \dot{R}(t)r = \frac{\dot{R}}{R} d > c?$$



What does this mean?

Locally things are not moving (just Special Relativity).

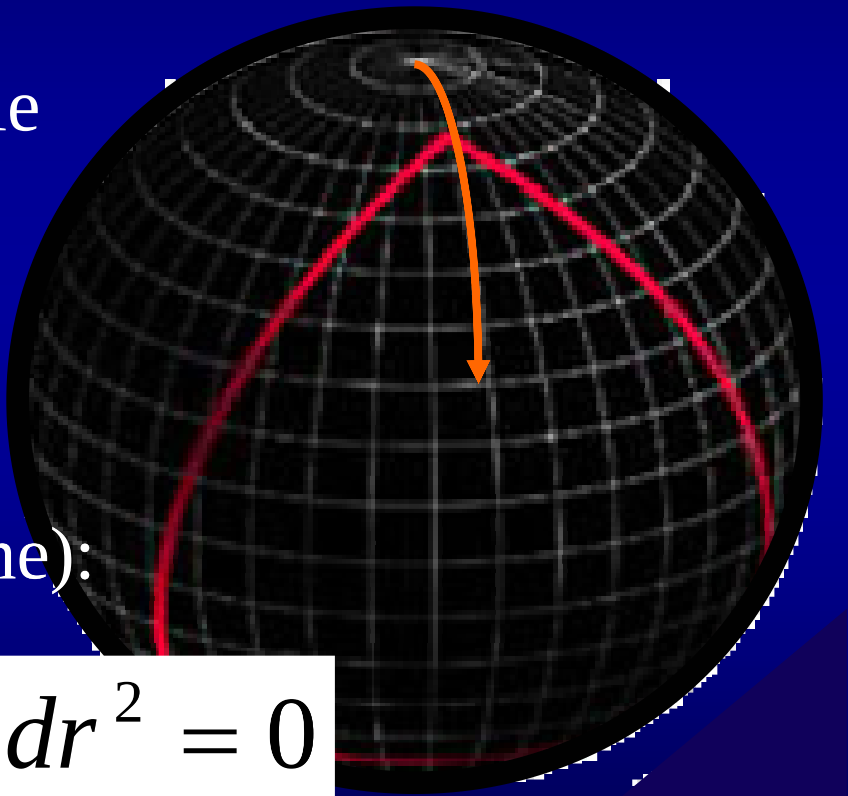
But distance (geometry) is changing.

No superluminal information exchange.

Light Propagation

- How does light propagate through the expanding Universe?

- Let a photon travel from the pole (r=0) along a line of constant longitude (dθ=0,dϕ=0).



- The line element for a photon is a null geodesic (zero proper time):

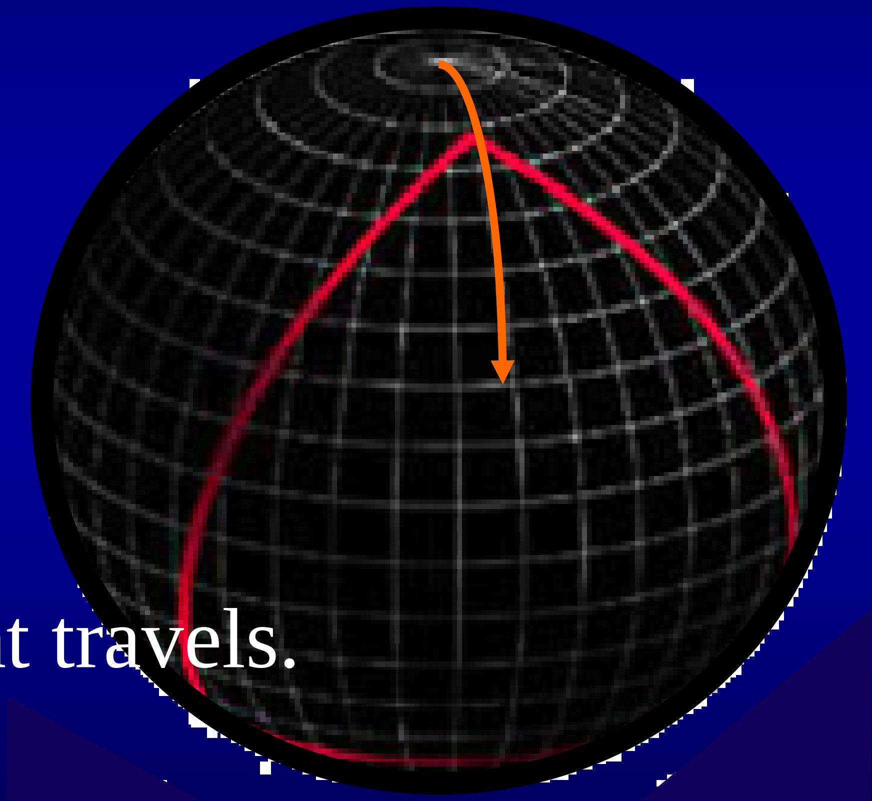
$$c^2 d\tau^2 = c^2 dt^2 - R^2(t) dr^2 = 0$$

Light Propagation

- Equation of motion of a photon:

$$c^2 dt^2 = R^2(t) dr^2$$

$$r(t) = \int_0^t \frac{c\,dt}{R(t)}$$



The comoving distance light travels.

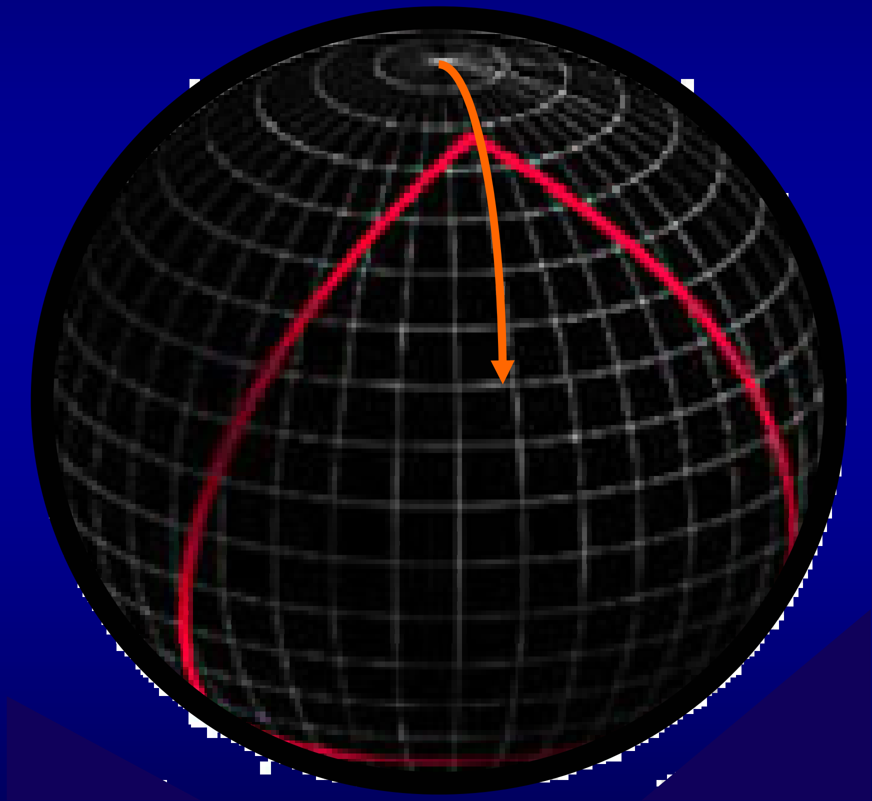
Light Propagation

- Let's assume $R(t)=R_0(t/t_0)^{\alpha}$:

$$r(t) = \int_0^t \frac{c\,dt'}{R(t')}$$

$$= \frac{c t_0^{\alpha}}{R_0} \int_0^t t'^{-\alpha}\, dt'$$

$$= \frac{c t_0^{\alpha}}{R_0} \left[\frac{t'^{1-\alpha}}{1-\alpha} \right]_0^t, \quad \alpha \neq 1$$



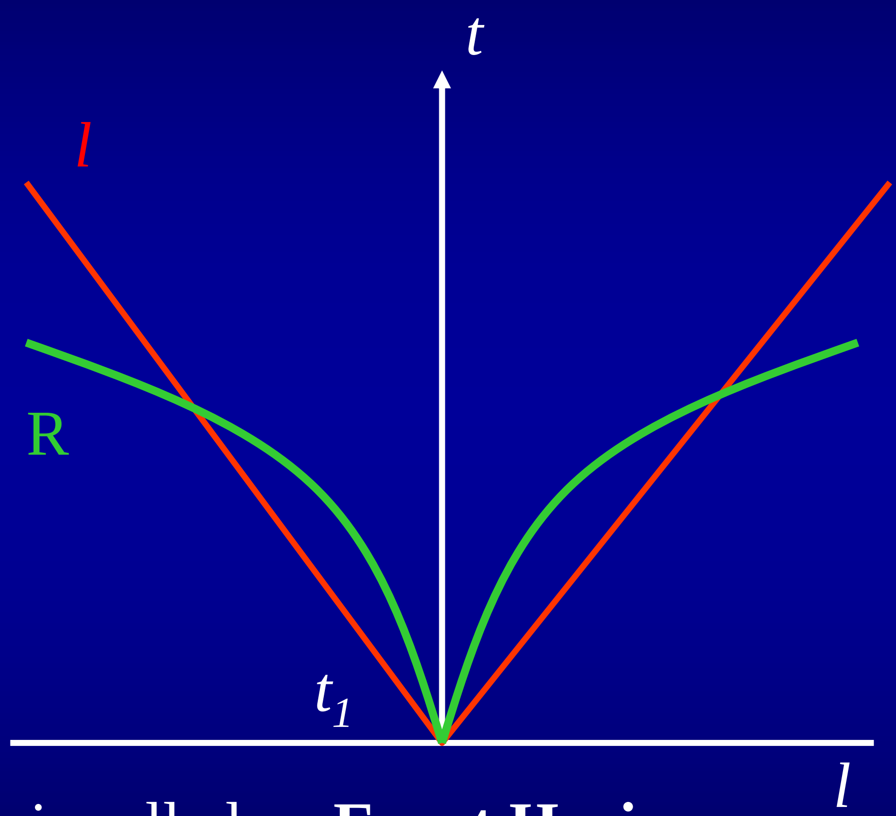
Causal structure

- Lets assume $\alpha > 1$:

$$\underline{R(t) = R_0 (t/t_0)^2} :$$

$$r(t) = \frac{c t_0^2}{R_0}\left(\frac{1}{t_1} - \frac{1}{t}\right)$$

$$l(t) = R(t) r(t) = c\left(\frac{t^2}{t_1} - t\right)$$



For $t >> t_1$, r is constant. This is called an **Event Horizon.**
As t_1 tends to 0, *l*(t) diverges, everywhere is causally connected.

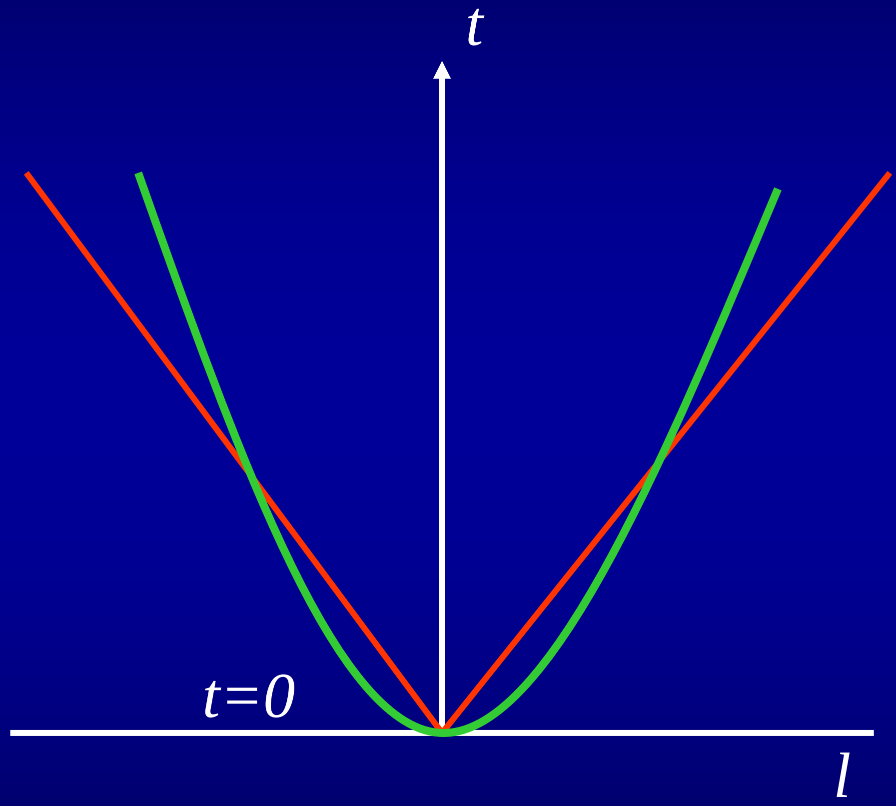
Causal structure

- Lets assume $\alpha < 1$:

$$R(t) = R_0 (t/t_0)^{1/2}:$$

$$r(t) = 2c\left(\frac{t_0^{1/2}}{R_0}\right) t^{1/2}$$

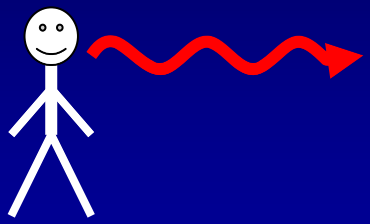
$$l(t) = R(t)r(t) = 2ct$$



At early times all points are causally disconnected.
The furthest that light can have travelled is called the **Particle Horizon.**

Cosmological Redshifts

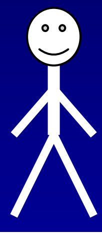
- Consider the emission and observation of light:



$$t = t_0$$

$$r = 0$$

$$d = Rr = 0$$



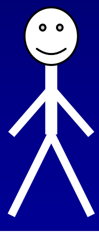
$$t = t_1$$

$$r = r_1 = \int_{t_0}^{t_1} \frac{dt}{R(t)}$$

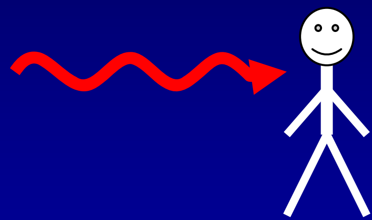
$$d_1 = R(t_1)r_1$$

Cosmological Redshifts

- Consider the emission and observation of light:

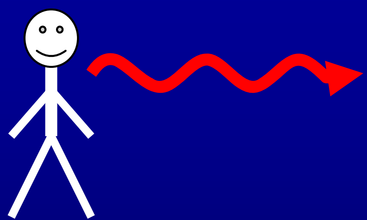


$$t = t_0$$
$$r = 0$$
$$d = Rr = 0$$

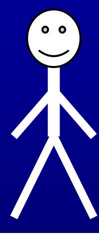


$$t = t_1$$
$$r = r_1 = \int_{t_0}^{t_1} \frac{dt}{R(t)}$$
$$d_1 = R(t_1) r_1$$

A bit later:



$$t = t_0 + \delta t_0$$
$$r = 0$$
$$d = Rr = 0$$



$$t = t_1 + \delta t_1$$
$$r = r_1 = \int_{t_0+\delta t_0}^{t_1+\delta t_1} \frac{dt}{R(t)}$$
$$d_1 = R(t_1 + \delta t_1) r_1$$

Cosmological Redshifts

- But the comoving position of an observers is a constant:

$$r_1 = \int_{t_0}^{t_1} \frac{dt}{R(t)} = \int_{t_0+\delta t_0}^{t_1+\delta t_1} \frac{dt}{R(t)} = \int_{t_0}^{t_1} \frac{dt}{R(t)} + \left(\int_{t_1}^{t_1+\delta t_1} \frac{dt}{R(t)} - \int_{t_0}^{t_0+\delta t_0} \frac{dt}{R(t)} \right)$$

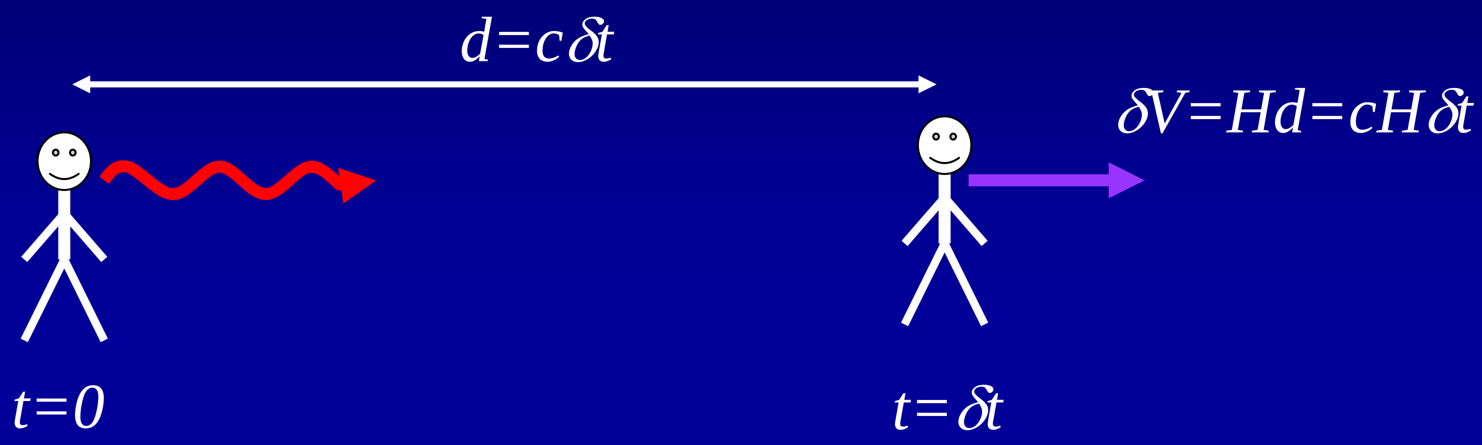
$$\frac{\delta t_1}{R(t_1)} = \frac{\delta t_0}{R(t_0)}$$

Say the wavelength of light is $\lambda = c\delta t$:

$$\frac{\lambda_0}{R_0} = \frac{\lambda_1}{R_1}$$ so $$(1+z) \equiv \frac{\nu_0}{\nu_1} = \frac{R_1}{R_0}$$

Cosmological Redshifts

- Can also understand as a series of small Doppler shifts:



$$\frac{\delta v}{v} = \frac{+\delta V}{c} = -H\delta t = -\frac{\dot{R}}{R}\delta t = -\frac{\delta R}{R}$$

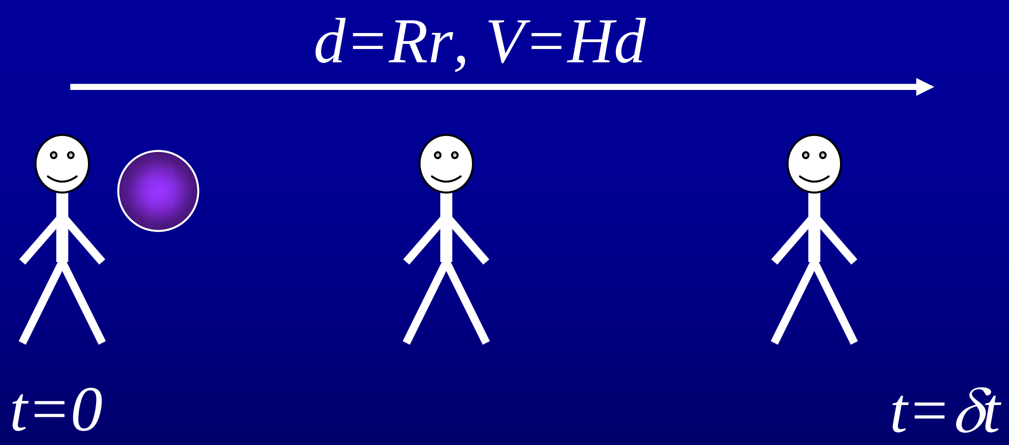
$$v \propto R^{-1}$$

Decay of particle momentum

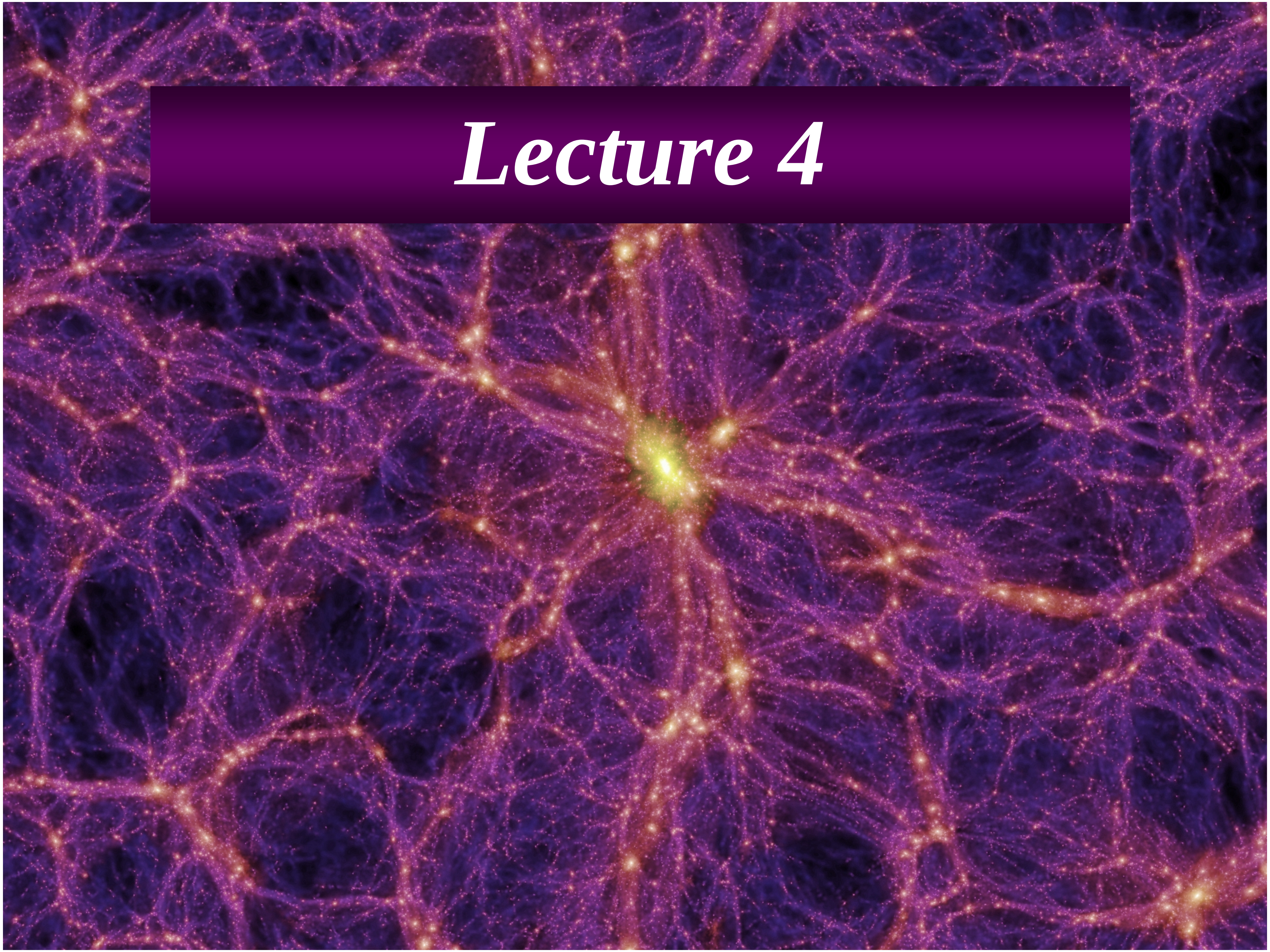
- Every particle has a de Broglie wavelength:
- So momentum (seen by FO's) is redshifted too:

$$p = \hbar \nu$$
$$\Rightarrow \; p \propto 1/R$$

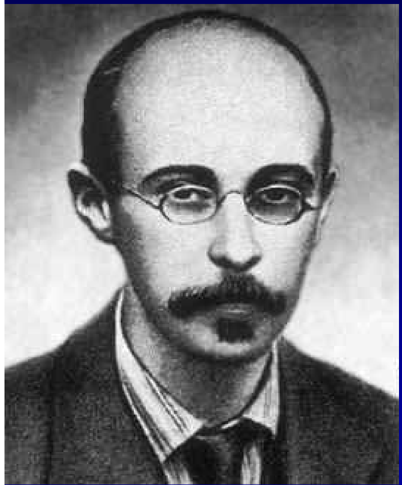
- Why? ("Hubble drag", "expansion of space"?)



Lecture 4

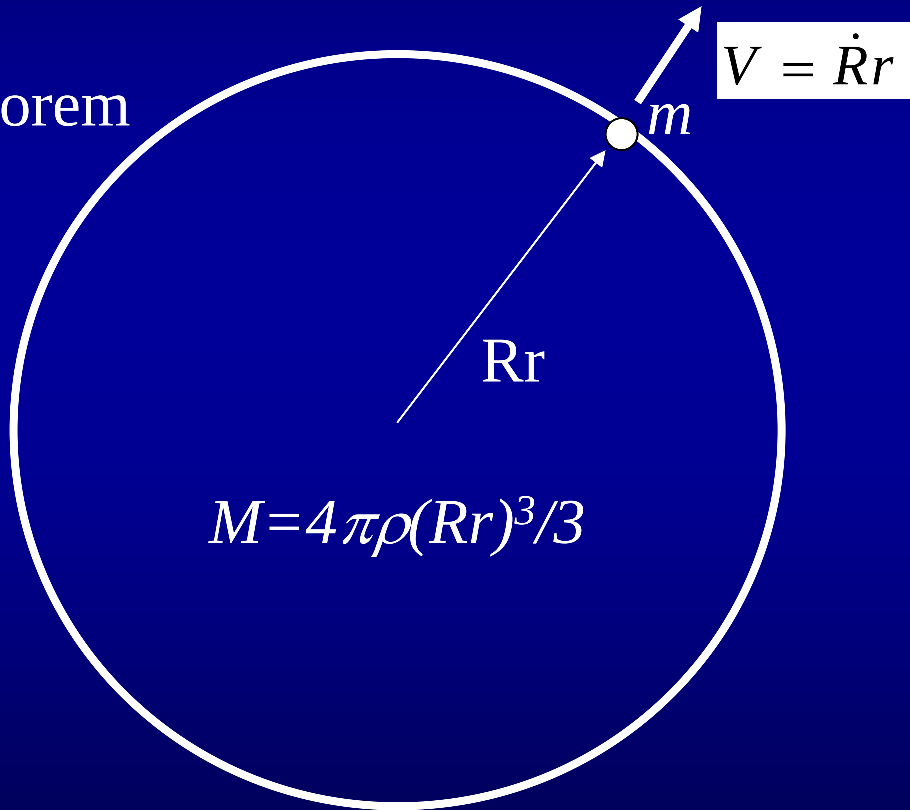


The Dynamics of the Expansion



In 1922 Russian physicist Alexandre Friedmann predicted the expansion of the Universe

Birkhoff's Theorem

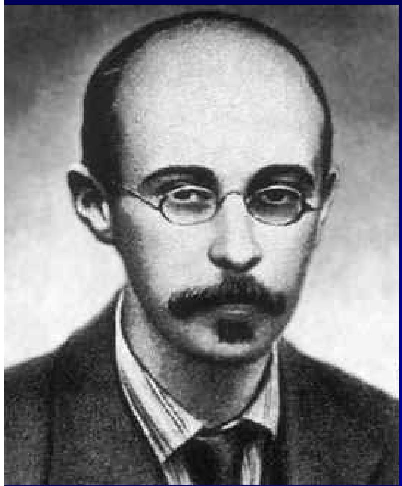


Newtonian Derivation:

$$Total \quad Energy = K.E. + P.E$$

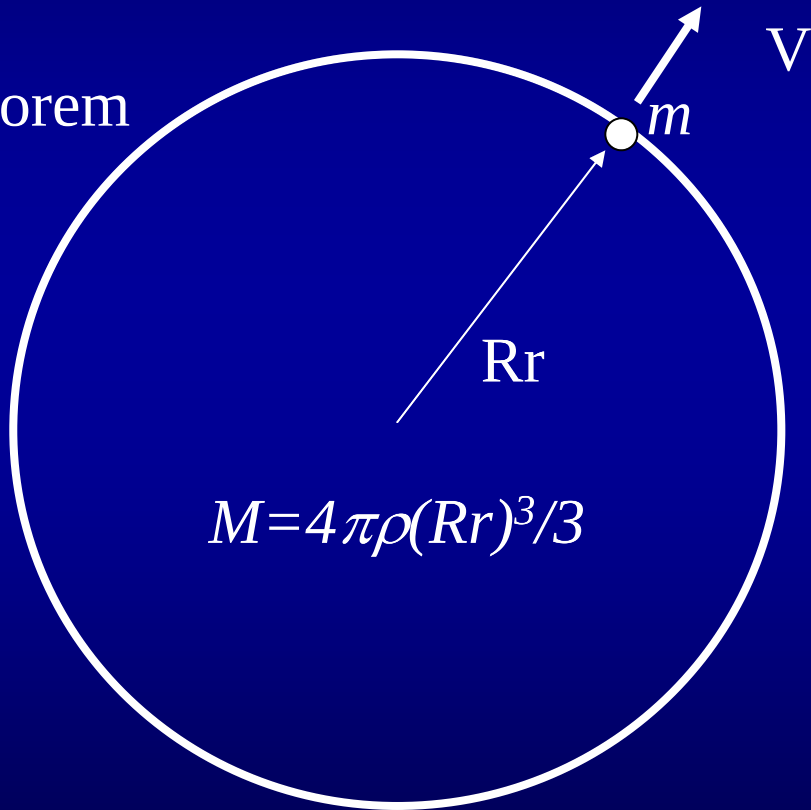
$$E_{TOT} = \frac{1}{2} m(\dot{R}r)^2 - \frac{GMm}{Rr}$$

The Dynamics of the Expansion



In 1922 Russian physicist Alexandre Friedmann predicted the expansion of the Universe

Birkhoff's Theorem

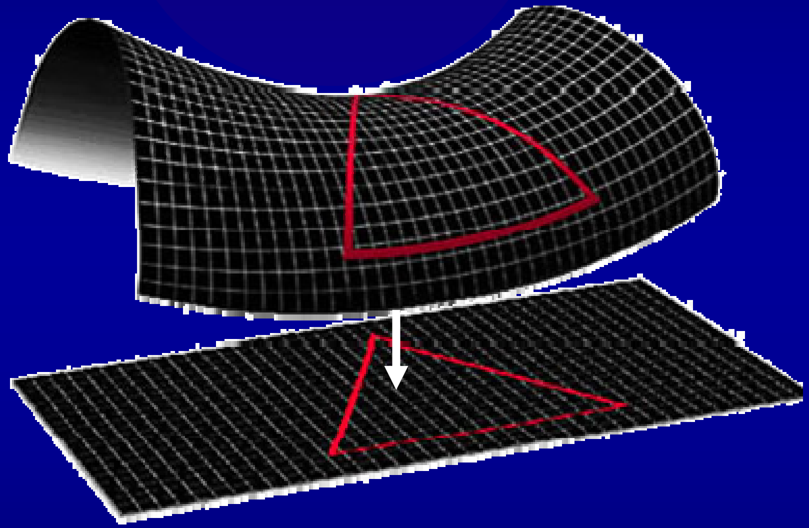


Friedmann Equation:

$$H^2 = \left(\frac{\dot{R}}{R}\right)^2 = \frac{4\pi G\rho}{3} - \frac{kc^2}{R^2}$$

Geometry & Density

- There is a direct connection between density & geometry:



$$\dot{R}^2 = \frac{8\pi G\rho}{3}R^2 - kc^2$$

$$\rho R^2 \to 0$$

$$\dot{R}^2 = -kc^2$$

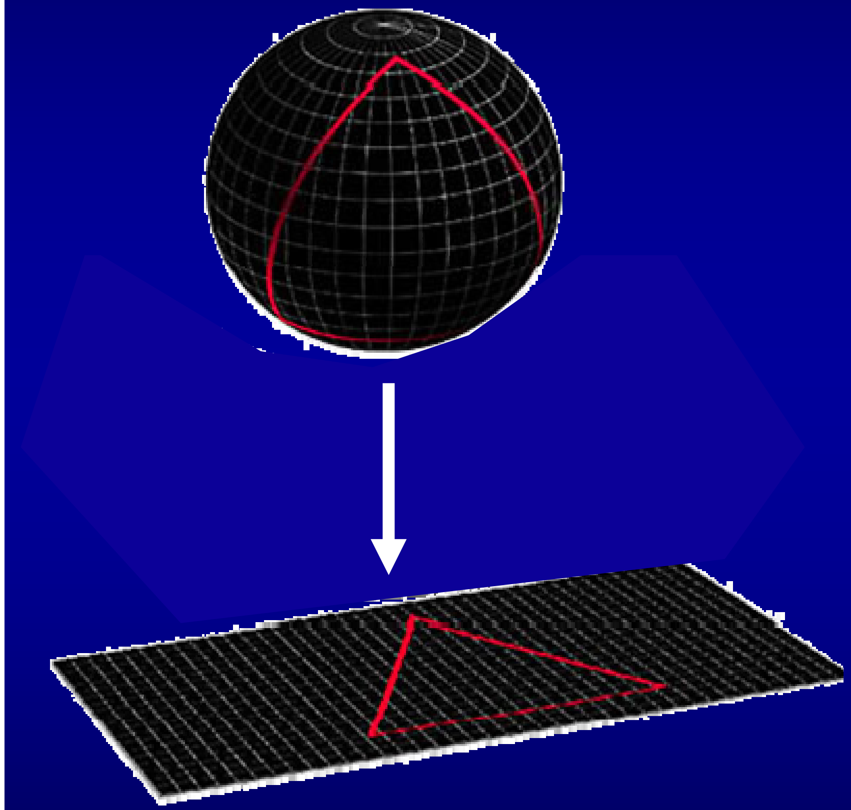
$$k = -1$$

$$R = ct$$

- So a low-density model will evolve to an empty, flat expanding universe.

Geometry & Density

- There is a direct connection between density & geometry:



$$H^2 = \frac{8\pi G\rho}{3} - \frac{c^2}{R^2}$$

$$R \to \infty$$

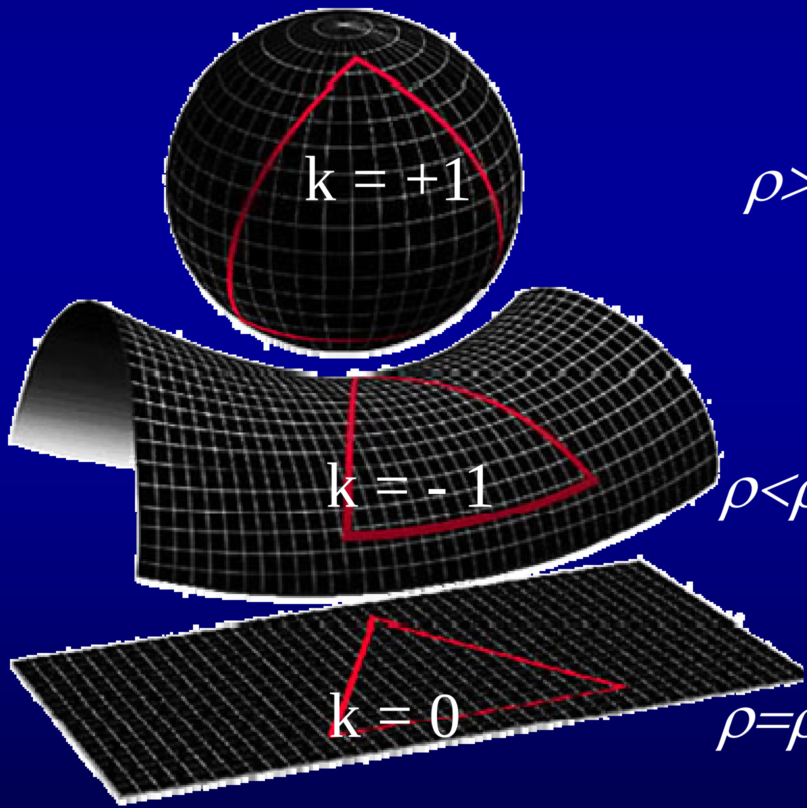
$$H^2 = \frac{8\pi G\rho}{3}$$

$$k = 0$$

- So with the right balance between H and ρ, we have a flat model.

Critical density & density parameter

- We can define a critical density for flat models and hence a density parameter which fixes the geometry.



$\rho > \rho_c$ $\Omega > 1$

$\rho < \rho_c$ $\Omega < 1$

$\rho = \rho_c$ $\Omega = 1$

$$\rho_c = \frac{3H^2}{8\pi G}$$

$$\Omega = \frac{\rho}{\rho_c} = \frac{8\pi G\rho}{3H^2}$$

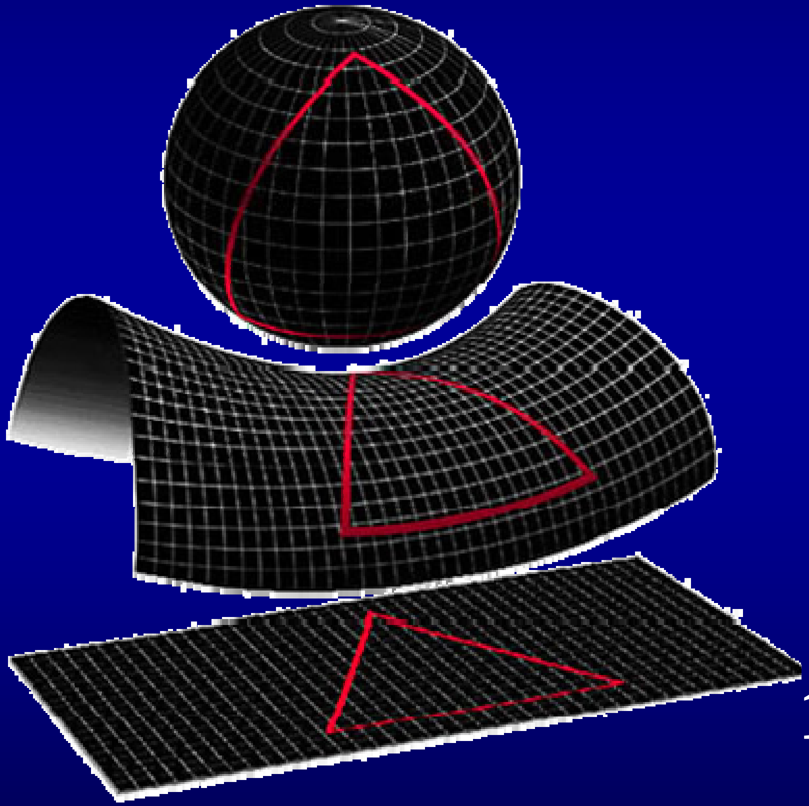
Critical density & density parameter

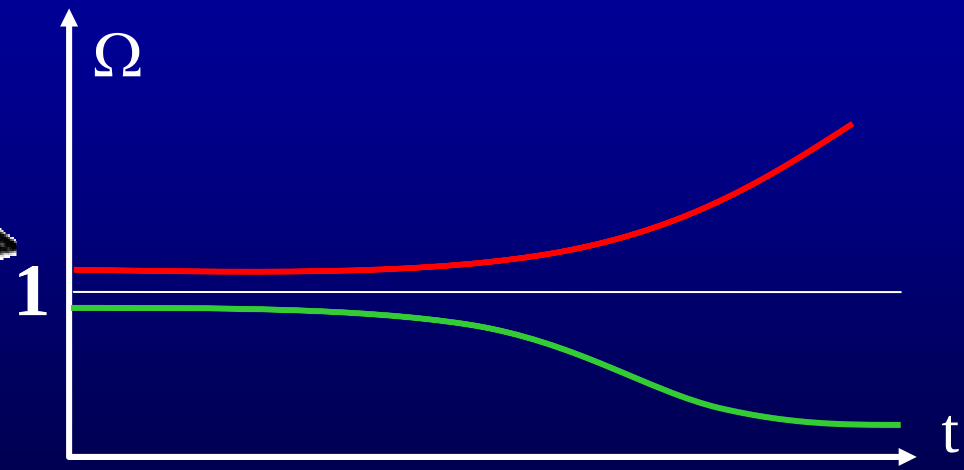
- How does Ω evolve with time?

$$H^2 = \frac{8\pi G\rho}{3} - \frac{kc^2}{R^2}$$

$$1 = \frac{8\pi G\rho}{3H^2} - \frac{kc^2}{R^2H^2}$$

$$\Omega(t) = 1 + \frac{kc^2}{R^2(t)H^2(t)}$$

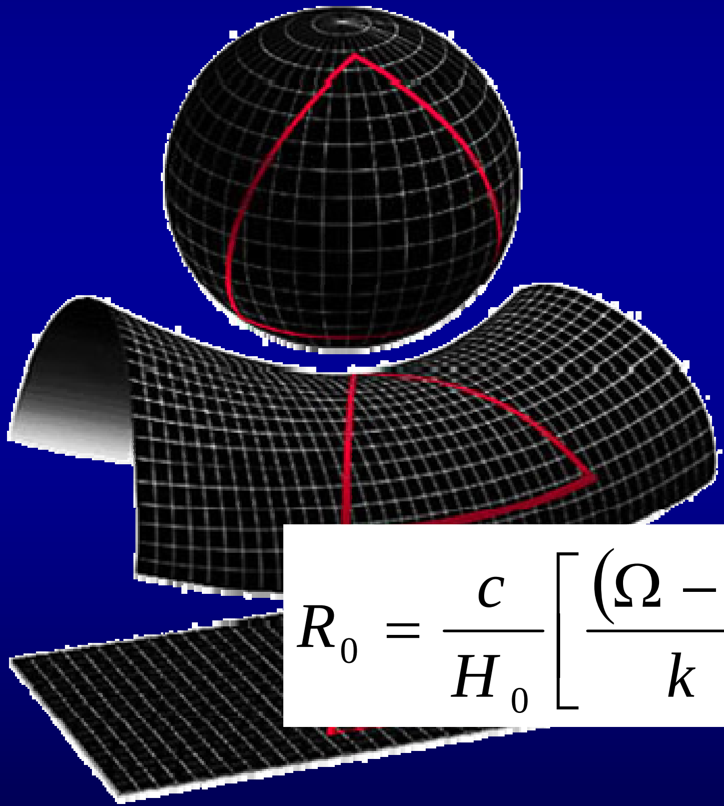




Critical density & density parameter

- What is present curvature length?

Define a dimensionless Hubble parameter:

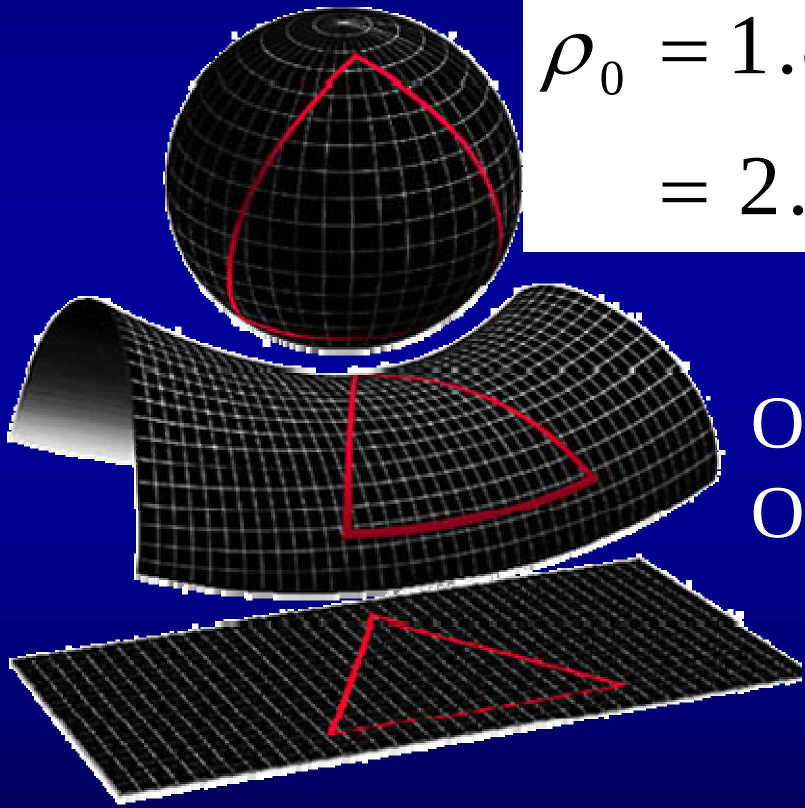


$$h = \frac{H}{100\, kms^{-1} Mpc^{-1}}$$

$$R_0 = \frac{c}{H_0}\left[\frac{(\Omega - 1)}{k}\right]^{-1/2} = 3000\left[\frac{(\Omega - 1)}{k}\right]^{-1/2} h^{-1} Mpc$$

Critical density & density parameter

- What is present density?



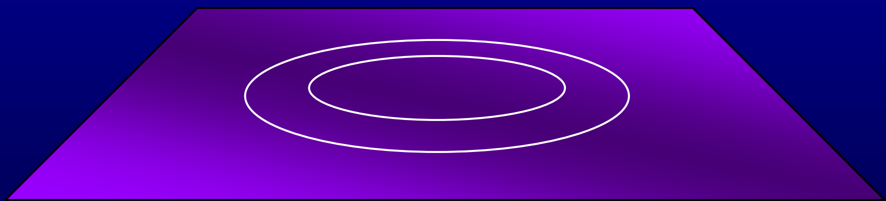
$$\rho_0 = 1.88 \times 10^{-26} (\Omega h^2)\ kg\ \ m^{-3}$$

$$= 2.78 \times 10^{11} (\Omega h^2) \mathrm{M}_{\mathrm{SUN}}\ Mpc^{-1}$$

Or 1 small galaxy per cubic Mpc.
Or 1 proton per cubic meter.

The meaning of the expansion of space

- Consider an expanding empty, spatially flat universe. c.f. a relativistic Grenade Model:
- Minkowski metric: $$c^2 d\tau^2 = c^2 dt^2 - (dr^2 + r^2 d\psi^2)$$
- Let v=Hr, H=1/t so v=r/t.
- Switch to comoving frame: $$t' = t/\gamma = t\sqrt{1-\left(V/c\right)^2} = t\sqrt{1-\left(r/tc\right)^2}$$



The meaning of the expansion of space

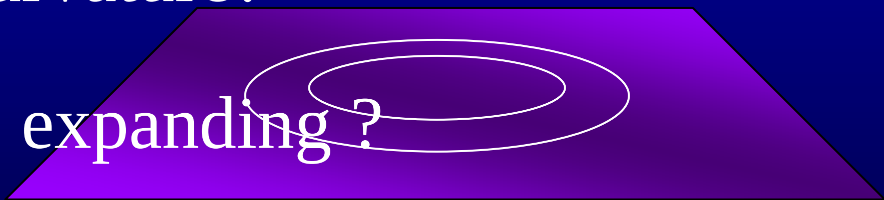
- Rewrite in terms of t' (comoving time):

$$\gamma^2 = 1 + \left(\frac{r}{ct'}\right)^2$$

- Hence in the comoving frame:

$$c^2 d\tau^2 = c^2 dt'^2 - \left(\frac{dr^2}{1 + (r/ct')^2} + r^2 d\psi^2\right)$$

- but this is a k=-1 open model with R=ct!
- So what is curvature?
- And is space expanding ?



The matter dominated universe

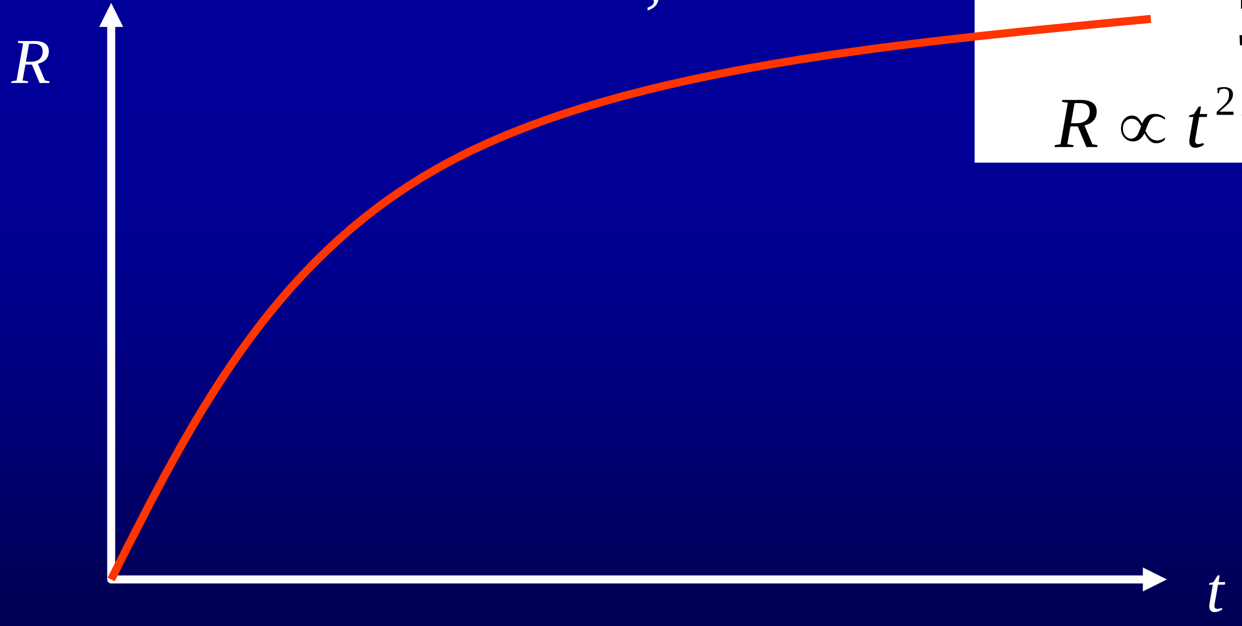
- Consider a universe with pressureless matter (dust, galaxies, or cold dark matter).
- As Universe expands, density of matter decreases:

$$\rho=\rho_0(R/R_0)^{-3}.$$

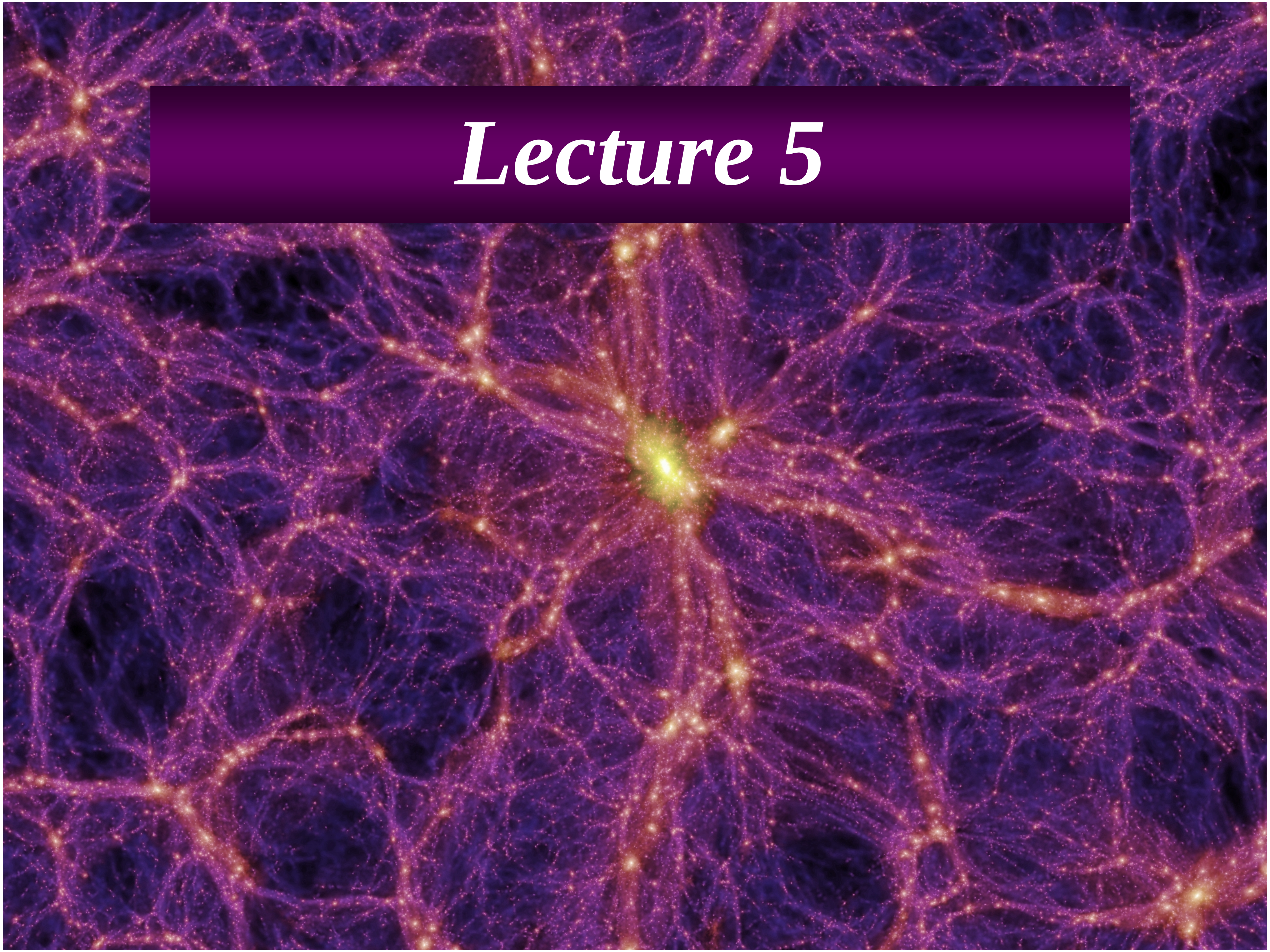
- Consider a flat model: k=0, Ω=1.

$$H^2 = \frac{8\pi G\rho}{3}$$

$$R \propto t^{2/3}$$



Lecture 5



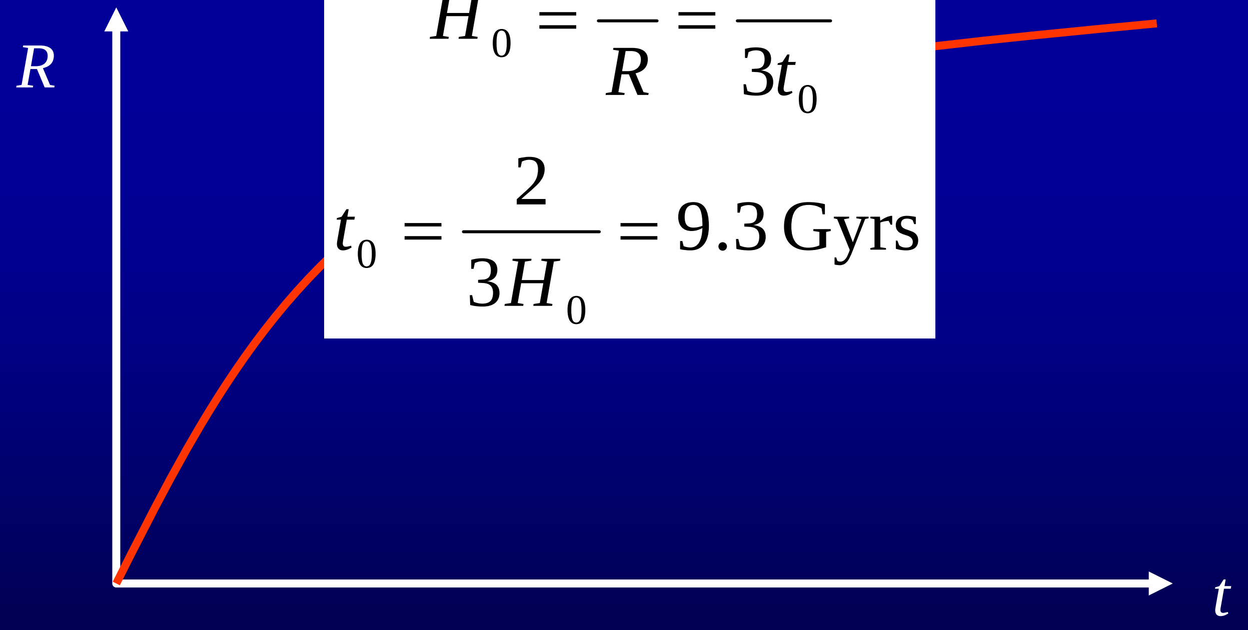
The matter dominated universe

- The spatially flat, matter-dominated model is called the Einstein-de Sitter model.

$$R \propto t^{2/3}$$

$$H_0 = \frac{\dot{R}}{R} = \frac{2}{3t_0}$$

$$t_0 = \frac{2}{3H_0} = 9.3\,\text{Gyrs}$$



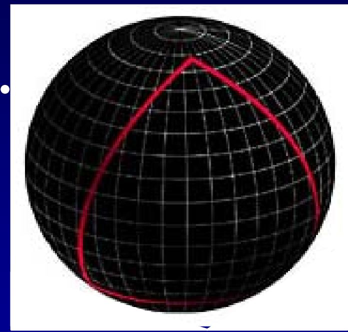
The matter dominated universe

- Consider an open or closed, matter-dominated universe.
- Define a conformal time, d$\eta = cdt/R(t)$.

$$c^2 d\tau^2 = R^2(t)[d\eta^2 - (dr^2 + S_k^2(r)d\psi^2)]$$

The matter dominated universe

- Consider a closed, matter-dominated universe.
- Define a conformal time, d$\eta = cdt/R(t)$.

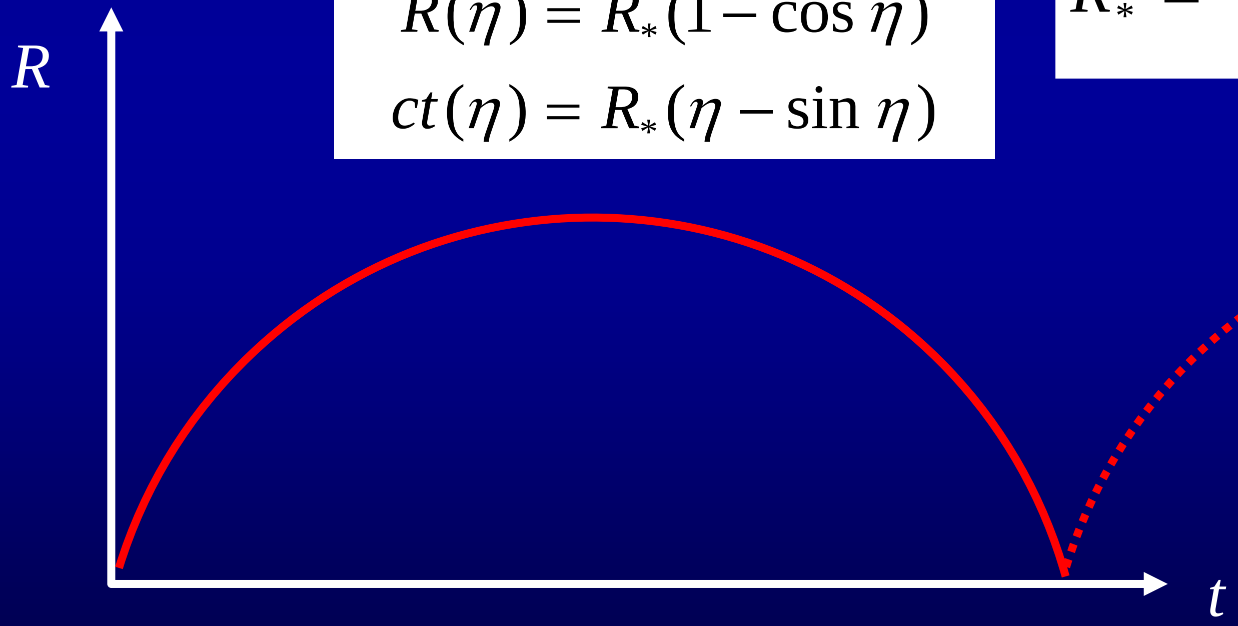


$$\left(\frac{R'}{R_*}\right)^2 = 2\left(\frac{R}{R_*}\right) - \left(\frac{R}{R_*}\right)^2$$

$$R(\eta) = R_*(1 - \cos\eta)$$

$$ct(\eta) = R_*(\eta - \sin\eta)$$

$$R_* = \frac{4\pi G \rho_0 R_0^3}{3c^2}$$



The matter dominated universe

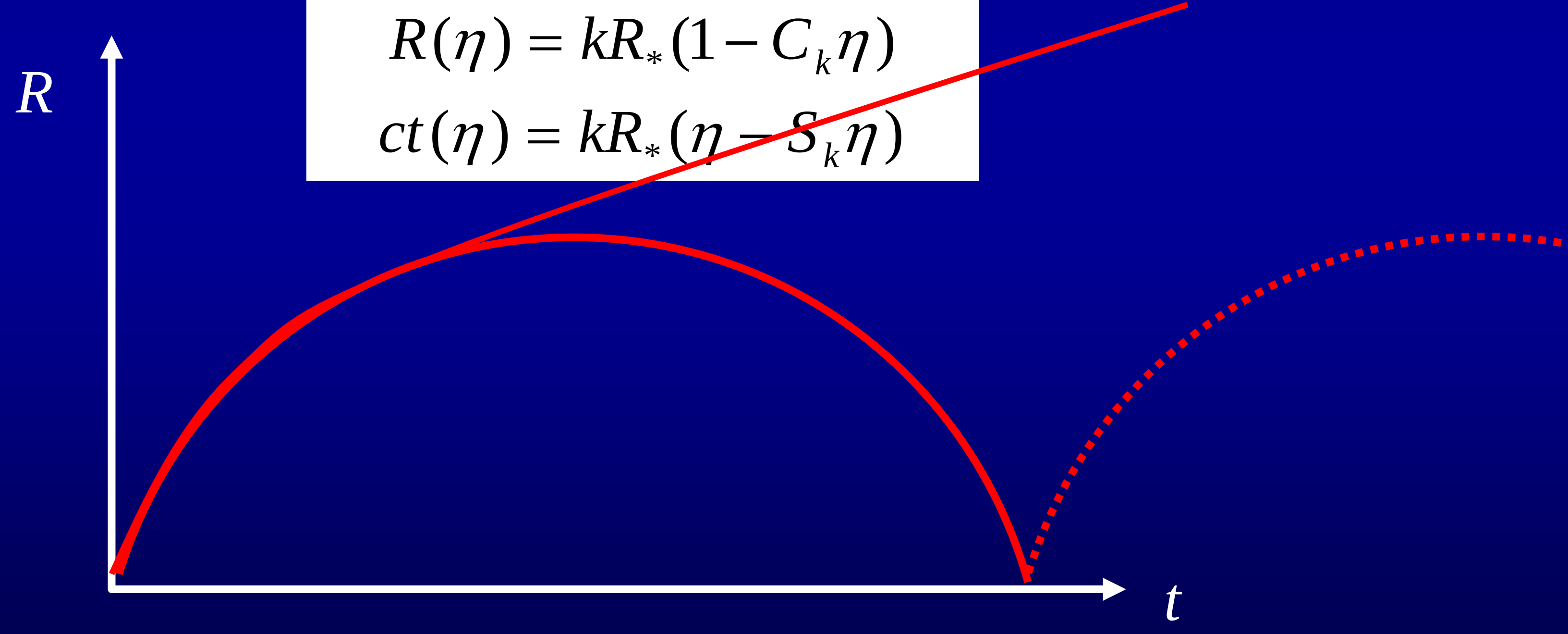
- Consider an open or closed, matter-dominated universe.
- Define a conformal time, d$\eta = cdt/R(t)$.

$$\left(\frac{R'}{R_*}\right)^2 = 2\left(\frac{R}{R_*}\right) - k\left(\frac{R}{R_*}\right)^2$$

$$R_* = \frac{4\pi G\rho_0 R_0^3}{3c^2}$$

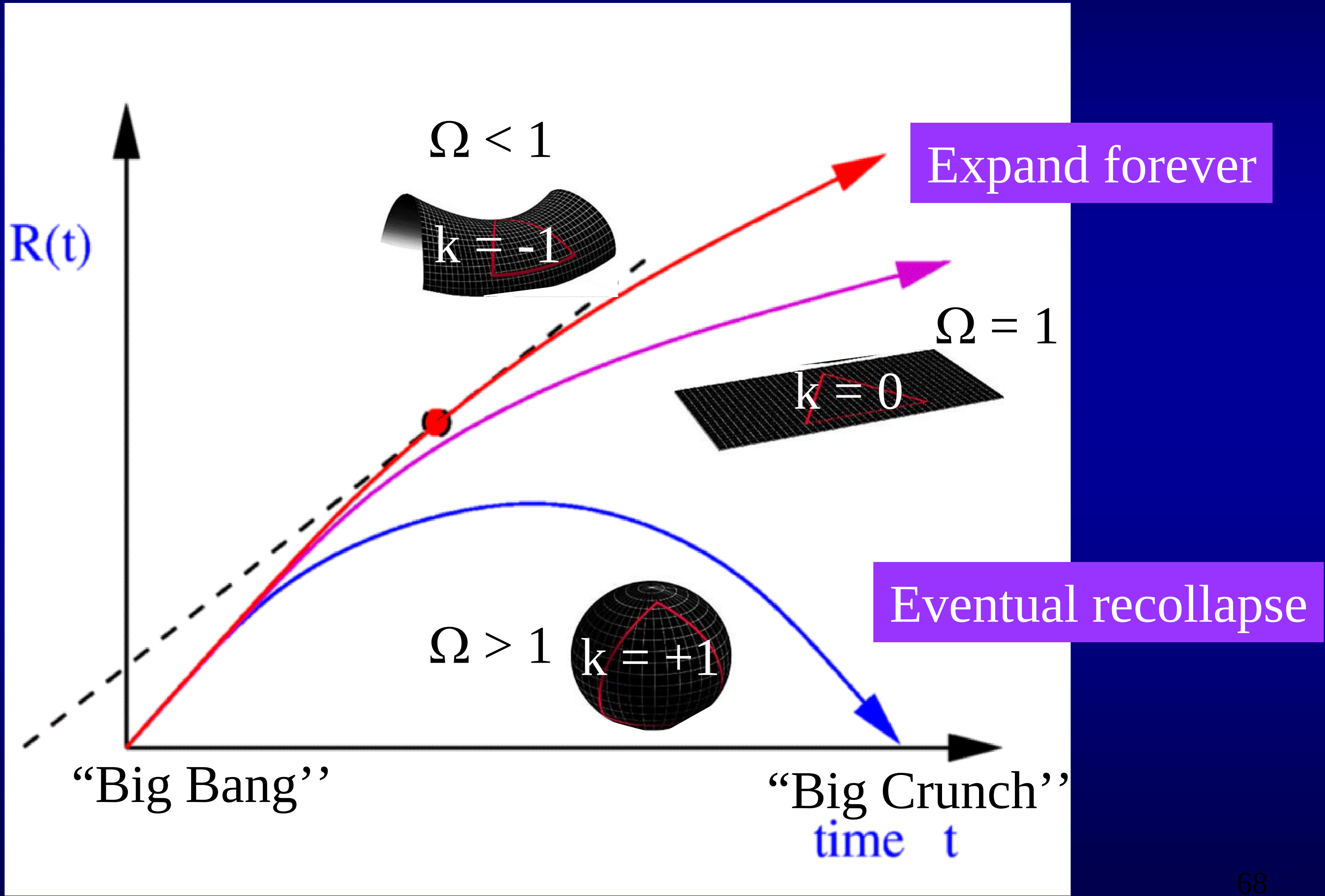
$$R(\eta) = kR_*(1 - C_k\eta)$$

$$ct(\eta) = kR_*(\eta - S_k\eta)$$



The matter dominated universe

- So for matter-dominated models geometry/density=fate.

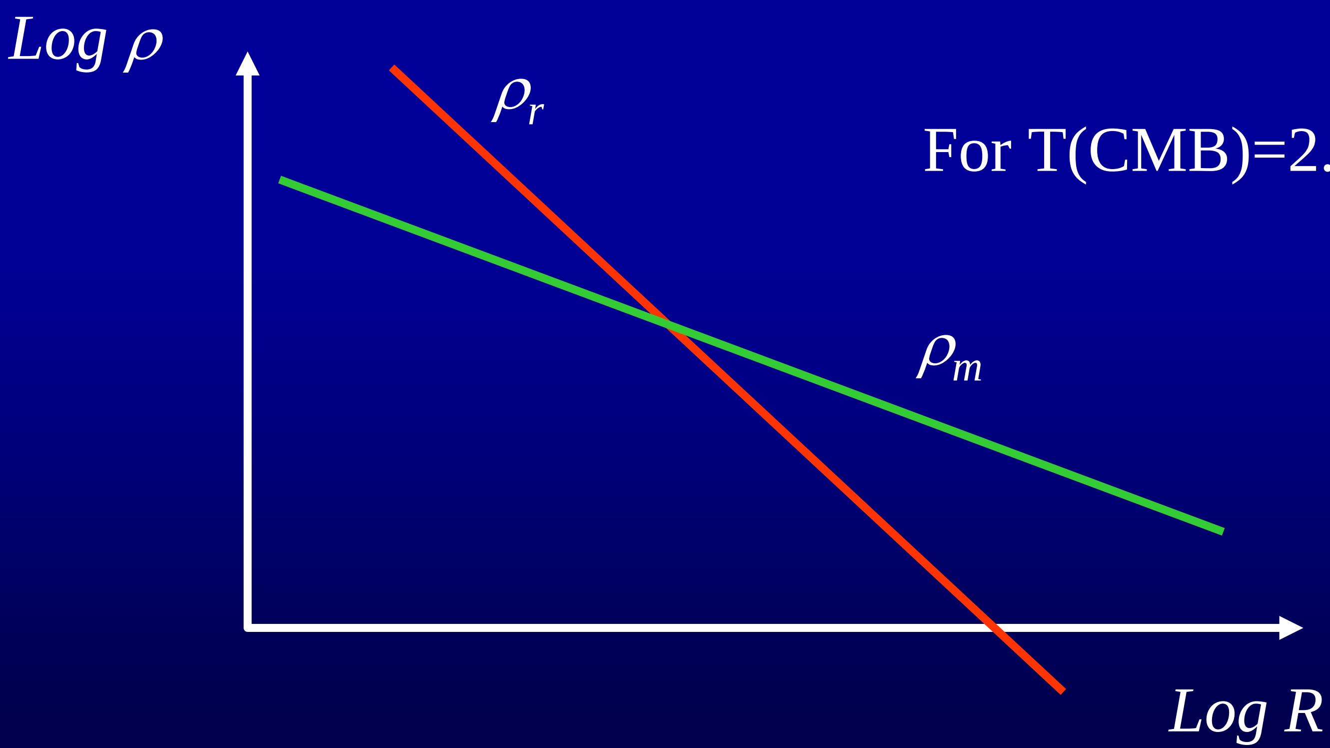


The radiation dominated universe

- As Universe expands, density of matter decreases:

$$\rho_m=\rho_{0m}(R/R_0)^{-3}.$$

- Radiation energy density: $\rho_r=\rho_{0r}(R/R_0)^{-4}$.
- At early enough times we have radiation-dominated Universe



For T(CMB)=2.73K, z_{eq}=1000.

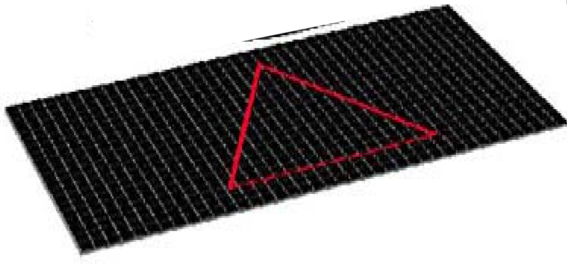
The radiation dominated universe

- At early enough times we also have a flat model: k=0

$$\dot{R}^2 = \frac{8\pi G}{3}\left(\rho_{m,o}(R_0/R)^3 + \rho_{\gamma,o}(R_0/R)^4\right)R^2 + c^2 k$$

$$R \to 0, \qquad \Omega \to 1$$

$$\dot{R}^2 = \frac{8\pi G}{3}\left(\rho_{m,o}(R_0/R)^3 + \rho_{\gamma,o}(R_0/R)^4\right)R^2$$



$$\dot{R}^2 = \frac{8\pi G}{3}\left(\rho_{\gamma,o}R_0{}^4\right)R^{-2}, \quad \Rightarrow \qquad R \propto t^{1/2}$$

So Particle Horizon!

The radiation dominated universe

- Timescales:

- Matter-dominated: R~t$^{2/3}$

$$H=\frac{2}{3t}=\sqrt{\frac{8\pi G\rho_m}{3}}\Rightarrow t=\frac{1}{\sqrt{6\pi G\rho_m}}$$

- Radiation dominated: R~t$^{1/2}$

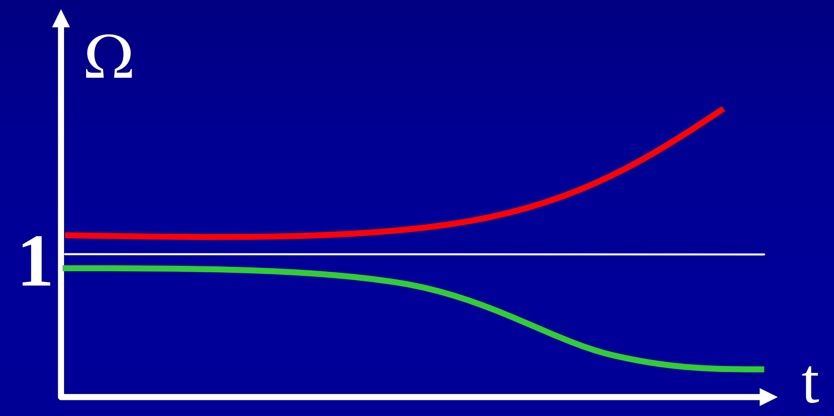
$$H=\frac{1}{2t}=\sqrt{\frac{8\pi G\rho_\gamma}{3}}\Rightarrow t=\frac{1}{\sqrt{32\pi G\rho_\gamma}}$$

The radiation dominated universe

- Spatial flatness at early times:

Recall:

$$\Omega(t) = 1 + \frac{kc^2}{(RH)^2} = 1 + k\left(\frac{t}{t_0}\right)$$



How close to 1 can this be? At Planck time ($t=10^{-43}$s)?

$$\Omega(t) = 1 \pm \frac{t_{pl}}{t_0} = 1 \pm 10^{-60}$$

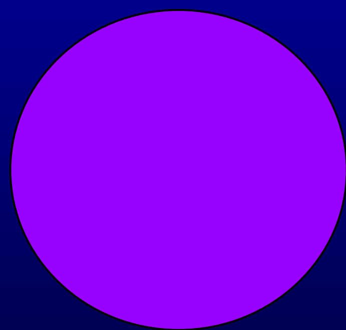
Energy density and Pressure

- Thermodynamics and Special Relativity:

$$dE = -pdV$$
$$d(\rho R^3 c^2) = -pd(R^3)$$
$$\dot{\rho} + 3H(\rho + p/c^2) = 0$$

- So energy-density changes due to expansion.

$$\dot{\rho} + 3H\rho = -3Hp/c^2$$



Energy density and Pressure

- Conservation of energy: $\dot{\rho} = -3H(\rho + p/c^2)$

- For pressureless matter (CDM, dust, galaxies):

$$p = 0 \quad \Rightarrow \quad \rho \propto R^{-3}$$

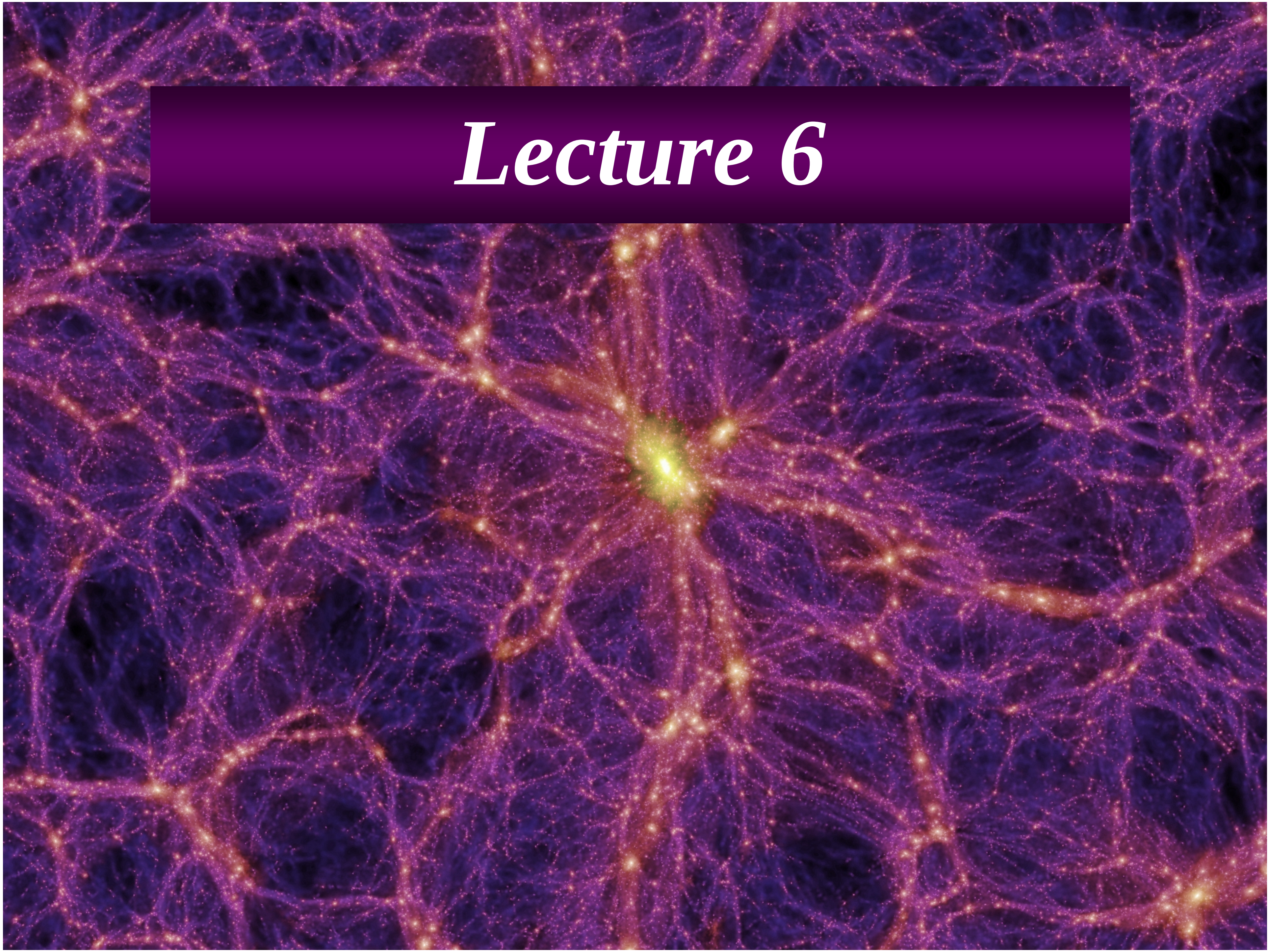
- Radiation pressure:

$$\rho_\gamma \propto R^{-4} \quad \Rightarrow \quad \dot{\rho}_\gamma = 4H\rho_\gamma$$

$$p_\gamma = \frac{1}{3}\rho_\gamma c^2$$

- Cf. electromagnetism.

Lecture 6



Pressure and Acceleration

- Time derivative of Friedmann equation:

$$\frac{d}{dt}\dot{R}^2 = \frac{d}{dt}\left(\frac{8\pi G\rho}{3}R^2 - kc^2\right)$$

$$2\dot{R}\ddot{R} = \frac{8\pi G}{3}(\dot{\rho}R^2 + 2\rho R\dot{R})$$

$$\dot{\rho} = -3H(\rho + p/c^2)$$

- Acceleration equation for R:

$$\ddot{R} = -\frac{4\pi G}{3}\left(\rho + 3p/c^2\right)R$$

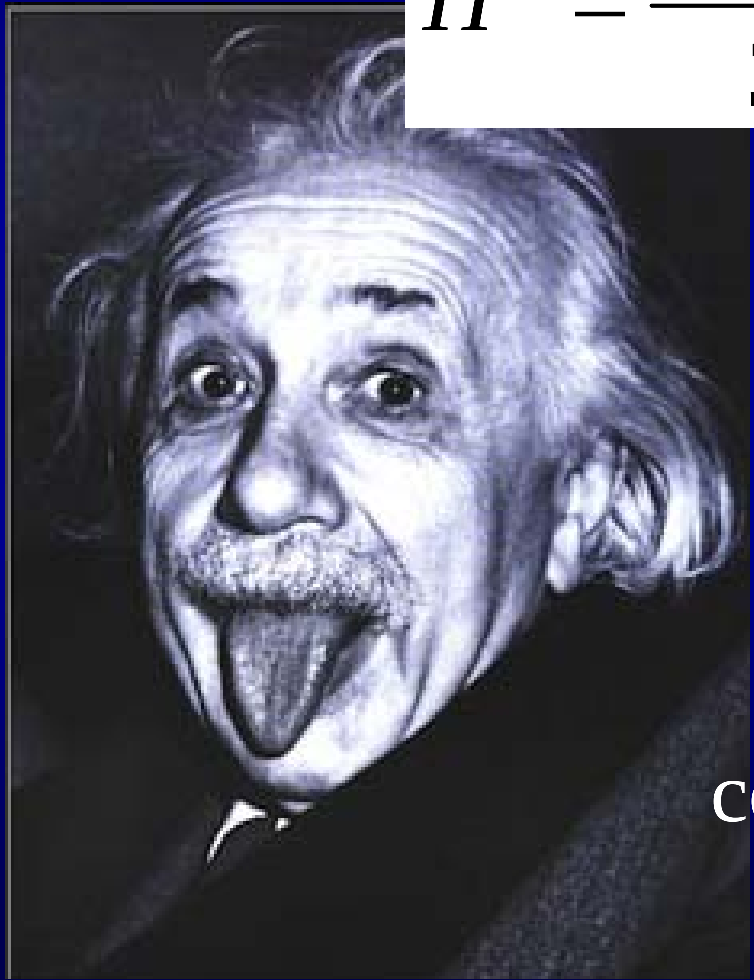
Vacuum energy and acceleration

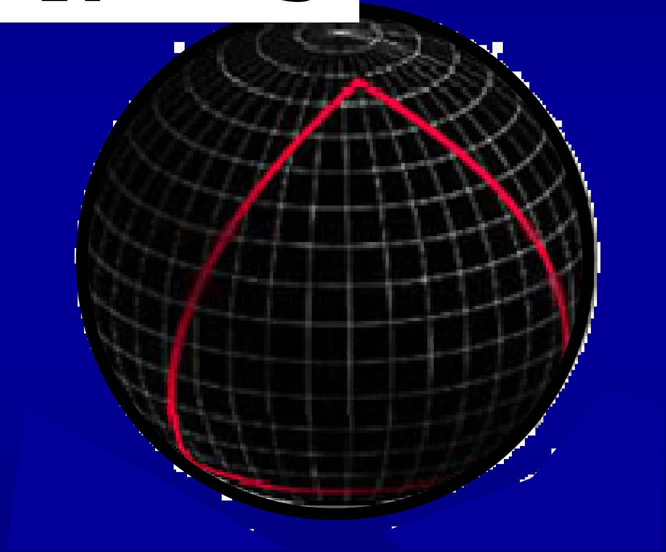
- Gravity responds to all energy.
- What about energy of the vacuum?

- Two possibilities:
  1. Einstein's cosmological constant.
  2. Zero-point energy of virtual particles.

Einstein's Cosmological Constant

$$H^2 = \frac{8\pi G\rho}{3} - \frac{kc^2}{R^2} + \frac{\Lambda}{3}$$





Einstein introduced
constant to make Universe static.

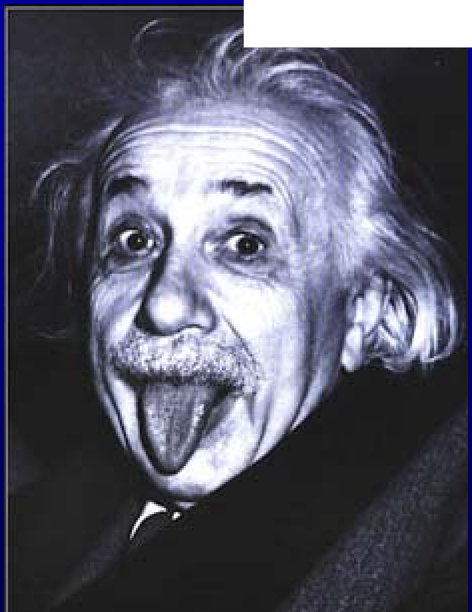
Einstein's Cosmological Constant

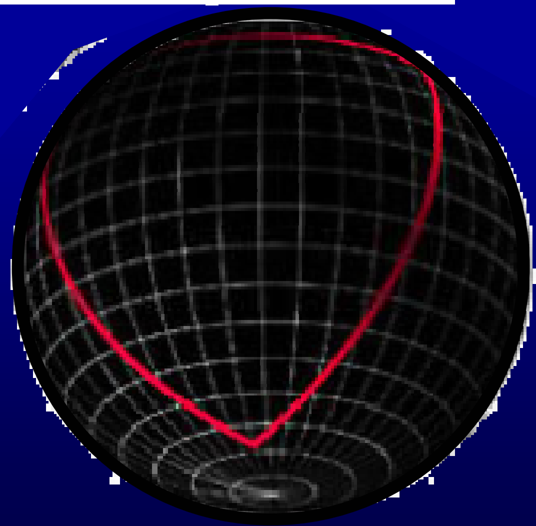
- Problem goes back to Newton (1670's).

$$\nabla^2\Phi = 4\pi G\rho \quad \Rightarrow \quad \Phi = 4\pi G\rho r^2 \quad \Rightarrow \quad g = \nabla\Phi \propto r$$

- Einstein's 1917 solution:

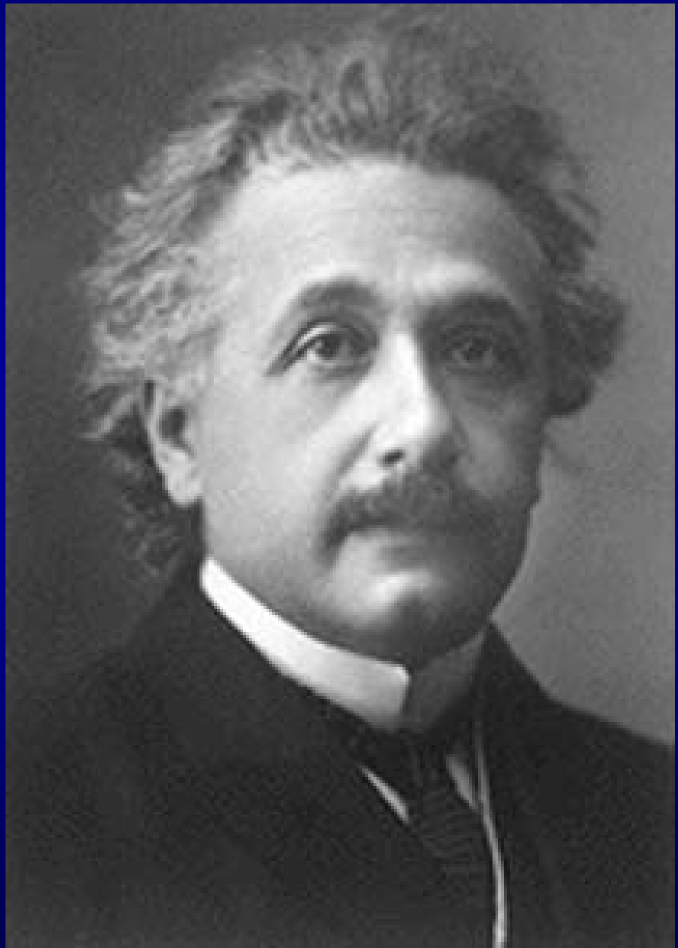
$$\nabla^2\Phi + \lambda\Phi = 4\pi G\rho \quad \Rightarrow \quad \Phi = \frac{4\pi G\rho}{\lambda} \quad \Rightarrow \quad g = 0$$

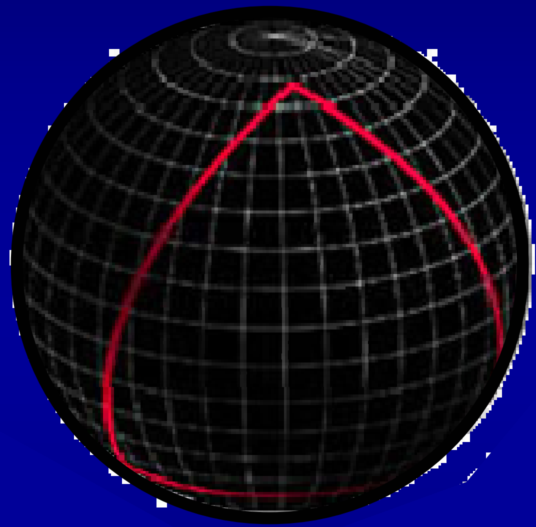




Einstein's Cosmological Constant

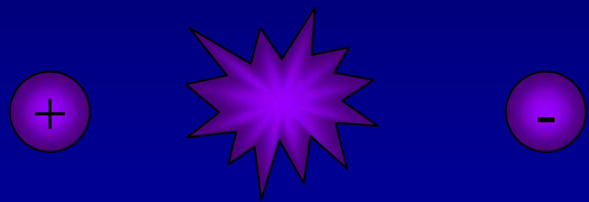
- But this is not stable to expansion/contraction.





Einstein called this:
"My greatest blunder."

Zero-point vacuum energy



$$E_n = \left(n + \frac{1}{2} \right) \hbar v$$

- British physicist Paul Dirac predicted antiparticles.
- Werner Heisenberg's Uncertainty Principle: Vacuum is filled with virtual particles.
- Observable (Casmir Effect) for electromagnetism.

The Vacuum Energy Problem

- So Quantum Physics predicts vacuum energy.
- But summation diverges.
- If we cut summation at Planck energy it predicts an energy 10^{120} times too big.

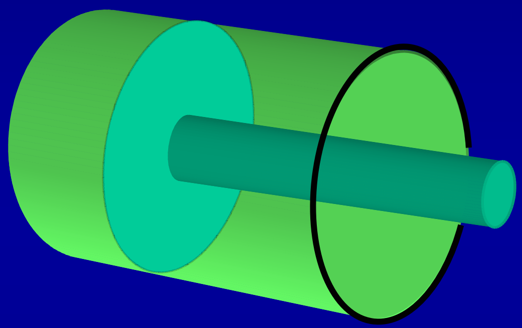
Density of Universe = 10 atoms/m^3

Density predicted = 1 million x mass of the Universe/m^3

- Perhaps the most inaccurate prediction in science? Or is it right?

Vacuum energy

- Vacuum energy is a constant everywhere: $\rho_V \sim R^0$
- Thermodynamics: Consider a piston:



$$d\left(\rho_V c^2 R^3\right) = \rho_V c^2 d\left(R^3\right) = -p_V d\left(R^3\right)$$

$$p_V = -\rho_V c^2$$

The equation of state of the vacuum.

Vacuum energy and acceleration

- Effect of negative pressure on acceleration:

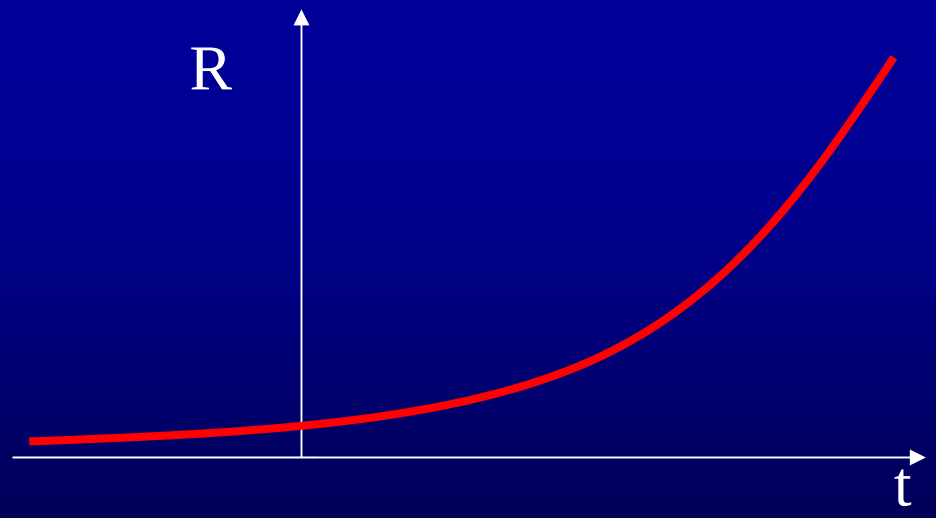
$$\ddot{R} \propto -(\rho_V + 3p_V / c^2) = +2\rho_V$$

- So vacuum energy leads to acceleration.

$$H^2 \propto \rho_V = \text{const}$$

$$\dot{R} = HR$$

$$R = R_0 e^{Ht}$$



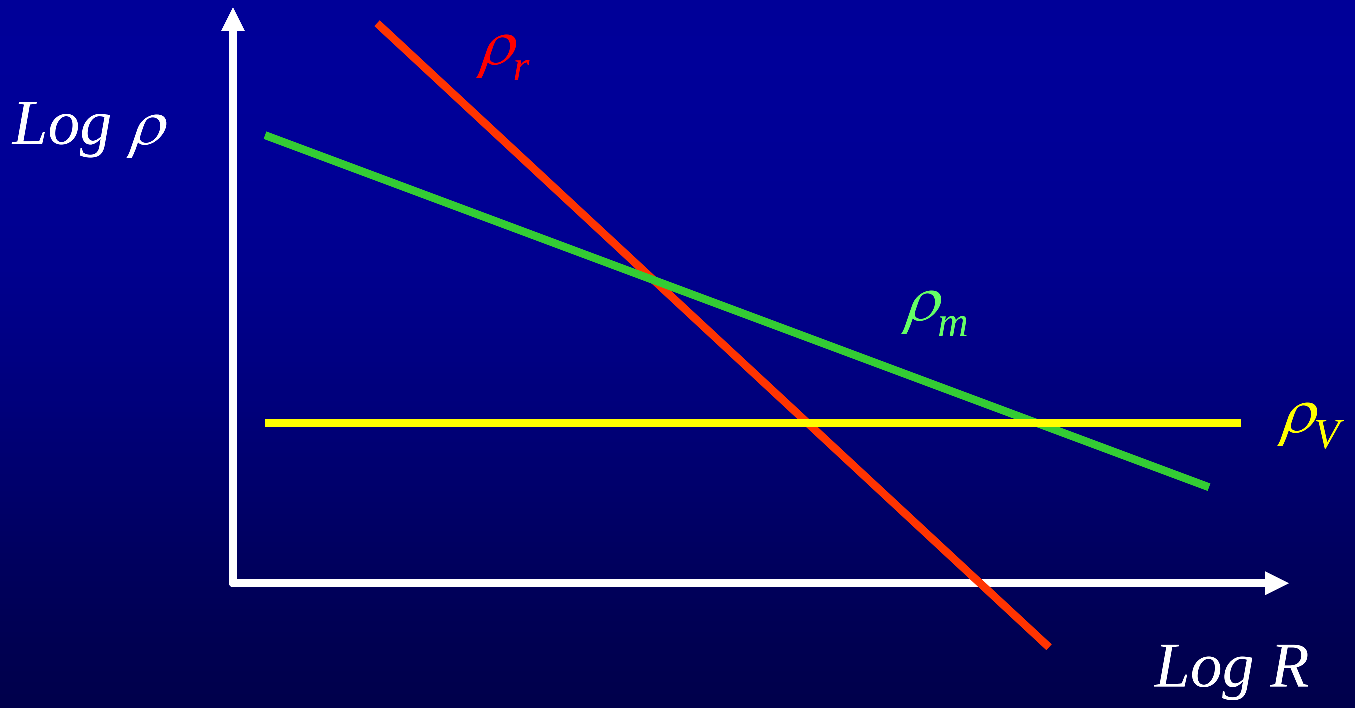
- Eddington: Λ is the cause of the expansion.

General equation of State

- In general should include all contributions to energy-density.

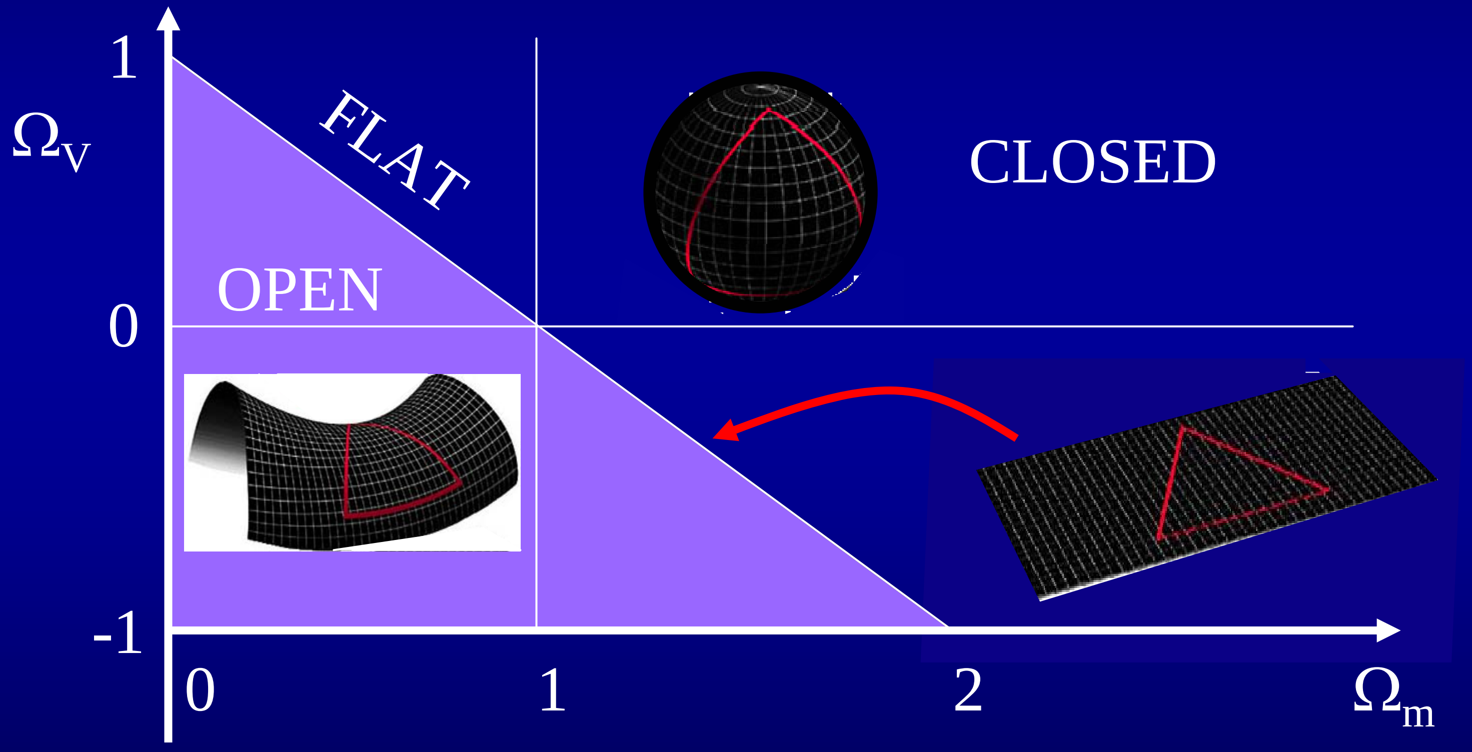
$$H^2 = \frac{8\pi G}{3}\rho + \frac{c^2 k}{R^2}, \qquad a = R(t)/R_0$$

$$= H_0^2\left(\Omega_V + \Omega_{m,o}a^{-3} + \Omega_{\gamma,o}a^{-4} + (1-\Omega_V - \Omega_{m,0} - \Omega_{\gamma,0})a^{-2}\right)$$



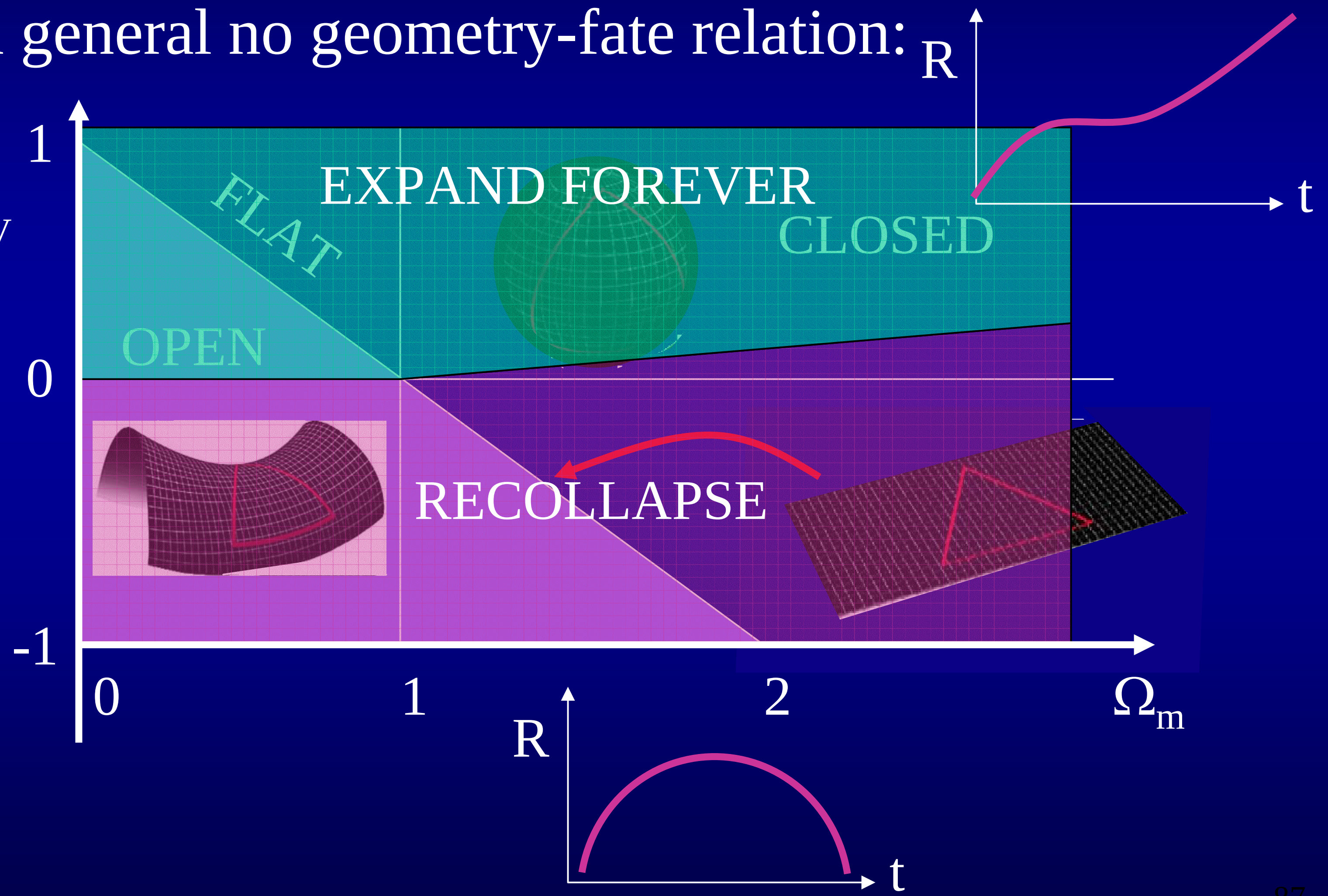
General equation of State

- In general must solve F.E. numerically.
- Geometry is still governed by total density:

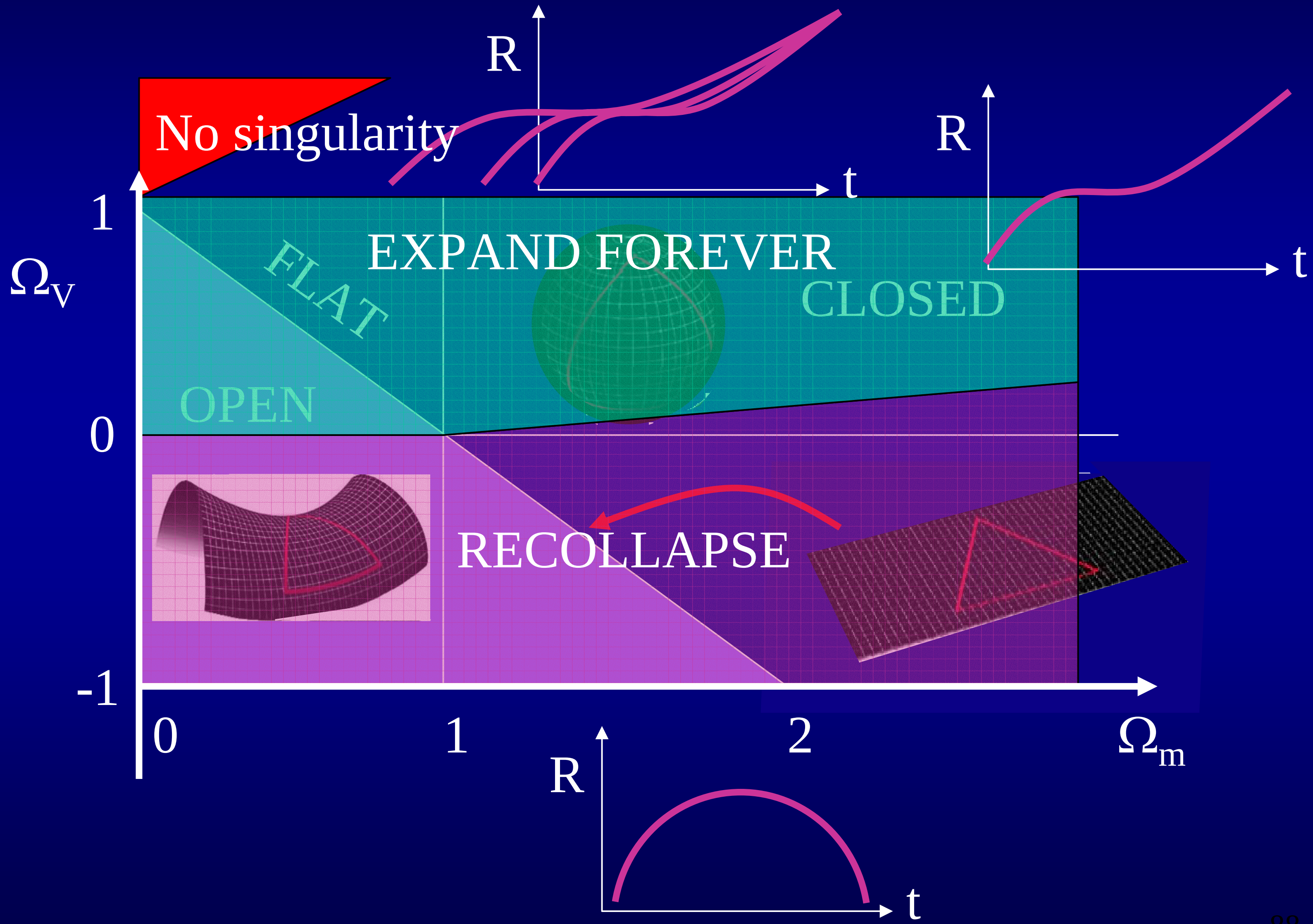


General equation of State

- In general must solve F.E. numerically.
- But in general no geometry-fate relation:



General equation of State



Age and size of Universe

- Evolution of redshift:

$$(1+z) = \frac{R_0}{R(t)}$$

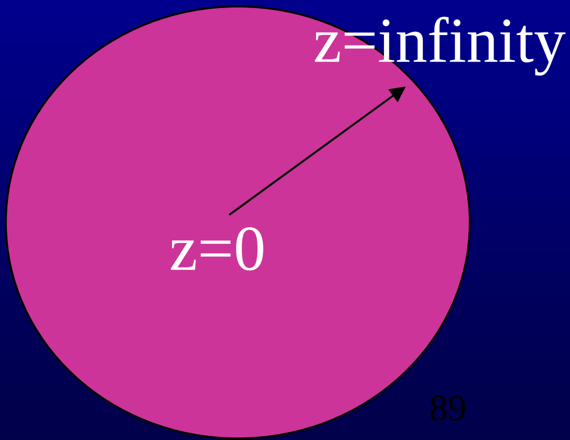
$$\Rightarrow \qquad \frac{dz}{dt} = -R_0 \frac{\dot{R}}{R^2} = -(1+z)H(z)$$

where

$$H^2(z) = H_0^2\left(\Omega_V + \Omega_{m,o}(1+z)^3 + \Omega_{\gamma,o}(1+z)^4 + (1-\Omega_V - \Omega_{m,0} - \Omega_{\gamma,0})(1+z)^2\right)$$

Age of the universe:

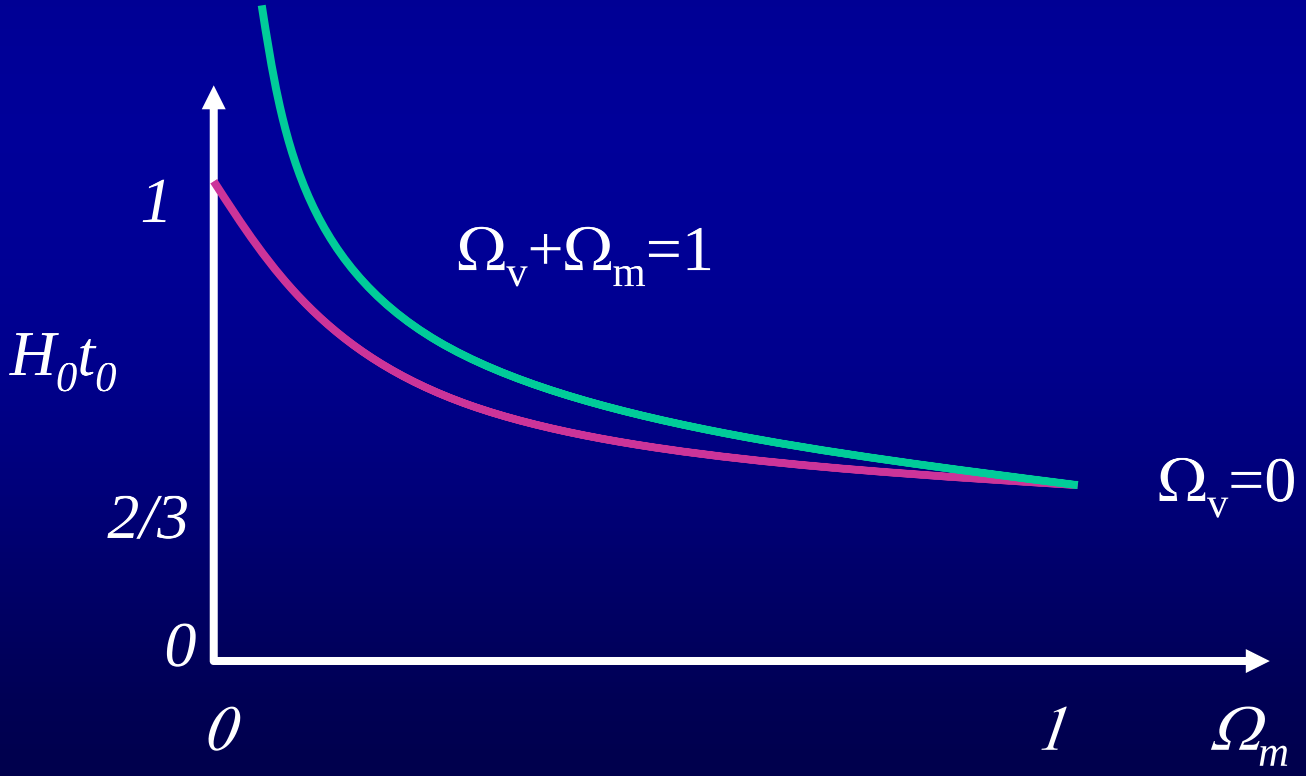
$$t(z) = \int_0^\infty \frac{dz}{(1+z)H(z)}$$



Age and size of Universe

- Usually evaluate t_0 numerically, but approximately:

$$t_0 \approx \frac{2}{3H_0}\left[0.7\Omega_m - 0.3(\Omega_V - 1)\right]^{-0.3}$$



Age and size of Universe

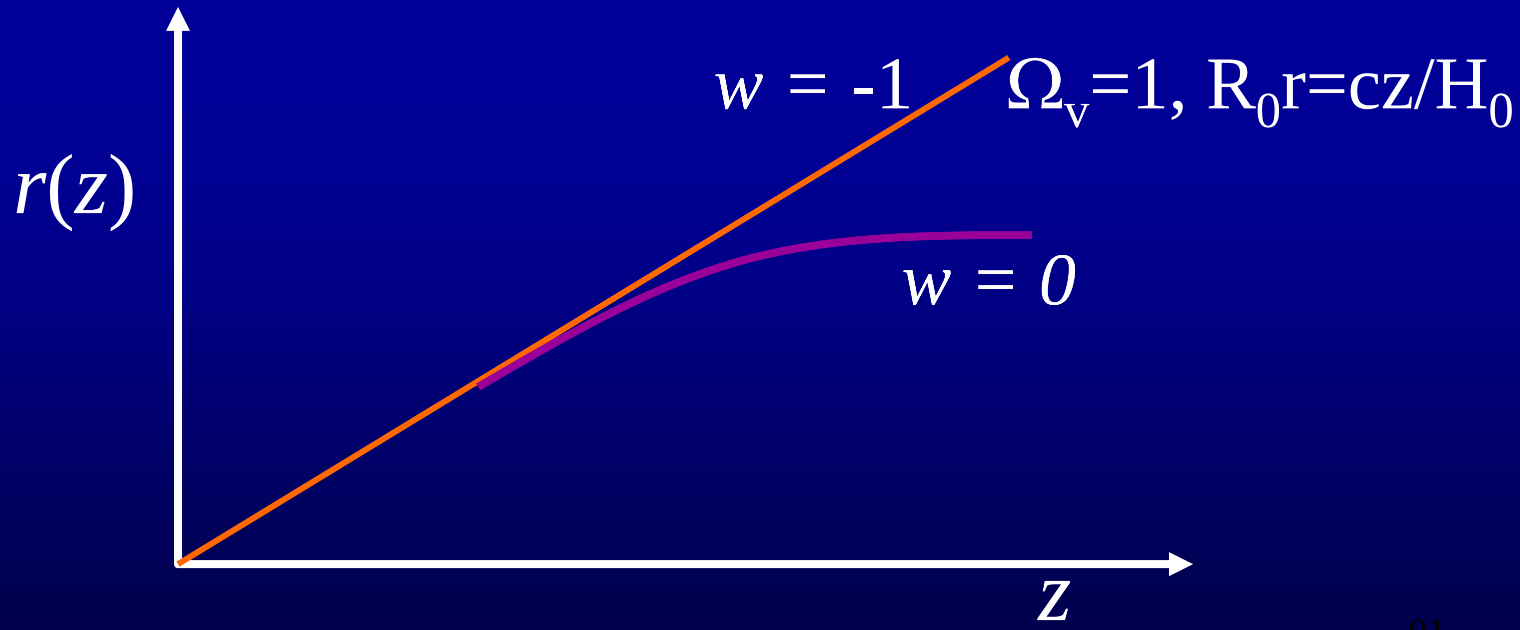
- Comoving distance-redshift relation: $dr = c\,dt/R = c\,dz/R_0H(z)$.

$\Omega_m=1$

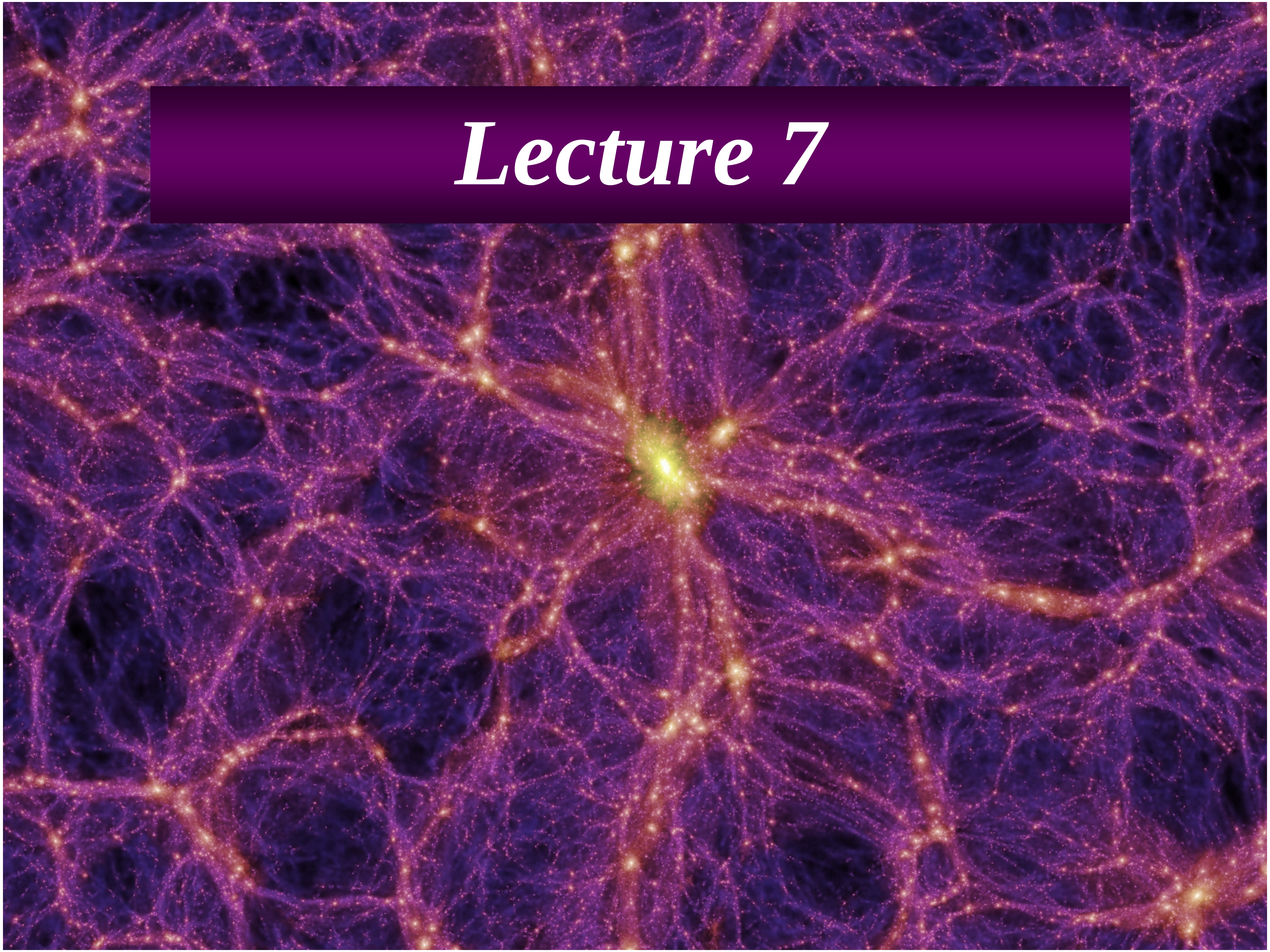
$$R_0 r = \frac{2c}{H_0}\left(1-(1+z)^{-1/2}\right)$$

$\Omega_m=0$

$$R_0 r = \frac{c}{2H_0}\left((1+z)-(1+z)^{-1}\right)$$



Lecture 7



Age and size of Universe

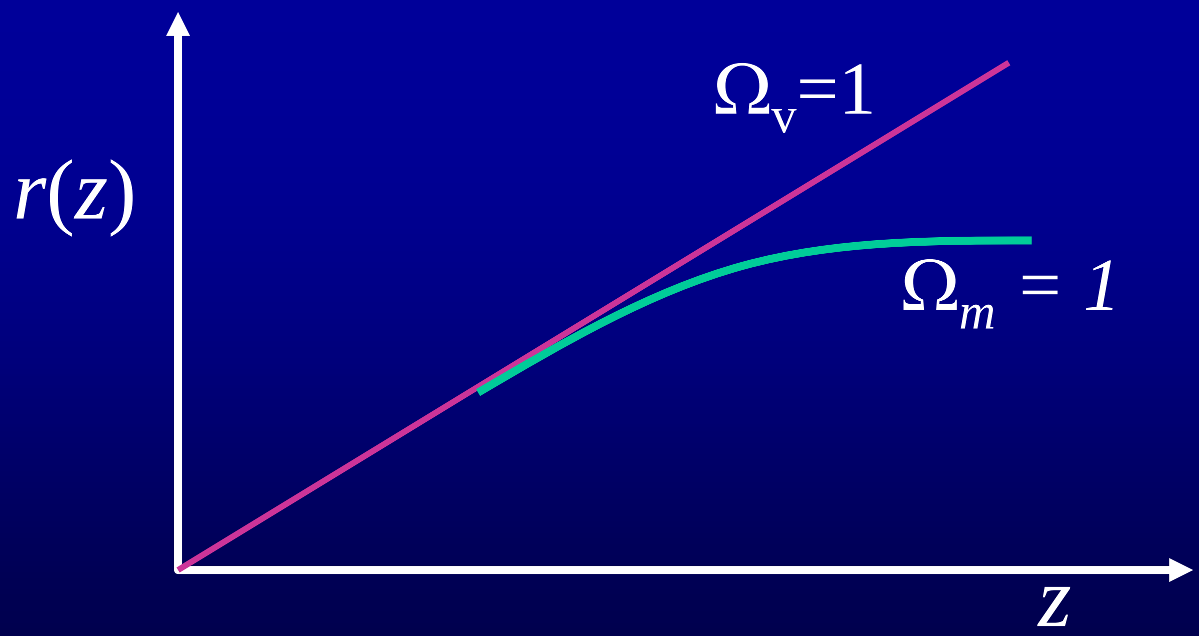
- Comoving distance-redshift relation: $dr = c\,dt/R = c\,dz/R_0H(z)$.

Einstein de Sitter $\Omega_m = 1$

$$R_0 r = \frac{2c}{H_0}\left(1 - (1+z)^{-1/2}\right)$$

de Sitter $\Omega_V = 1$

$$R_0 r = \frac{c}{H_0} z$$



Observational Cosmology

- Size and Volume:
  - Start from line element:

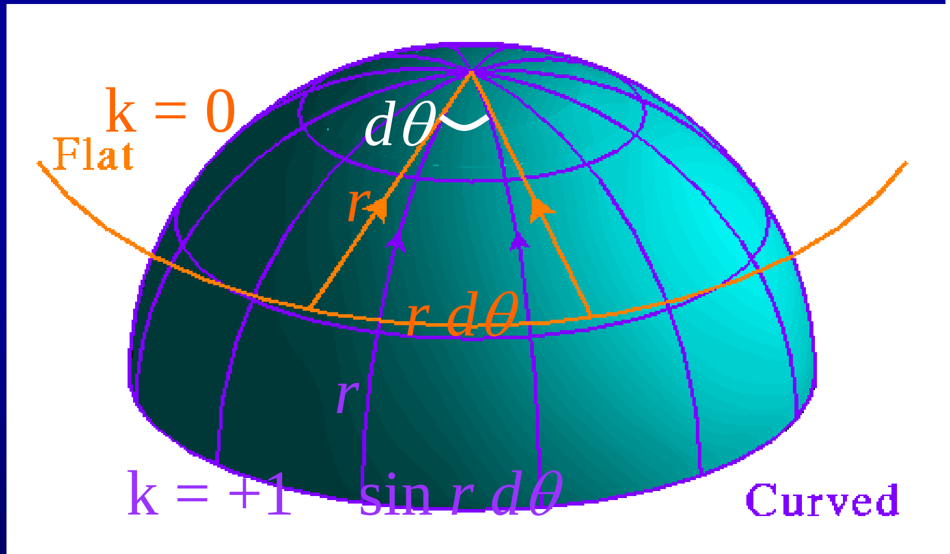
$$c^2 d\tau^2 = c^2 dt^2 - R^2(t)\left(dr^2 + S_k^2(r)\, d\psi^2\right)$$

- Angular sizes:

$$dl_{\perp} = R(z) S_k\left[r(z)\right] d\psi$$

- Volumes:

$$dV(z) = R^3(z) S_k^2[r(z)]\, dr\, d\psi$$



Observational Cosmology

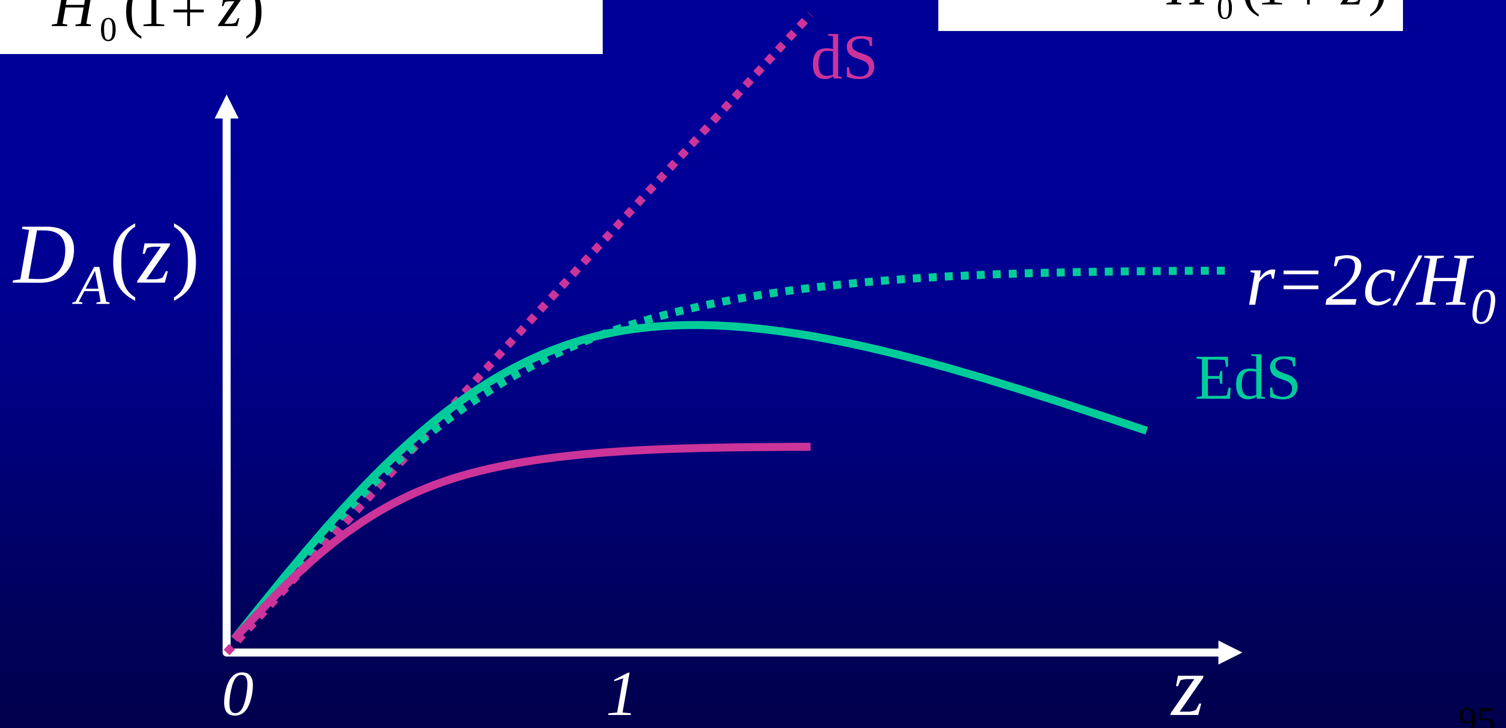
- Angular diameter distance: $$D_A(z) = R(z) S_k[r(z)]$$

- Einstein-de Sitter universe

$$D_A(z) = \frac{2c}{H_0(1+z)}\left(1-(1+z)^{-1/2}\right)$$

de Sitter universe

$$D_A(z) = \frac{cz}{H_0(1+z)}$$



Observational Cosmology

- Angular size:

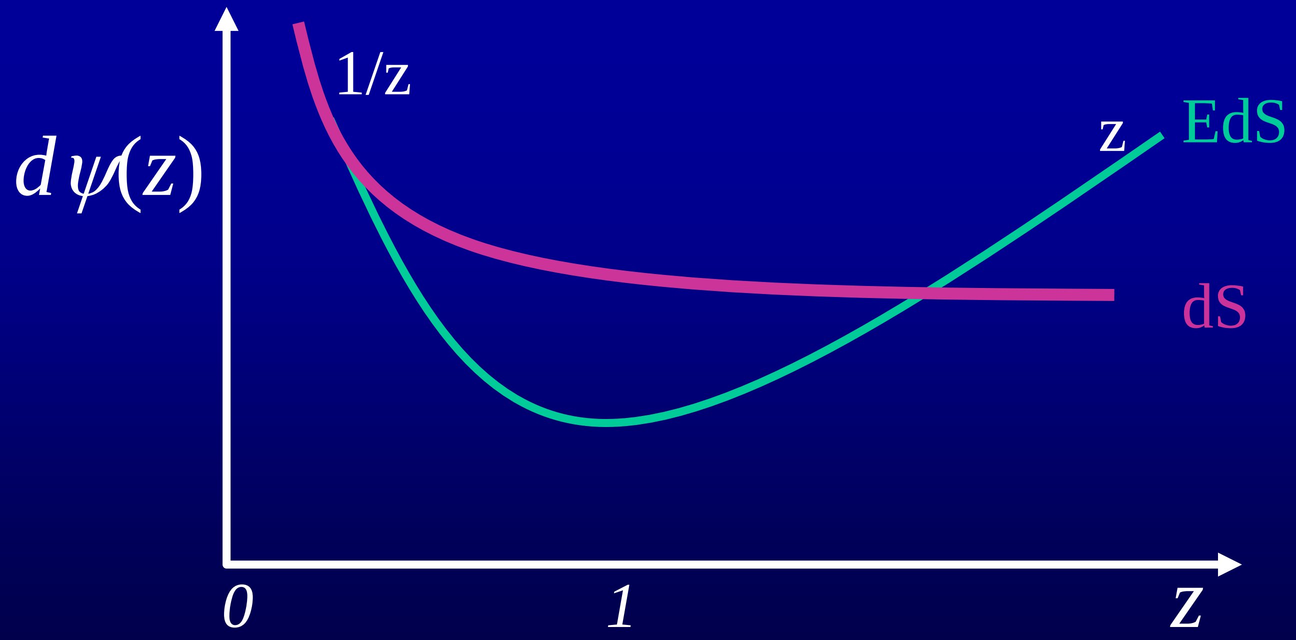
$$d\psi(z) = \frac{dl_\perp}{D_A(z)}$$

- Einstein-de Sitter universe

$$D_A(z) = \frac{2c}{H_0(1+z)}\left(1-(1+z)^{-1/2}\right)$$

de Sitter universe

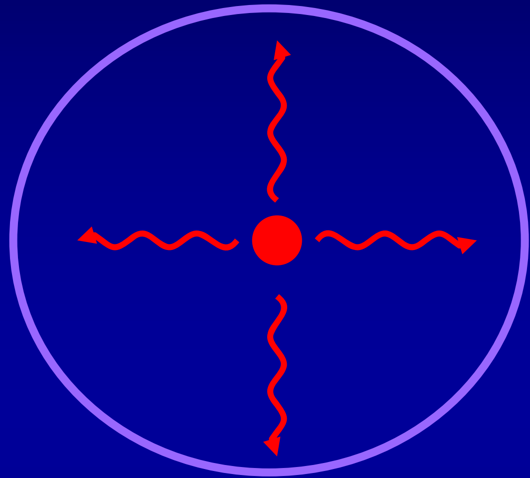
$$D_A(z) = \frac{cz}{H_0(1+z)}$$



Observational Cosmology

- Luminosity and flux density:
  - Euclidean space:

$$S_\nu = \frac{L_\nu(\nu)}{4\pi r^2}$$



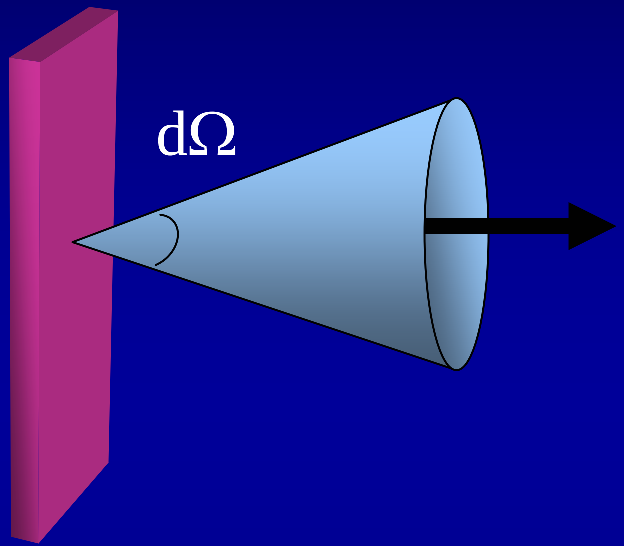
  - Curved, expanding space:
    - $L = E/t \sim (1+z)^{-2}$
    - $L_\nu = dL/d\nu \qquad d/d\nu_0 = (1+z)d/d\nu$
    - $\nu = (1+z)\nu_0$

$$S_\nu(\nu_0) = \frac{L_\nu[(1+z)\nu_0]}{4\pi R_0^2 S_k^2[r(z)](1+z)}$$

$$S_{TOT} = \int \frac{d\nu}{1+z} S_\nu = \frac{L_{TOT}}{4\pi R_0^2 S_k^2 (1+z)^2}$$

Observational Cosmology

- Surface brightness, I_ν:



$$S_\nu = I_\nu d\Omega$$

$$\frac{I_\nu}{\nu^3} = \frac{1}{\left(e^{h\nu/kT} - 1\right)} = n(\nu/T) = \frac{B_\nu[(1+z)\nu]}{(1+z)^3\nu^3}$$

So the high-redshift objects are heavily dimmed by expansion.

Observational Cosmology

- Luminosity distance:

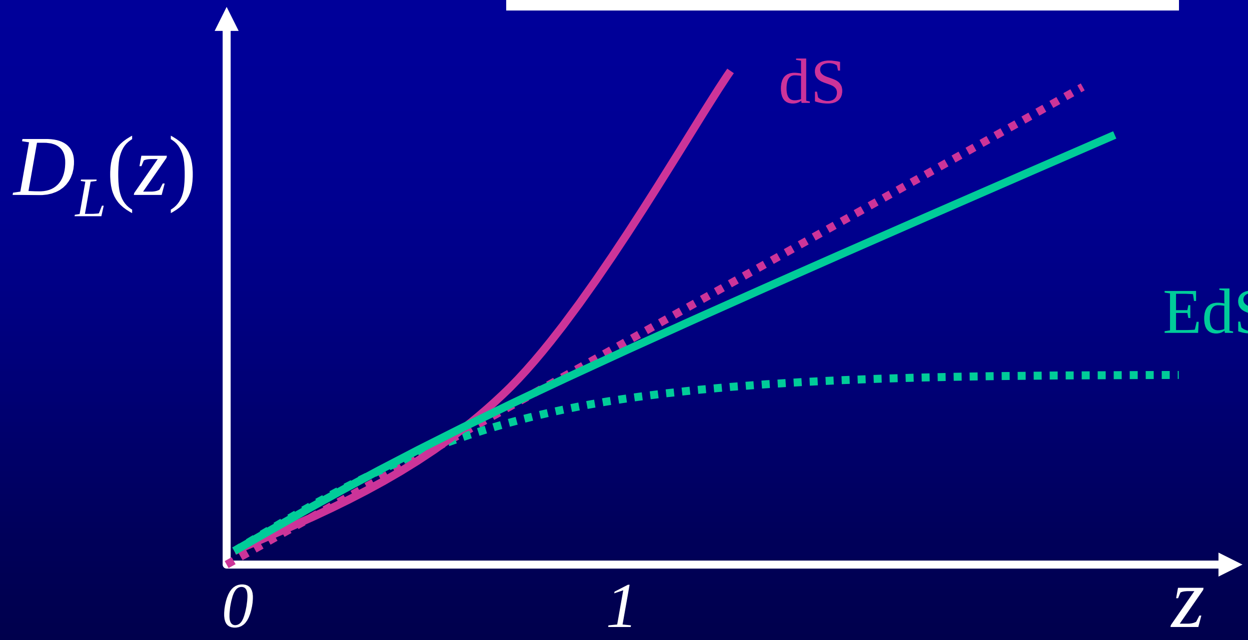
$$S_{TOT} = \frac{L_{TOT}}{4\pi D_L^2(z)}, \qquad D_L(z) = (1+z)R_0 S_k[r(z)]$$

- Einstein-de Sitter:

$$D_L(z) = \frac{2c(1+z)}{H_0}\left(1-(1+z)^{-1/2}\right)$$

- de Sitter:

$$D_L(z) = \frac{cz}{H_0}(1+z)$$



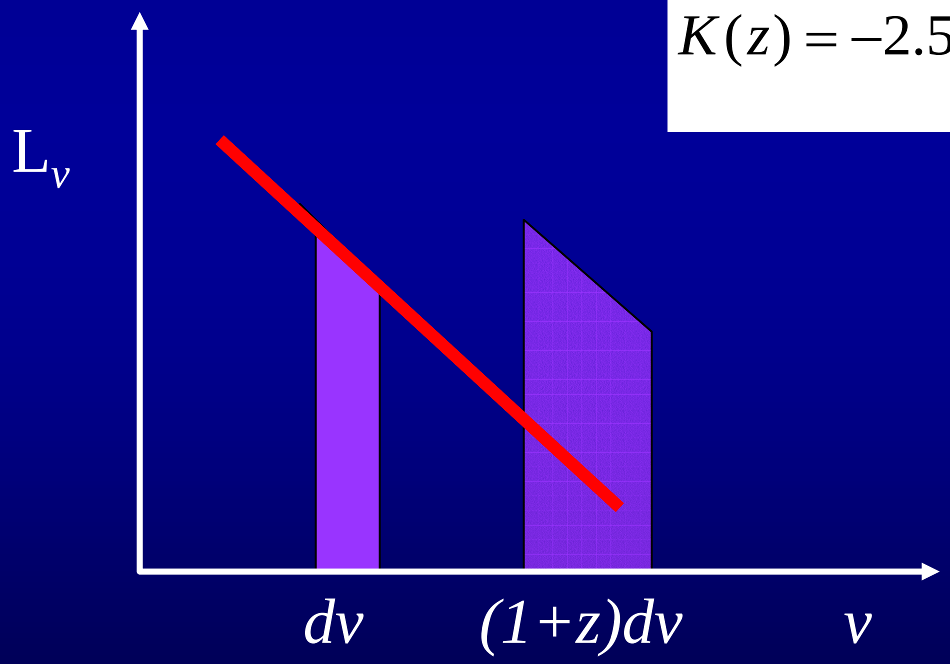
Observational Cosmology

- Magnitude-redshift relation:

$$m = M + 5\log\left(\frac{D_L(z)}{10\,pc}\right) + K(z)$$

- The K-correction: redshift shifts frequency & passbands.

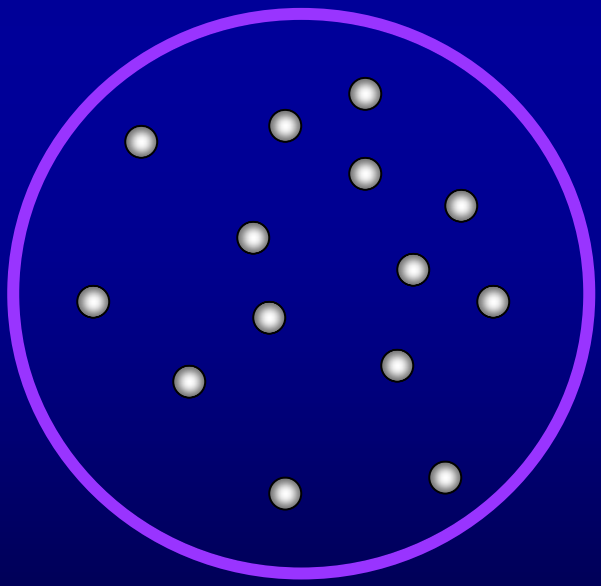
$$K(z) = -2.5\log\left(\frac{(1+z)L_\nu[(1+z)\nu]}{L_\nu[\nu]}\right)$$



Observational Cosmology

• Galaxy Counts: Number of galaxies on sky as function of flux, N(>S).

•Euclidean Model: Consider n galaxies per Mpc^3 with same luminosity, L, in a sphere of radius D.



$$D \propto S^{-1/2}$$

$$V \propto D^3 \propto S^{-3/2}$$

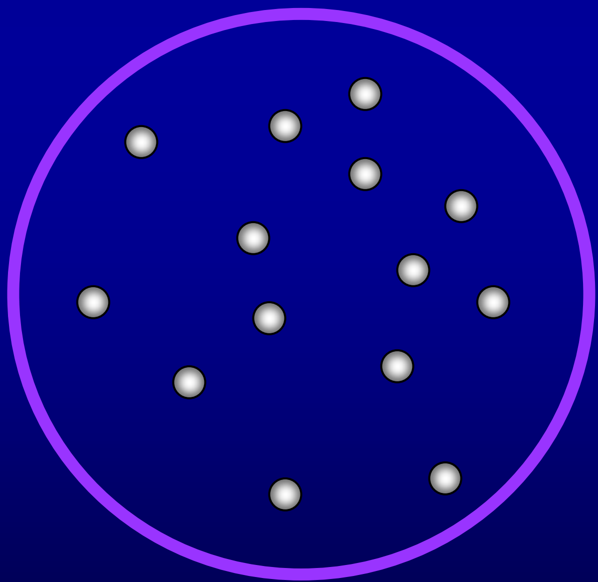
$$N(>S) = nV(S) \propto S^{-3/2}$$

Observational Cosmology

- Olbers Paradox:
  The Sky brightness:

$$I = \frac{1}{A}\int_0^\infty dS \, \frac{dN(>S)}{dS} S$$

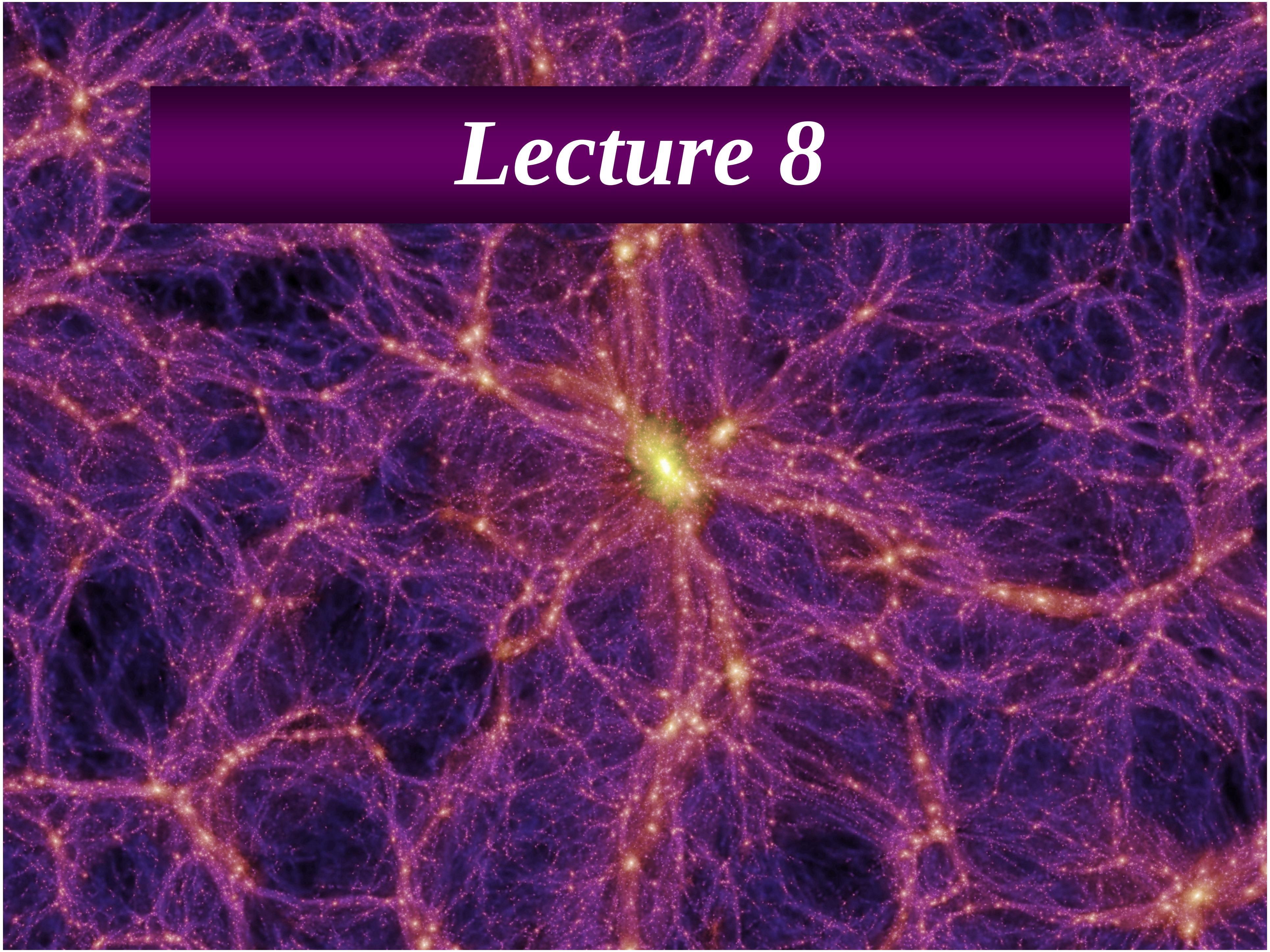
$$\propto \int_0^\infty dS S^{-3/2} = \left[S^{-1/2}\right]_0^\infty$$



which diverges as S goes to zero.

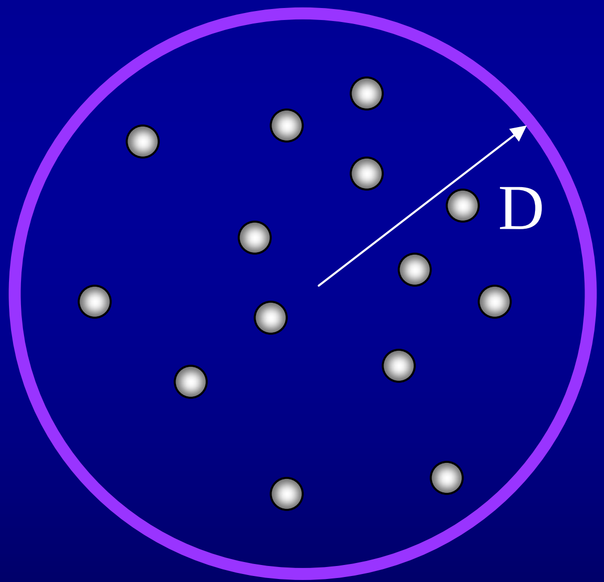
Too many sources as D increases, due to increase in volume.

Lecture 8



Observational Cosmology

- Olbers Paradox: Why is the night sky dark?



$$I = \frac{1}{A}\int_0^\infty dS \frac{dN(>S)}{dS} S$$

$$\propto \int_0^\infty dS S^{-3/2} = \left[S^{-1/2}\right]_0^\infty$$

which diverges as S goes to zero.

Too many sources as D increases, due to increase in volume.

Observational Cosmology

- Relativistic Galaxy Count Model:

  Ω=1, k=0 Einstein-de Sitter model:

$$r = 2(1-(1+z)^{-1/2})$$

$$V = \frac{A}{3}[R_0 r(z)]^3$$

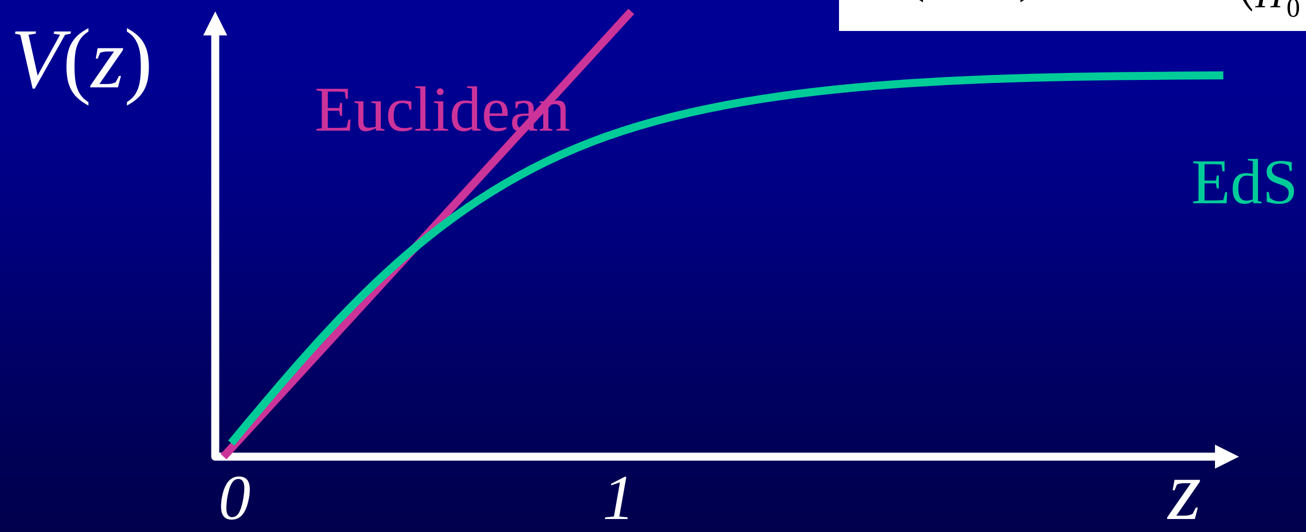
Flux density: $L_\nu \sim \nu^{-\alpha}$

$$S_\nu = \frac{L_\nu}{4\pi (R_0 r)^2 (1+z)^{1+\alpha}} \propto (1-(1+z)^{-1/2})^{-2}(1+z)^{-(1+\alpha)}$$

Number counts.

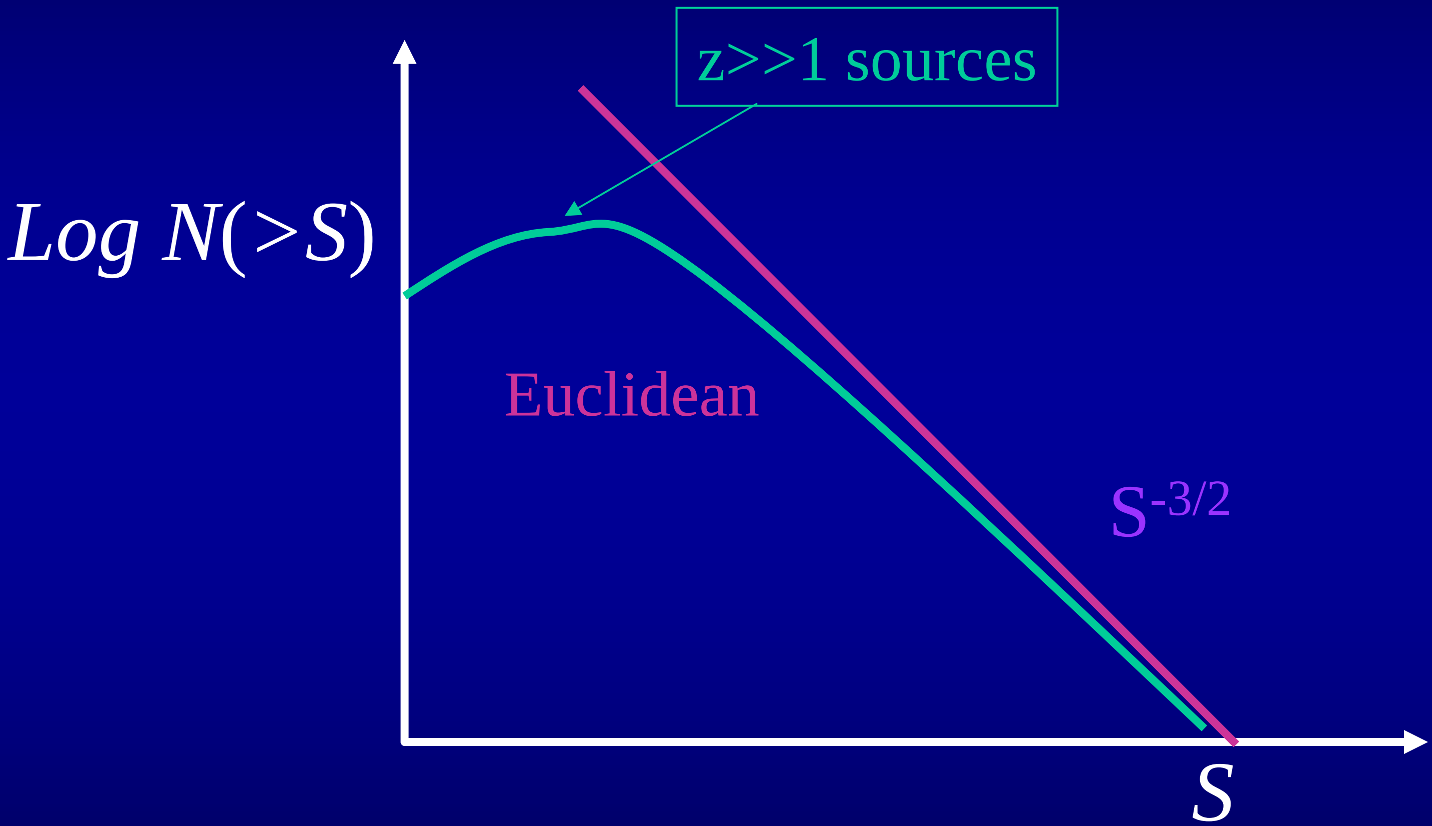
z<<1 $$N(>S) \propto V \propto z^3 \propto S^{-3/2}$$

z>>1 $$N(>S) \propto V \propto \left(\frac{2c}{H_0}\right)^3$$



Observational Cosmology

- Relativistic Galaxy Count Model:



Counts converge due to finite volume/age /distance at high-z.
So solves Olber's paradox.

Distances and age of the Universe

- Cosmological Distances: $c/H_0 = 3000h^{-1}$Mpc.
- Cosmological Time: $1/H_0$ = 14 Gyrs.

•Recall our solution for age of Universe:

$$t_0 = \int_0^\infty \frac{dz}{(1+z)H(z)}$$

$$\approx \frac{2}{3H_0}\left(0.7\Omega_m + 0.3(1-\Omega_V)\right)^{-0.3}$$

So if we know Ω_m, Ω_V and H_0, we can get t_0.
Or if we know H_0 and t_0 we can get Ω_m and Ω_V.

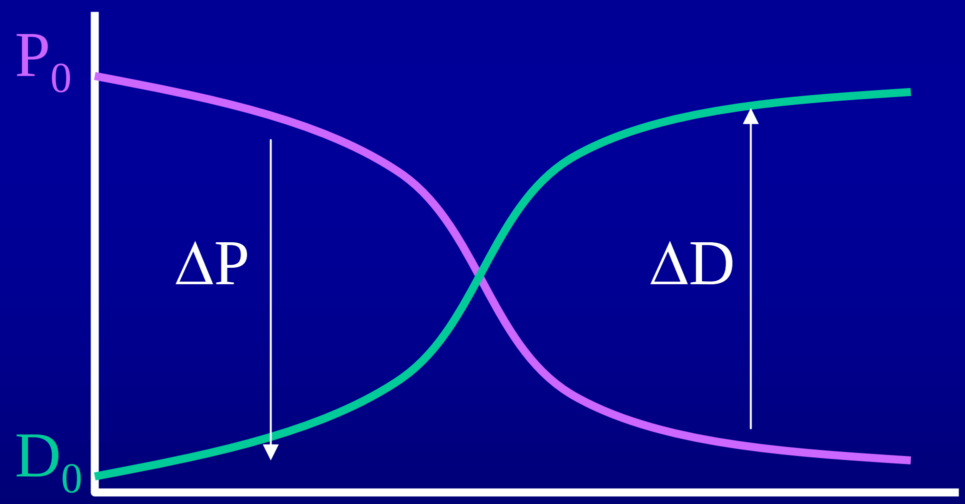
Distances and age of the Universe

- Estimating the age of the Universe, t_0:
  - Nuclear Cosmo-chronology:
    Natural clock of radioactive decay, $\tau \sim 10$Gyrs.
  - Heavy elements ejected from supernova into ISM:
    Thorium (232Th) => Lead (208Pb) 20 Gyrs
    Uranium (235U) => Lead (207Pb) 1 Gyr
    Uranium (238U) => Lead (206Pb) 6.5 Gyrs

Distances and age of the Universe

- Estimating the age of the Universe:
  - No new nuclei produced after solar system forms, just nuclear decay:

$$\Delta D = -\Delta P$$
$$= P_0(1 - e^{-t/\tau})$$
$$= P(e^{t/\tau} - 1)$$



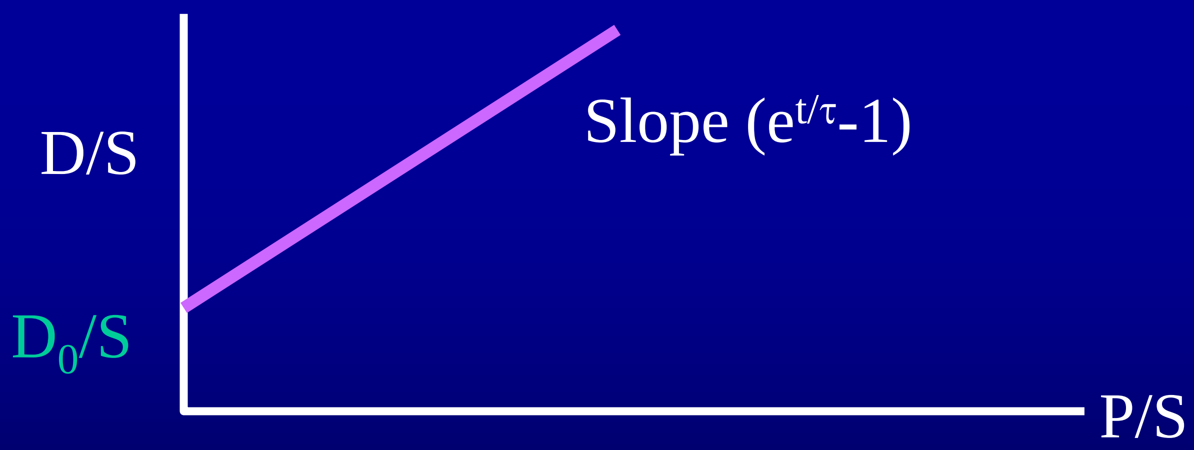
  - But don't know D_0, so how to measure ΔD?

Distances and age of the Universe

- Estimating the age of the Universe:
  - Take ratio with a stable isotope of D, S.

$$\frac{D}{S} = \frac{D_0}{S} + \frac{P}{S}(e^{t/\tau} - 1)$$

  - Plot D/S versus P/S



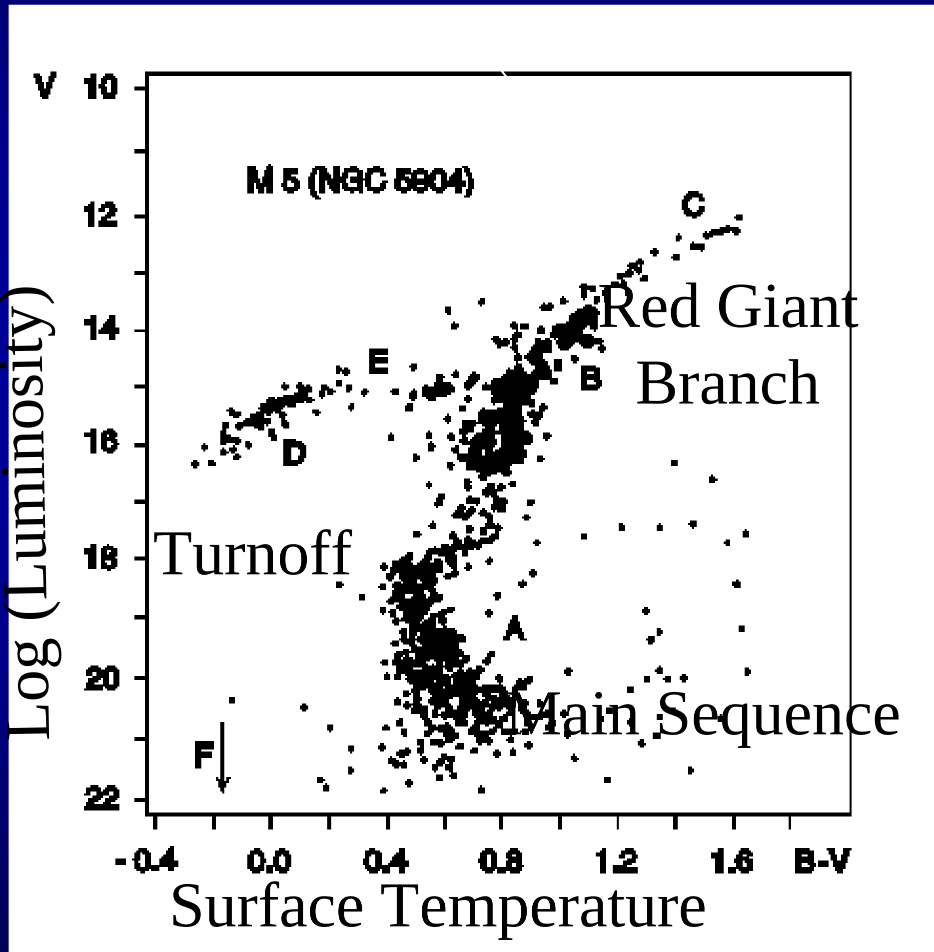
Meterorites: τ_{SS} = 4.57(+/-0.04) Gyrs

+ Nuclear theory: τ_{MW} = 9.5 Gyrs

Distances and age of the Universe

- Age from Stellar evolution:

$t \sim M/L$

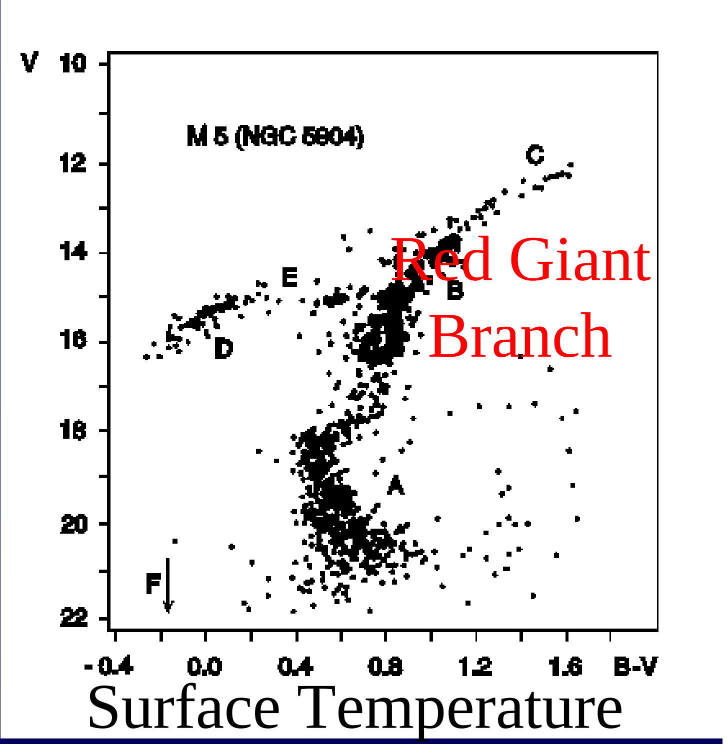


τ_{GC} = 13-17 Gyrs.

Recall for EdS t_0=9.3Gyrs!

Distances and age of the Universe

- Local distance:
  - Use Cepheid Variables (cf Hubbles measurement to M31).
  - Mass $M=3 – 9M_{sun}$
  - Moving onto Red Giant Branch



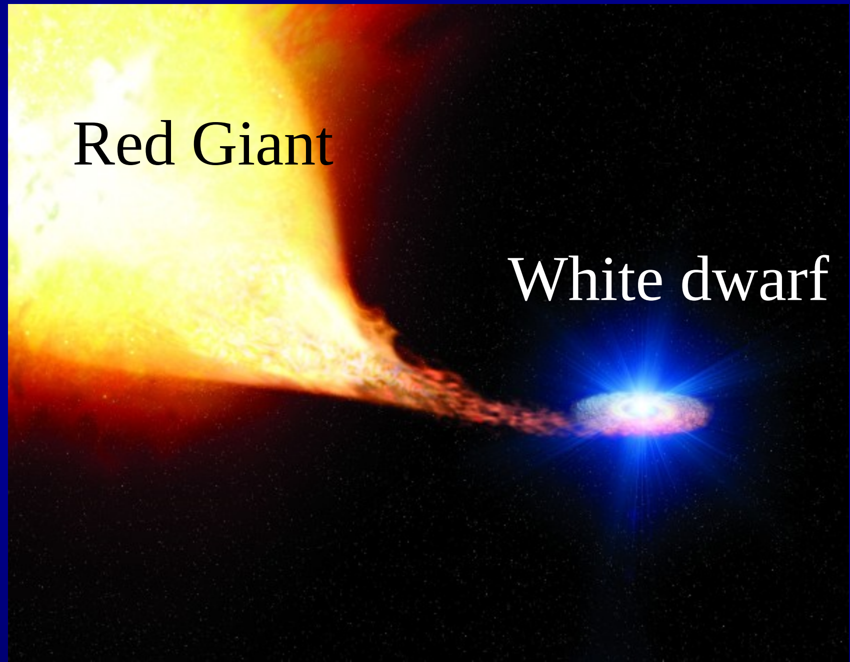
$$Luminosity \sim (Period)^{1.3}$$
$$L \sim 1/D^2$$
$$D \sim 1/L^{1/2}$$

Need to know D_{LMC}=51kpc +/- 6%.
From parallax, or SN1987a.

Distances and age of the Universe

- Larger distances:

- Use supernova Hubble diagram.
- SN Ia, Ib, II.
- SNIa standard candles.



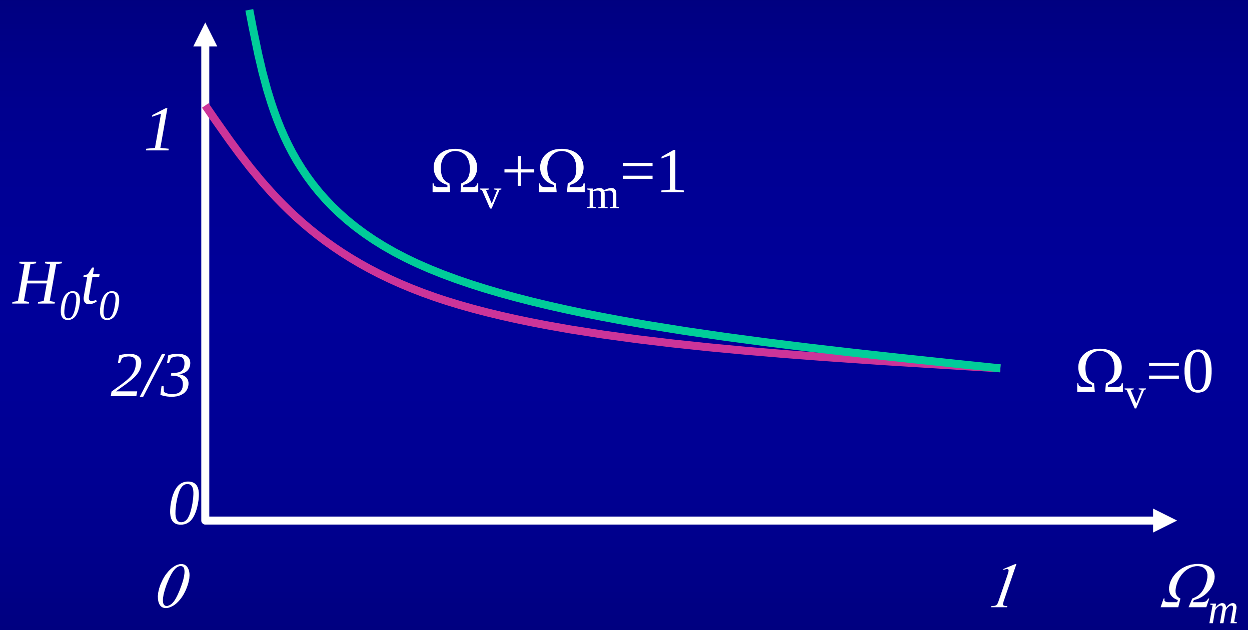
- Nuclear detonation of WD.

- HST Key programme:
  $H_0 = 72+/-8kms^{-1}$.

(error mainly distance to LMC)

Distances and age of the Universe

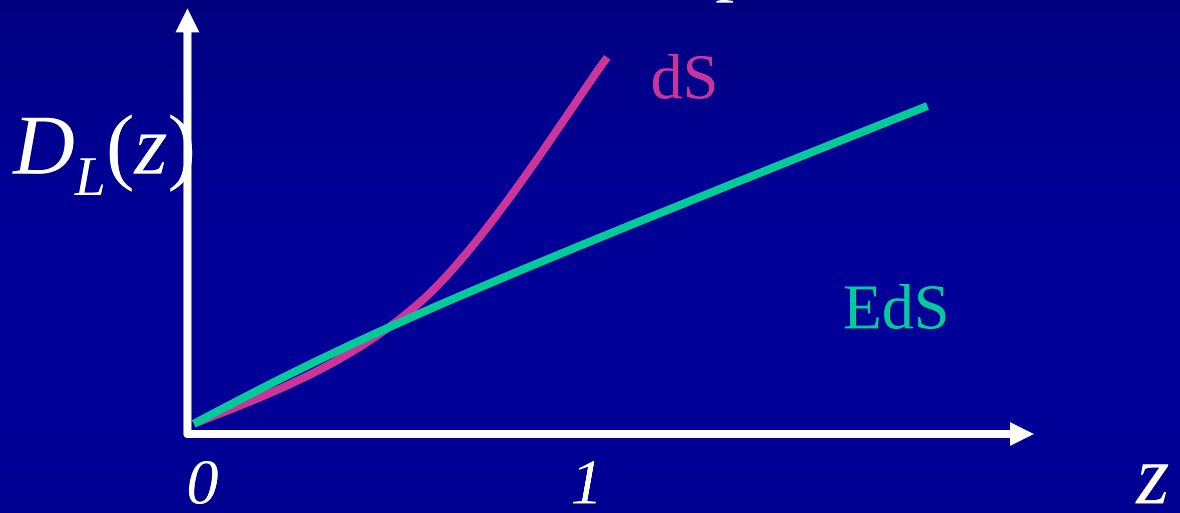
- So now know H_0 and t_0 so now know H_0t_0=0.96.
- What can we infer about Ω_m and Ω_V?



- This implies that if Ω_V=0, Ω_m=0….
- Or $\Omega_V=2.3\Omega_m$! Implies vacuum domination…
- And if flat (k=0) Ω_V=0.7, Ω_m=0.3.

Cosmological Geometry

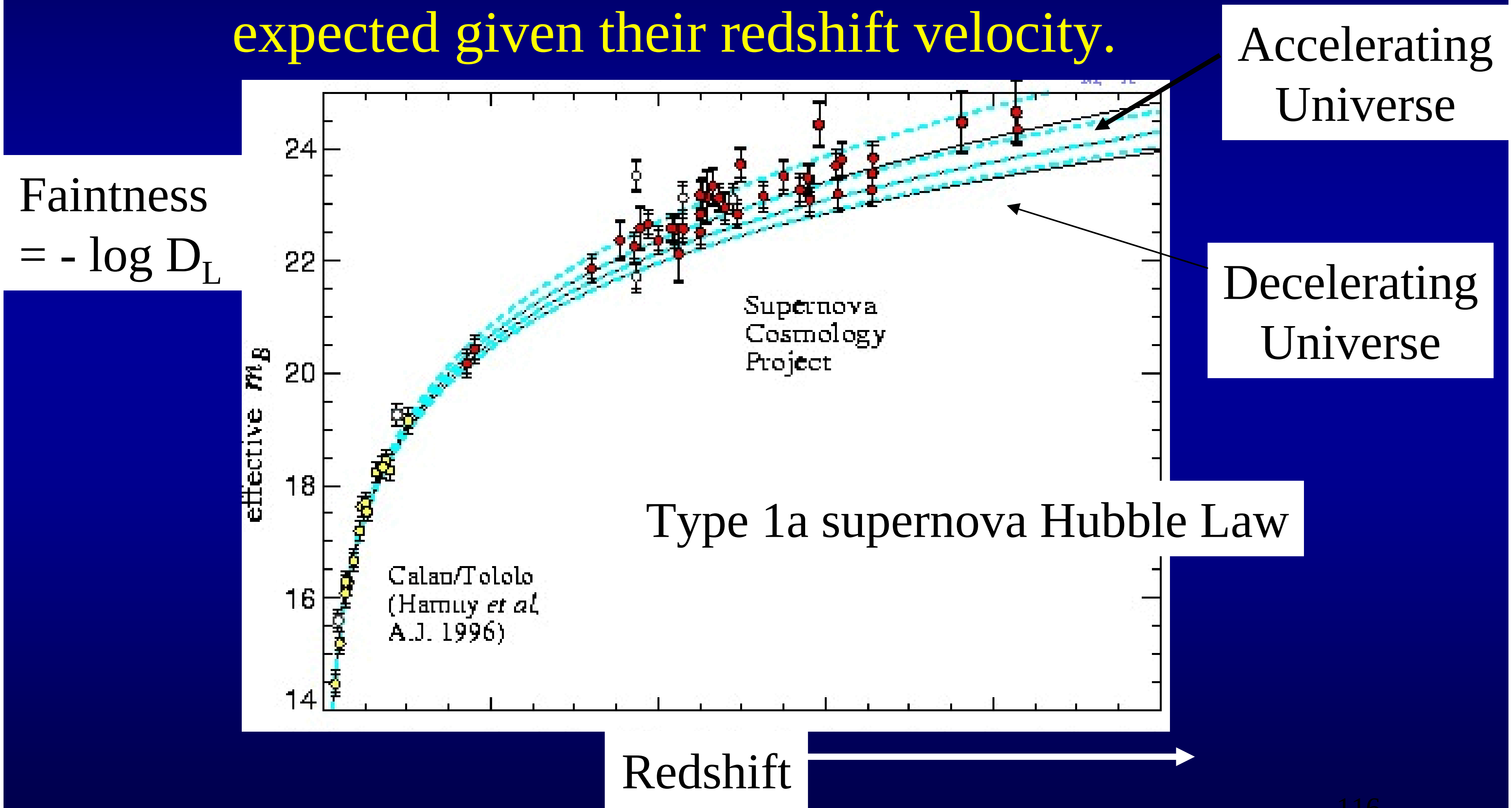
- Can measure Ω_m and Ω_V from luminosity distances to standard candles – the supernova Hubble diagram.



$$D_L(z) = (1+z)\int_0^z \frac{dz}{H(z)} \approx \frac{c}{H_0}\left(z + (2+\Omega_m - 2\Omega_V)z^2/4\right)$$

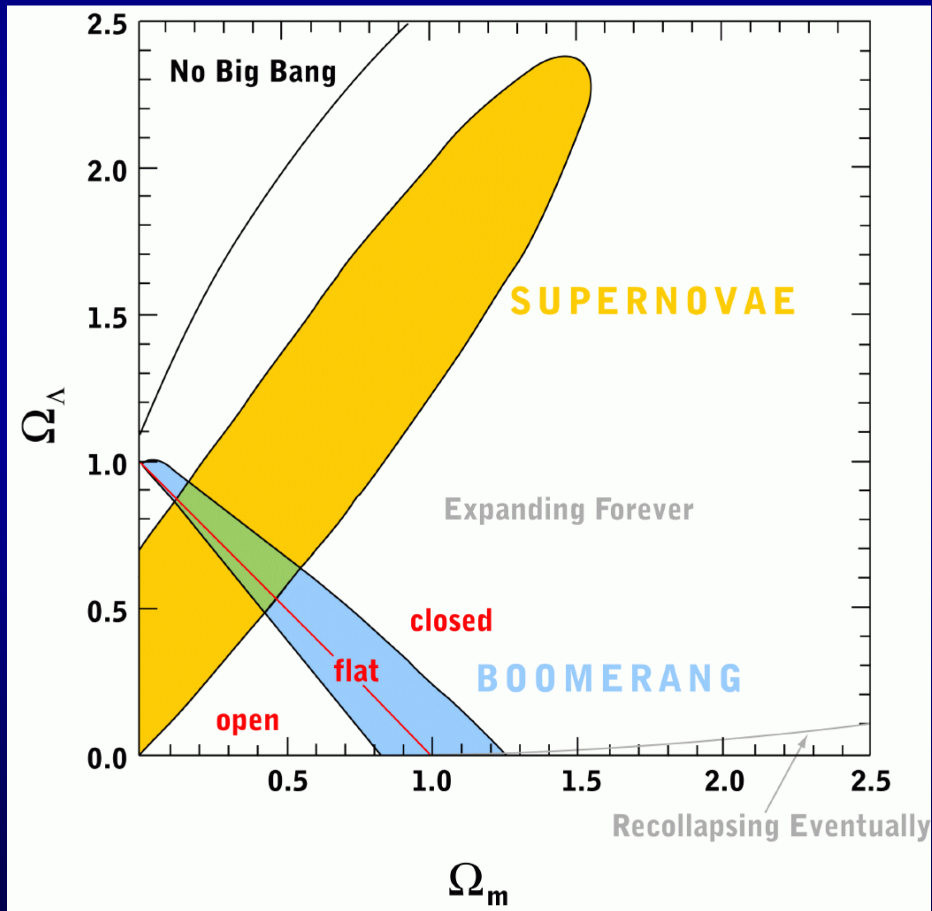
Cosmological Geometry

- The supernova Type Ia are fainter than expected given their redshift velocity.

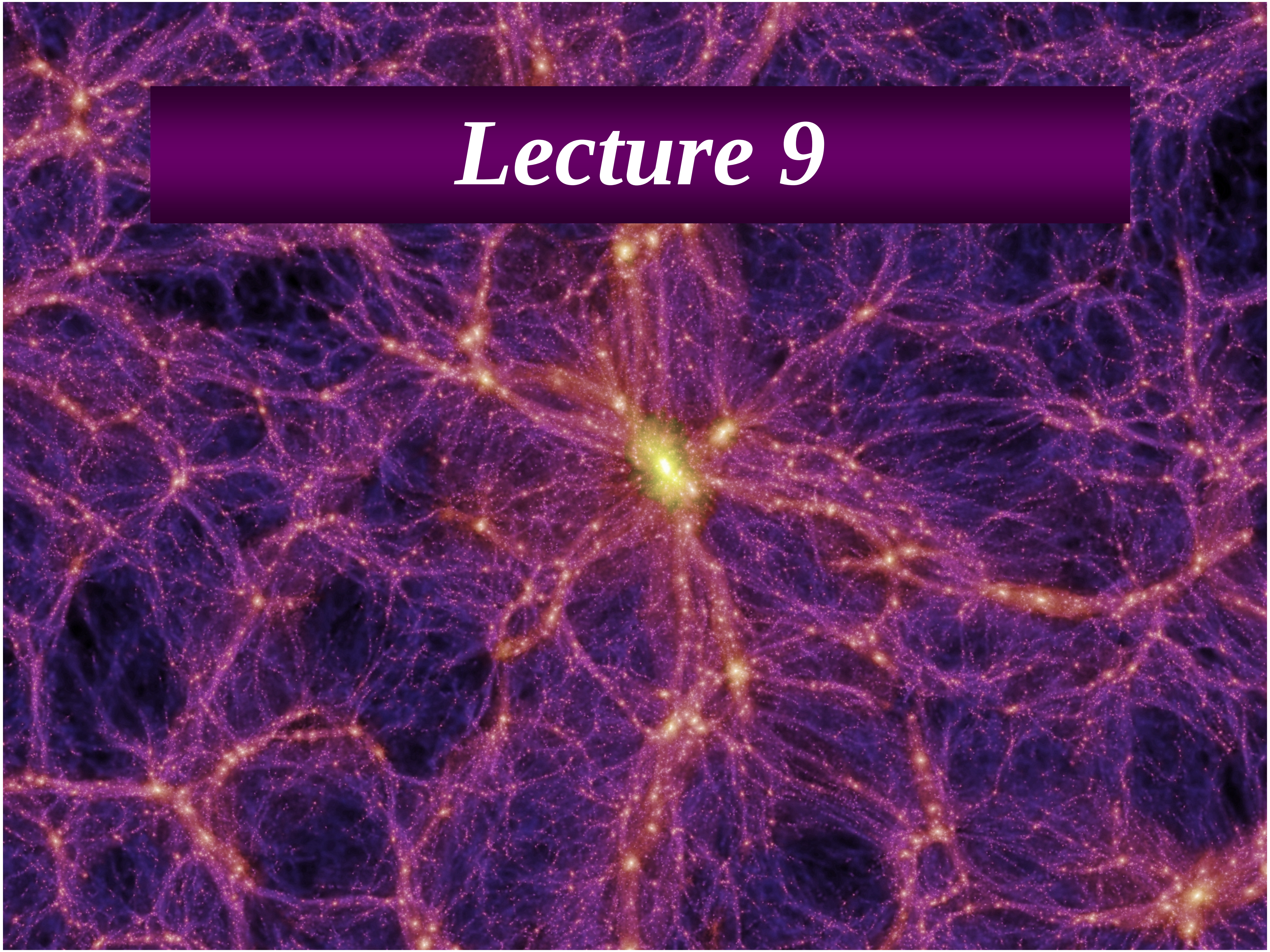


Cosmological Geometry

- Can measure Ω_m and Ω_V from luminosity distances to standard candles – the supernova Hubble diagram.



Lecture 9

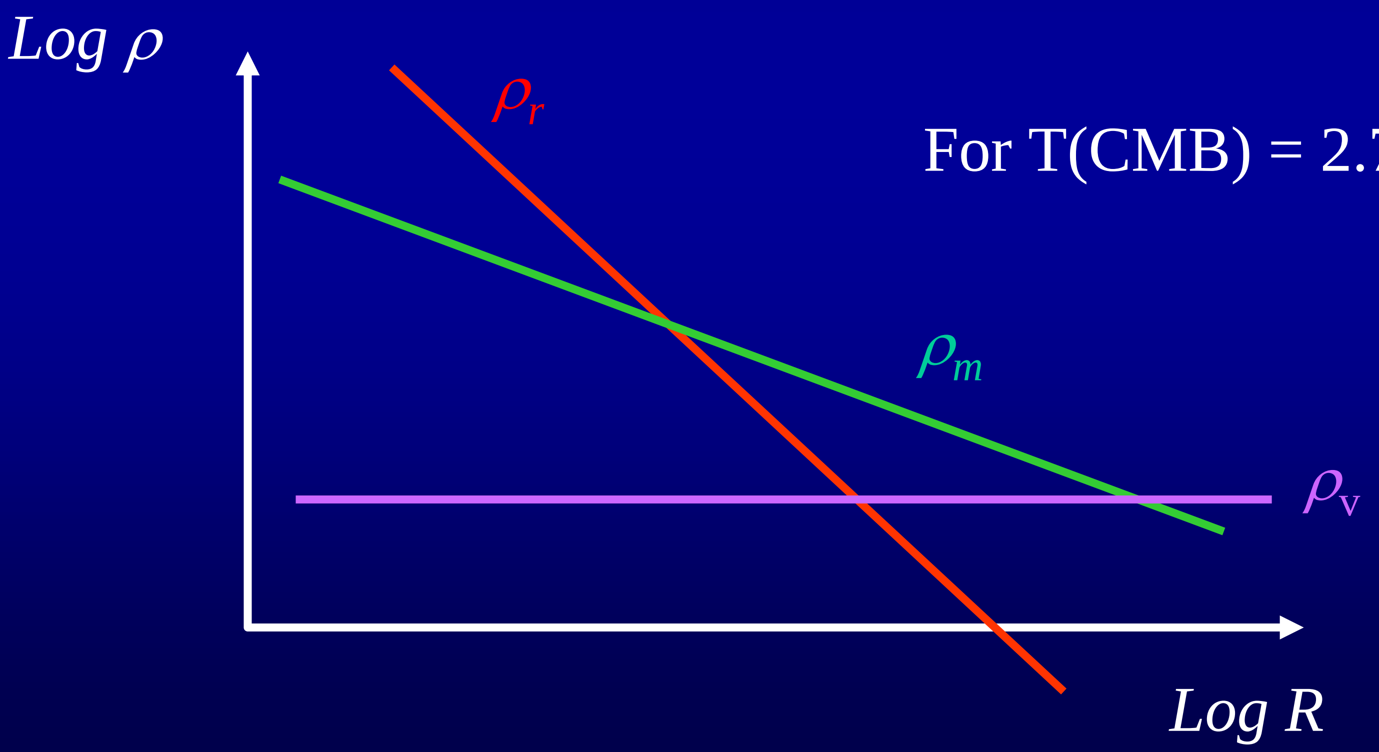


The thermal history of the Universe

- Recall that as Universe expands:

$$\rho_m=\rho_{0m}(R/R_0)^{-3} \quad \rho_r=\rho_{0r}(R/R_0)^{-4} \quad \rho = \rho_{v0}\,(R/R_0)^0$$

- At early enough times we have radiation-dominated Universe.



For T(CMB) = 2.73K today.

The thermal history of the Universe

- Also expect a neutrino background, $\rho_\nu = 0.68\rho_\gamma$ (see later).

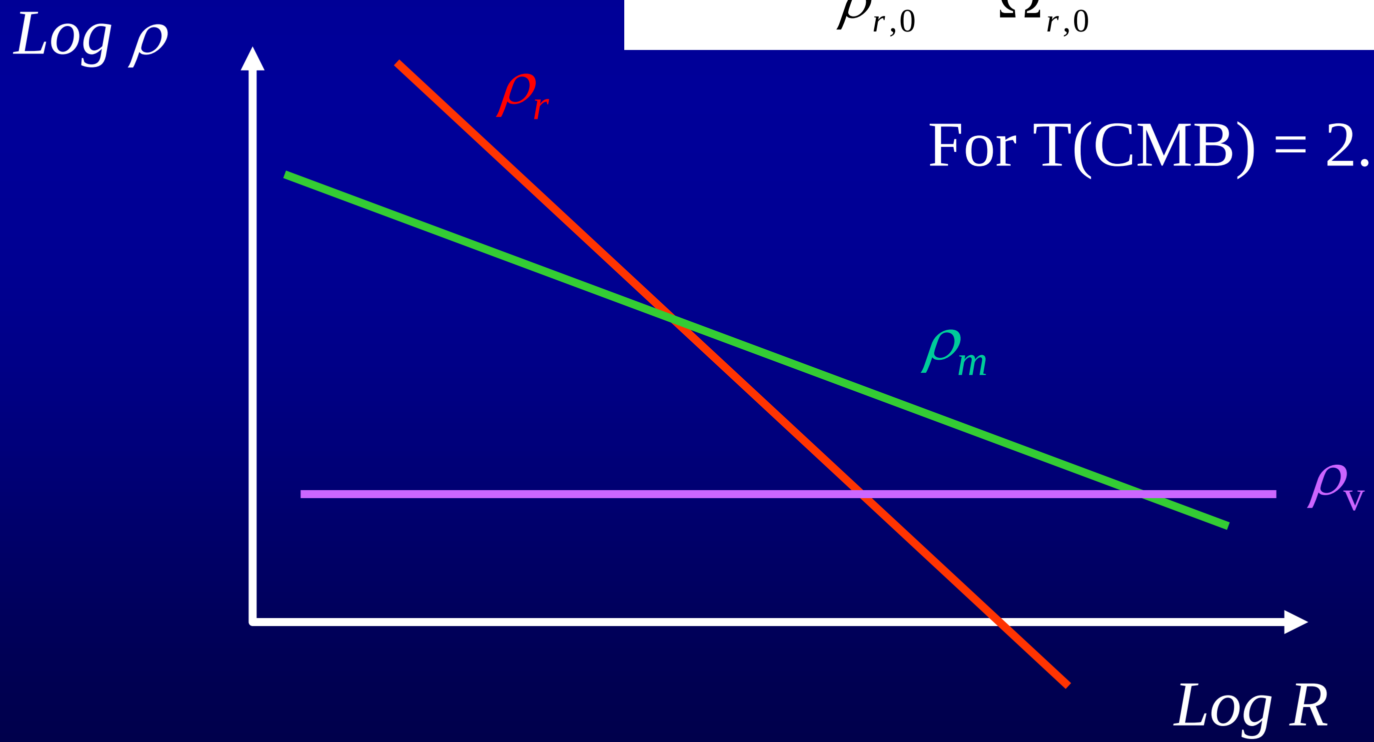
$$\rho_r = \rho_\gamma + \rho_\nu$$

$$\rho_r = aT^4$$

$$\Omega_r = 4.2 \times 10^{-3} h^{-2}$$

$$\frac{\rho_m}{\rho_r} = \frac{\rho_{m,0}(1+z)^3}{\rho_{r,0}(1+z)^4} = 1$$

$$1 + z_{eq} = \frac{\rho_{m,0}}{\rho_{r,0}} = \frac{\Omega_{m,0}}{\Omega_{r,0}} = 23{,}900(\Omega_m h^2)\left(T/2.73K\right)^{-4}$$



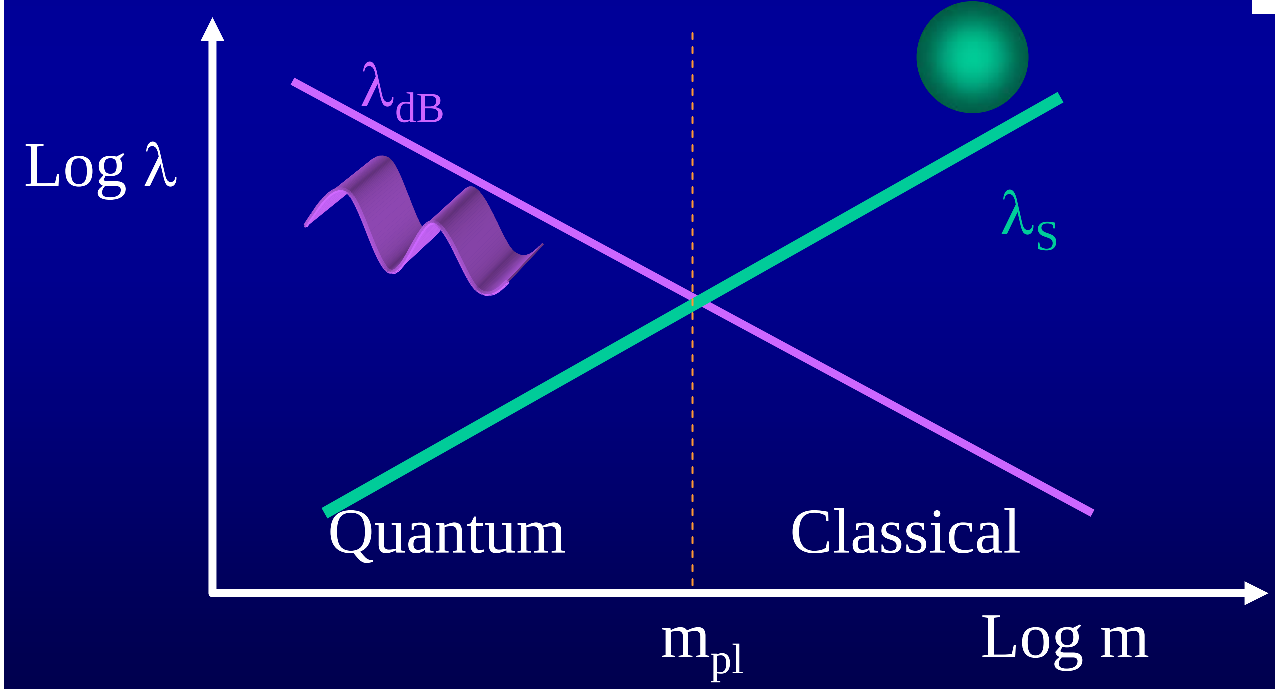
For T(CMB) = 2.73K today.

The thermal history of the Universe

- How far back do we think we can go to in time?
- To the Quantum Gravity Limit:
  - Quantum Mechanics: de Broglie:
  - General Relativity: Schwartzschild:

$$\lambda_{dB} = \frac{2\pi c}{v} = \frac{2\pi\hbar}{mc}$$

$$\lambda_S = \frac{2Gm}{c^2}$$



$$m_{pl} = \sqrt{\frac{\hbar c}{G}} = 10^{19}\, GeV$$

$$\ell_{pl} = \sqrt{\frac{\hbar G}{c^3}} = 10^{-35}\, m$$

$$t_{pl} = \sqrt{\frac{\hbar G}{c^5}} = 10^{-45}\, s$$

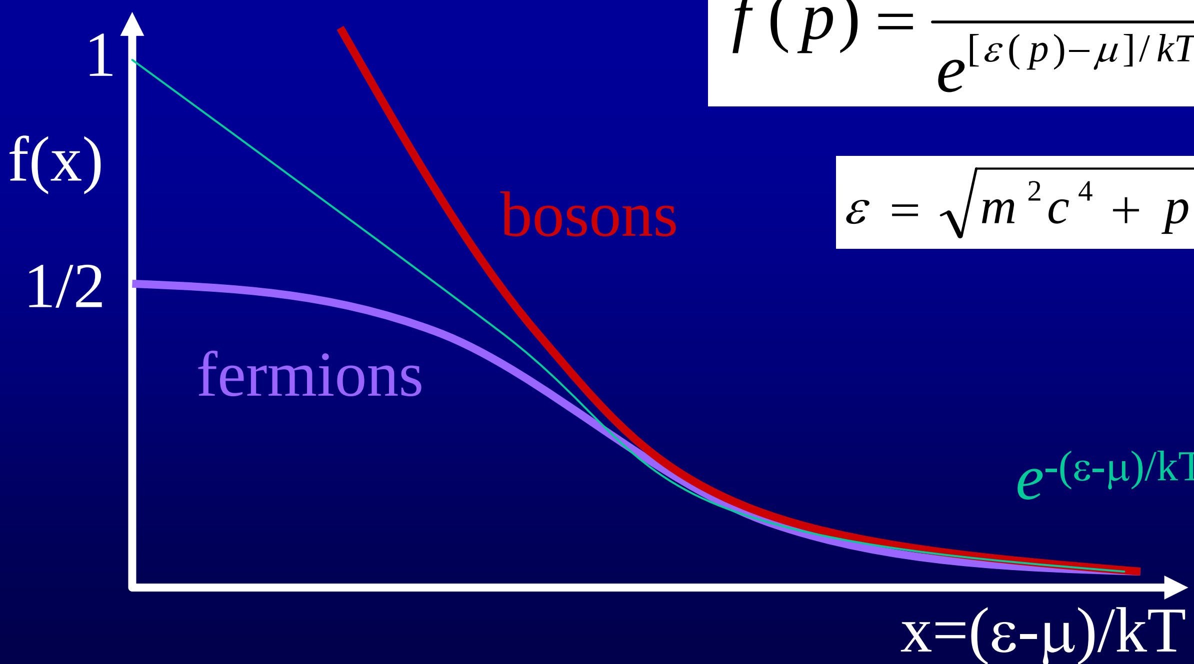
Thermal backgrounds

- If expansion rate < interaction rate we have thermal equilibrium.
- Shall also assume a we have perfect gas.
- Occupation number for relativistic quantum states is:

$$f(p) = \frac{1}{e^{[\varepsilon(p)-\mu]/kT} \pm 1}$$

+ fermions
– bosons

$$\varepsilon = \sqrt{m^2c^4 + p^2c^2}$$



$e^{-(\varepsilon-\mu)/kT}$ – Boltzmann

Thermal backgrounds

- The Chemical Potential, μ:

$$dE = TdS - pdV + \mu dN$$

  - A change of energy when change in number of particles.
  - As in equilibrium, expect total energy does not change:

$$\mu = 0$$

Thermal backgrounds

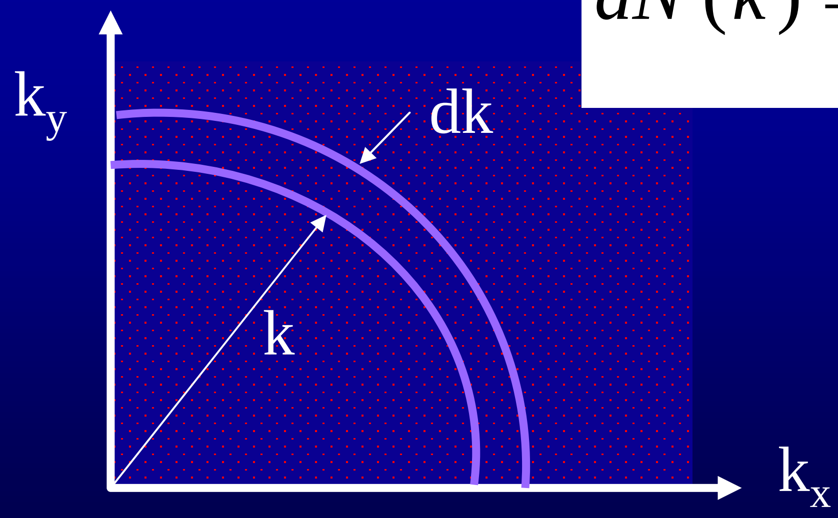
- The particle number density:

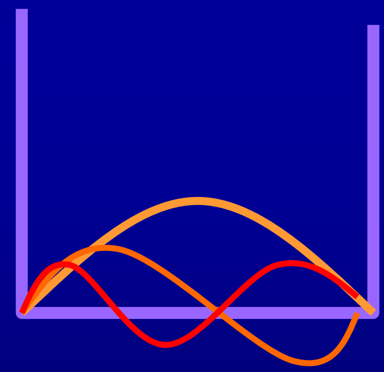
$$n = \frac{1}{V}\int dN\,(p)\,f\,(p)$$

  - N(p) is the density of discrete quantum states in a box of volume V with momentum p:

$$p = \hbar k$$

$$dN(k) = g\,\frac{V}{(2\pi)^3}\,d^3k$$





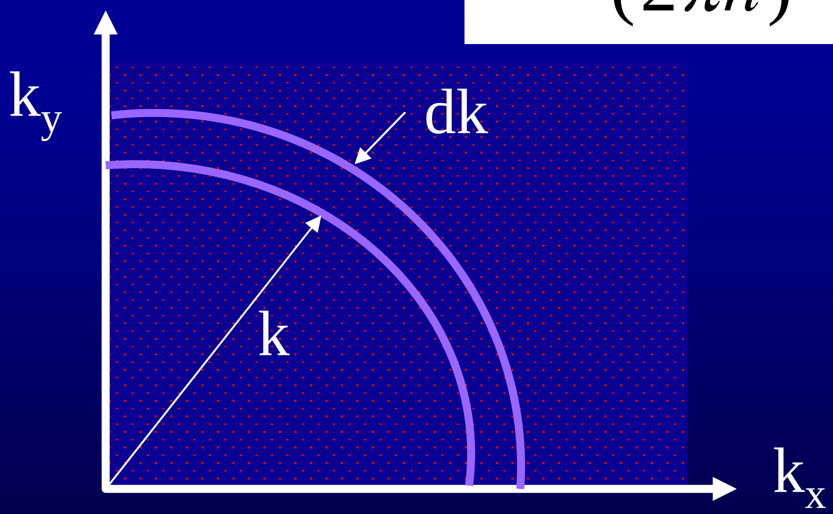
g = degeneracy factor
(eg spin states)

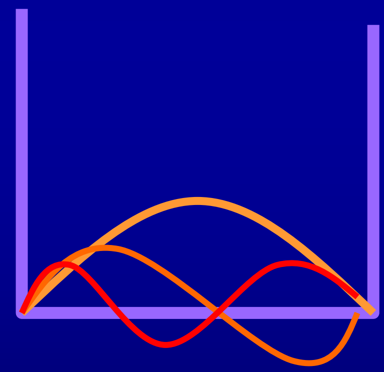
Thermal backgrounds

- The number density of relativistic quantum particles:

$$n = \frac{g}{\hbar^3}\int \frac{d^3p}{(2\pi)^3} f(p)$$

$$= \frac{g}{(2\pi\hbar)^3}\int_0^\infty \frac{4\pi p^2 dp}{e^{\varepsilon(p)/kT} \pm 1}$$



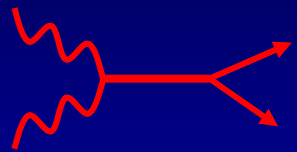


g = degeneracy factor
(eg spin states)

Thermal backgrounds

- The ultra-relativistic limit: p>>mc, $kT>>mc^2$ (bosons)

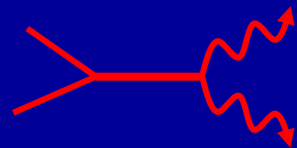
$$n = \frac{g}{(2\pi\hbar)^3}\int_0^\infty \frac{4\pi p^2 dp}{e^{pc/kT}-1} = \frac{\zeta(3)}{\pi^2} g \left(\frac{kT}{c\hbar}\right)^3$$



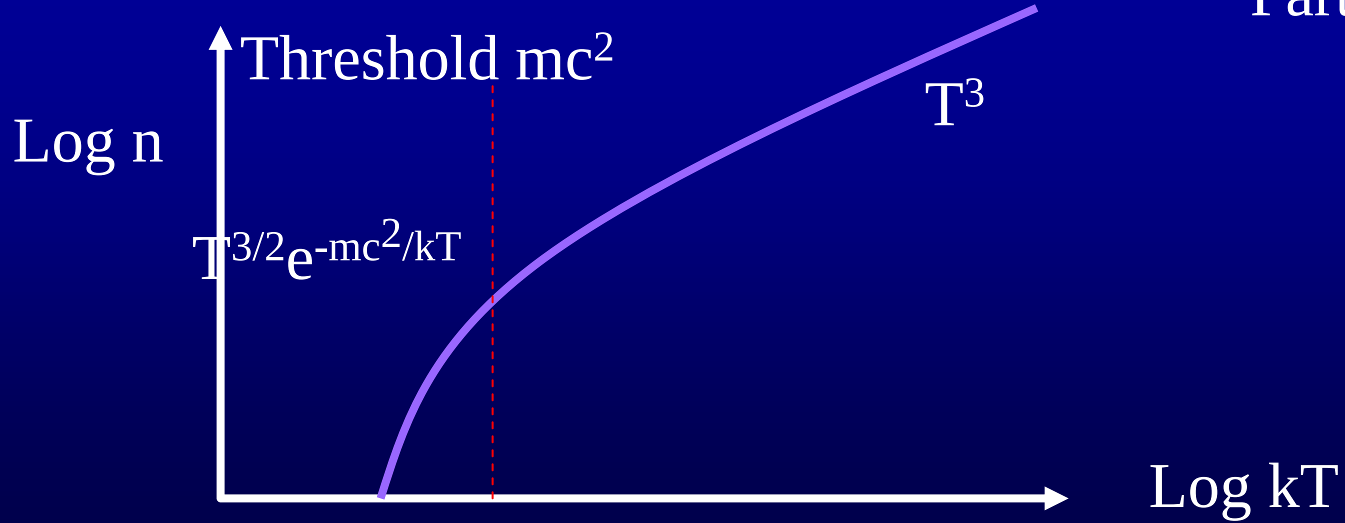
Particle production

- The non-relativistic limit: $kT<<mc^2$ (Boltzmann)

$$n = \frac{g}{(2\pi\hbar)^3}\int_0^\infty 4\pi p^2 dp e^{-(mc^2+p/2c)/kT} = g\left(\frac{mkT}{2\pi\hbar^2}\right)^{3/2} e^{-mc^2/kT}$$

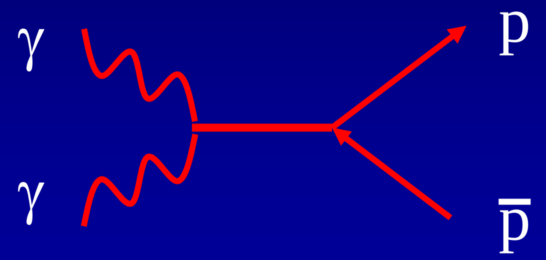


Particle annihilation



Thermal backgrounds

- Proton-antiproton production and annihilation:



- $m_p = 10^3$ MeV so for $T > 10^{13}$ K there is a thermal background of protons and antiprotons.
- But when $T<10^{13}$K annihilation to photons.
- Should annihilate to zero, but in fact $\Delta p/p=10^{-9}$! (or else we wouldn't be here.)
- So there must have been a Matter-Antimatter Asymmetry!!

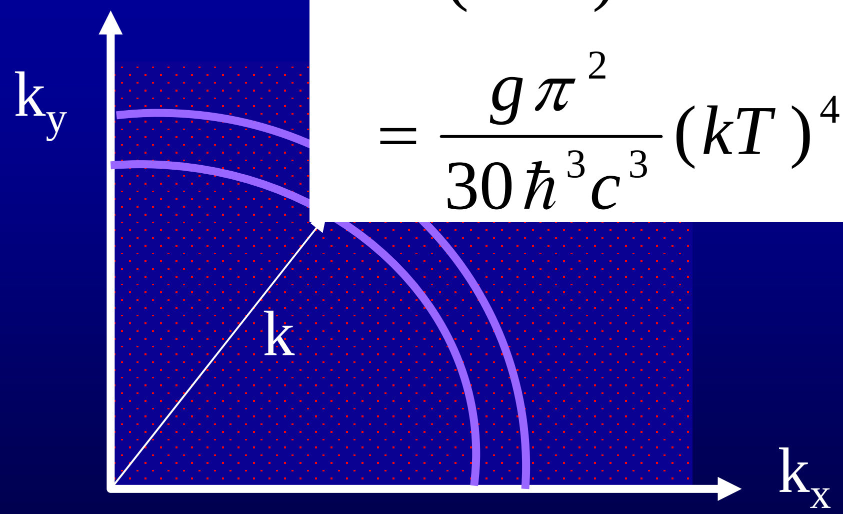
Thermal backgrounds

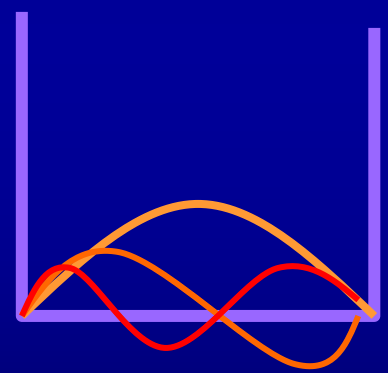
- The energy density of relativistic quantum particles (bosons):

$$u = \rho c^2 = \frac{g}{\hbar^3}\int_0^\infty \frac{d^3 p}{(2\pi)^3} f(p)\varepsilon(p)$$

$$= \frac{g}{(2\pi\hbar)^3}\int_0^\infty \frac{4\pi p^2 dp}{e^{\varepsilon(p)/kT}-1}\varepsilon(p)$$

$$= \frac{g\pi^2}{30\hbar^3 c^3}(kT)^4$$





g = degeneracy factor
(eg spin states)

Thermal backgrounds

- The entropy, S, of relativistic quantum particles:
  - The entropy is an extensive quantity (like E & V):

E_1 V_1 S_1	+	E_2 V_2 S_2	⟶	$E=E_1+E_2$ $V=V_1+V_2$ $S=S_1+S_2$

$$\frac{\partial E}{\partial V} = \frac{E}{V} \qquad \frac{\partial S}{\partial V} = \frac{S}{V}$$

so

$$dE(V,T) = TdS(V,T) - pdV$$

$$\frac{E}{V}dV + \frac{\partial E}{\partial T}dT = \left(T\frac{S}{V}dV + T\frac{\partial S}{\partial T}dT\right) - pdV$$

$$\Rightarrow \qquad \frac{E}{V} = \frac{TS}{V} - p$$

Hence

$$s = \frac{S}{V} = \frac{\rho + p/c^3}{T}$$

Thermal backgrounds

- So in the ultra-relativistic case:

$$n \propto T^3$$

$$u \propto T^4$$

$$p = \frac{1}{3}u$$

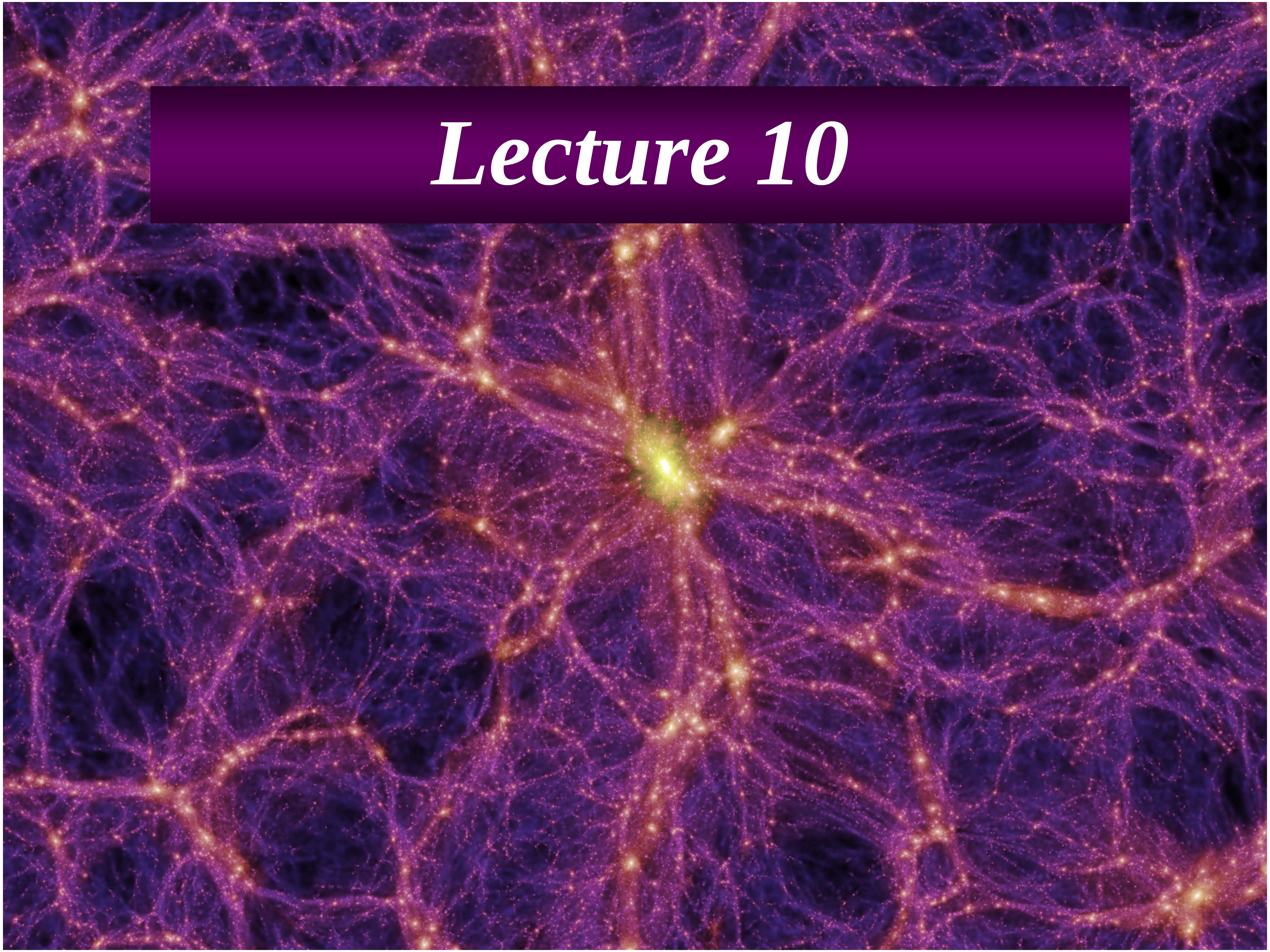
$$\Rightarrow s = \frac{4}{3}\frac{\rho}{T} \propto T^3$$

- So $s \propto n$

- But $\dot{s} = 0$ (entropy is a conserved quantity).

- Usual to quote ratios e.g. baryon density $n_B/s=10^{-9}$.

Lecture 10



Thermal backgrounds

- Given these simple scalings with T for bosons, what is the scaling for n, u and s for fermions when $kT >> mc^2$?

- Formally expand:

$$\frac{1}{e^x+1} = \frac{1}{e^x-1} - \frac{2}{e^{2x}-1}$$

- So occupation numbers:

$$f_F(T) = f_B(T) - 2 f_B(T/2)$$

Thermal backgrounds

- Given these simple scalings with T for bosons, what is the scaling for n, u and s for fermions when kT >> mc^2 ?

- For kT>>mc^2:

$$n_F \propto g_F T^3$$
$$n_B \propto g_B T^3$$

- Number densities:

$$n_F(T) = \frac{g_F}{g_B}\left[n_B(T) - 2n_B(T/2)\right]$$
$$= \frac{g_F}{g_B} n_B(T)\left[1 - \frac{2}{2^3}\right]$$
$$= \frac{3}{4}\frac{g_F}{g_B} n_B(T)$$

Thermal backgrounds

- Given these simple scalings with T for bosons, what is the scaling for n, u and s for fermions when kT >> mc^2 ?

- For kT>>mc^2:

$$u_F \propto g_F T^4$$
$$u_B \propto g_B T^4$$

- Energy densities:

$$u_F(T) = \frac{g_F}{g_B}\left[u_B(T) - 2u_B(T/2)\right]$$
$$= \frac{g_F}{g_B} u_B(T)\left[1 - \frac{2}{2^4}\right]$$
$$= \frac{7}{8}\frac{g_F}{g_B} u_B(T)$$

Thermal backgrounds

- Given these simple scalings with T for bosons, what is the scaling for n, u and s for fermions when kT >> mc^2 ?

- For kT>>mc^2:

$$s = \frac{u+p}{c^2 T}, \qquad u_F \propto g_F T^4, \quad p = u/3$$

$$u_B \propto g_B T^4$$

- Energy densities:

$$s_F(T) = \frac{g_F}{g_B c^2 T}\left[u_B(T)(1+1/3) - 2u_B(T/2)(1+1/3)\right]$$

$$= \frac{g_F}{g_B}\frac{u_B(T)}{c^2 T}\frac{4}{3}\left[1 - \frac{2}{2^4}\right]$$

$$= \frac{7}{8}\frac{g_F}{g_B} s_B(T)$$

Thermal backgrounds

- Given these simple scalings with T for bosons, what is the scaling for n, u and s for fermions when kT >> mc^2 ?

$$n_F(T) = \frac{3}{4}\frac{g_F}{g_B} n_B(T), \quad u_F(T) = \frac{7}{8}\frac{g_F}{g_B} u_B(T), \quad s_F(T) = \frac{7}{8}\frac{g_F}{g_B} s_B(T)$$

- Define an effective number of relativistic particles:

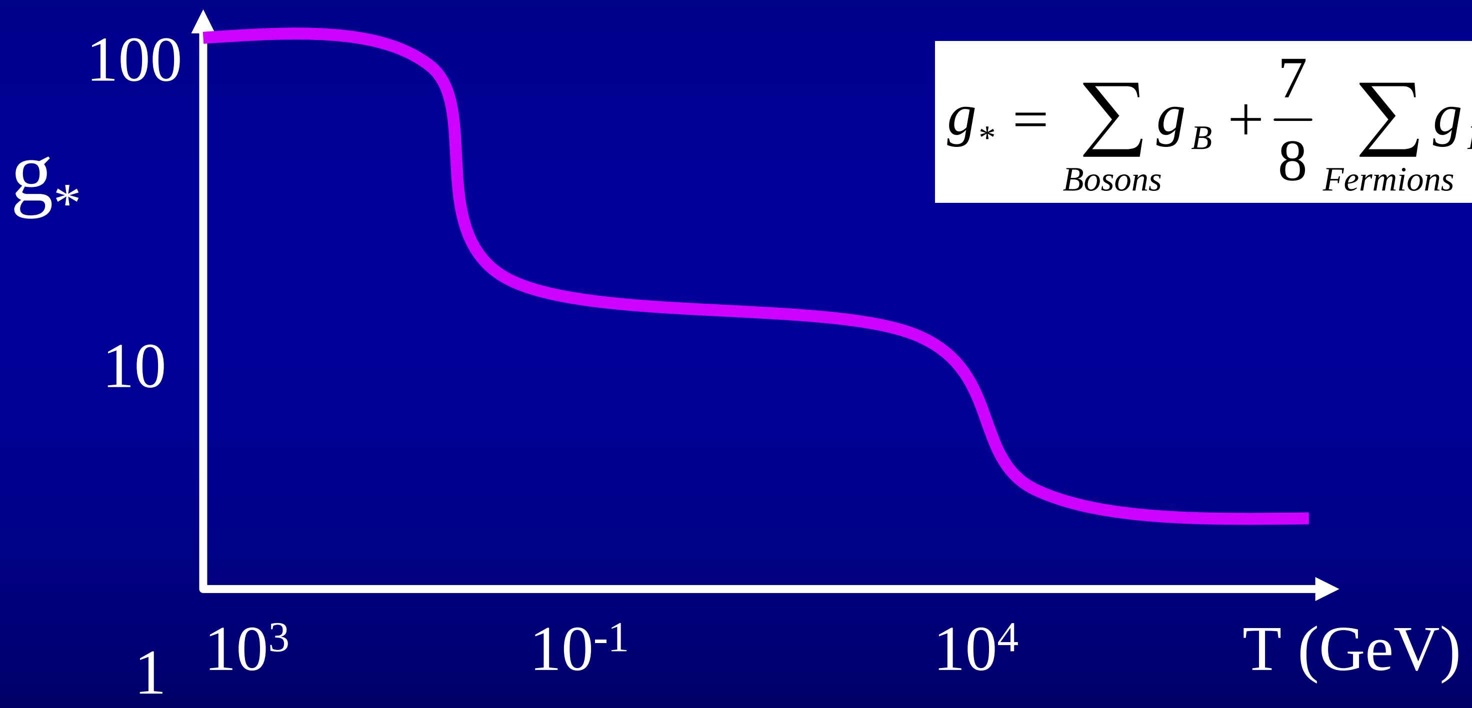
$$g_* = \sum_{Bosons} g_B + \frac{7}{8} \sum_{Fermions} g_F$$

- So energy of all relativistic particles is:

$$u_{Total} = \frac{g_* \pi}{30 (\hbar c)^3} (kT)^4$$

Thermal backgrounds

- The effective number of relativistic particles will change with time as $kT<mc^2$ and particles become non-relativistic.



$$g_* = \sum_{Bosons} g_B + \frac{7}{8} \sum_{Fermions} g_F$$

- For high-T $g_*=100$. If supersymmetric, $g_*=200$.

Time and Temperature

- At early times radiation and matter are strongly coupled and thermalized to temperature, T, of radiation.
- Recall: $t = \sqrt{\frac{3}{32\pi G\rho_r}}$ in a radiation-dominated universe,

and: $\rho_r \propto g_* T^4, \text{ so } t \propto g_*^{-1/2} T^{-2}$

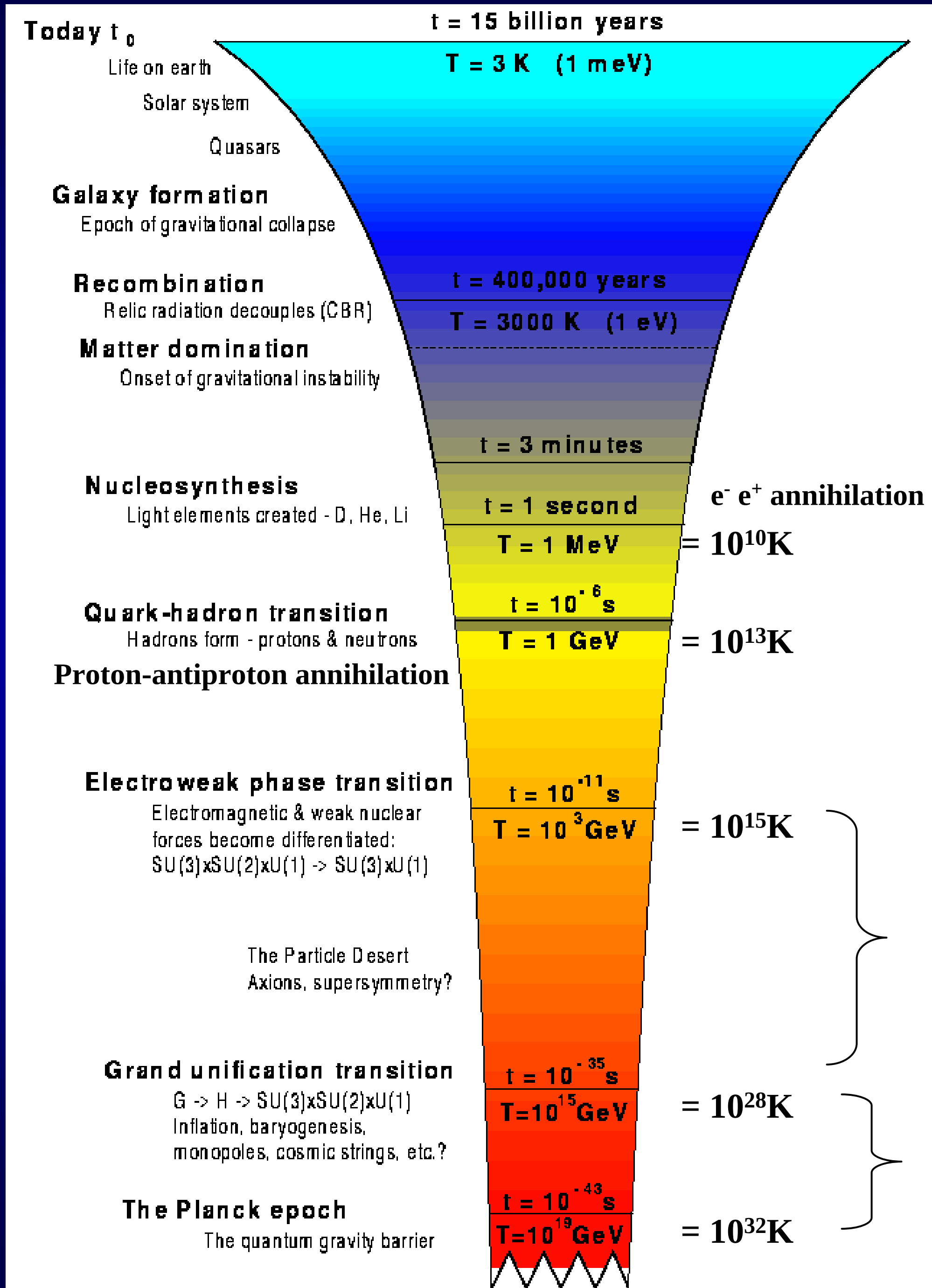
- Hence:

$$t = g_*^{-1/2}\left(\frac{T}{1.8\times10^{10}K}\right)^{-2} \text{seconds}$$

- Note also that T=2.73K(1+z), so z~T/(1 K).

A Thermal History of the Universe

- With time now related to temperature, and hence energy, we can map out the thermal history of the Universe.



- $z \sim T/(1K)$
- $E = kT$
  $\sim T/(3 \times 10^3)$eV

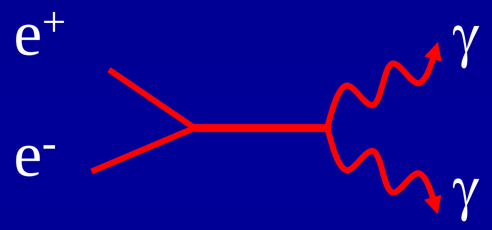
Dark Matter formed?

Inflation?

Freeze-out and Relic Particles

- Electrons - Positrons annihilation:

•For first 3 seconds we have $e^- e^+ \leftrightarrow \gamma\gamma$



- Then T drops and energy in γ becomes too low, so electrons and positrons annihilate.

- Stops when annihilation rate drops below expansion rate.

Freeze-out and Relic Particles

- Need the Boltzmann Equation to describe reactions:

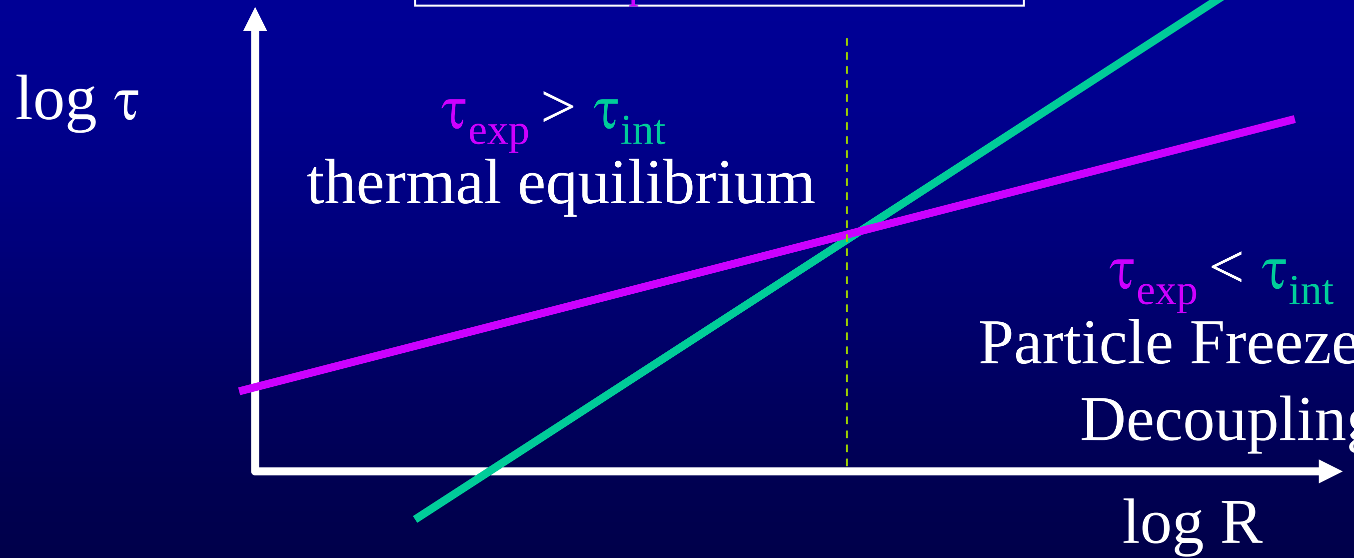
$$\dot{n} + 3Hn = -\langle \sigma v \rangle n^2$$

Rate of change

Dilution by expansion
$\tau_{exp} \sim 1/H \sim R^2$

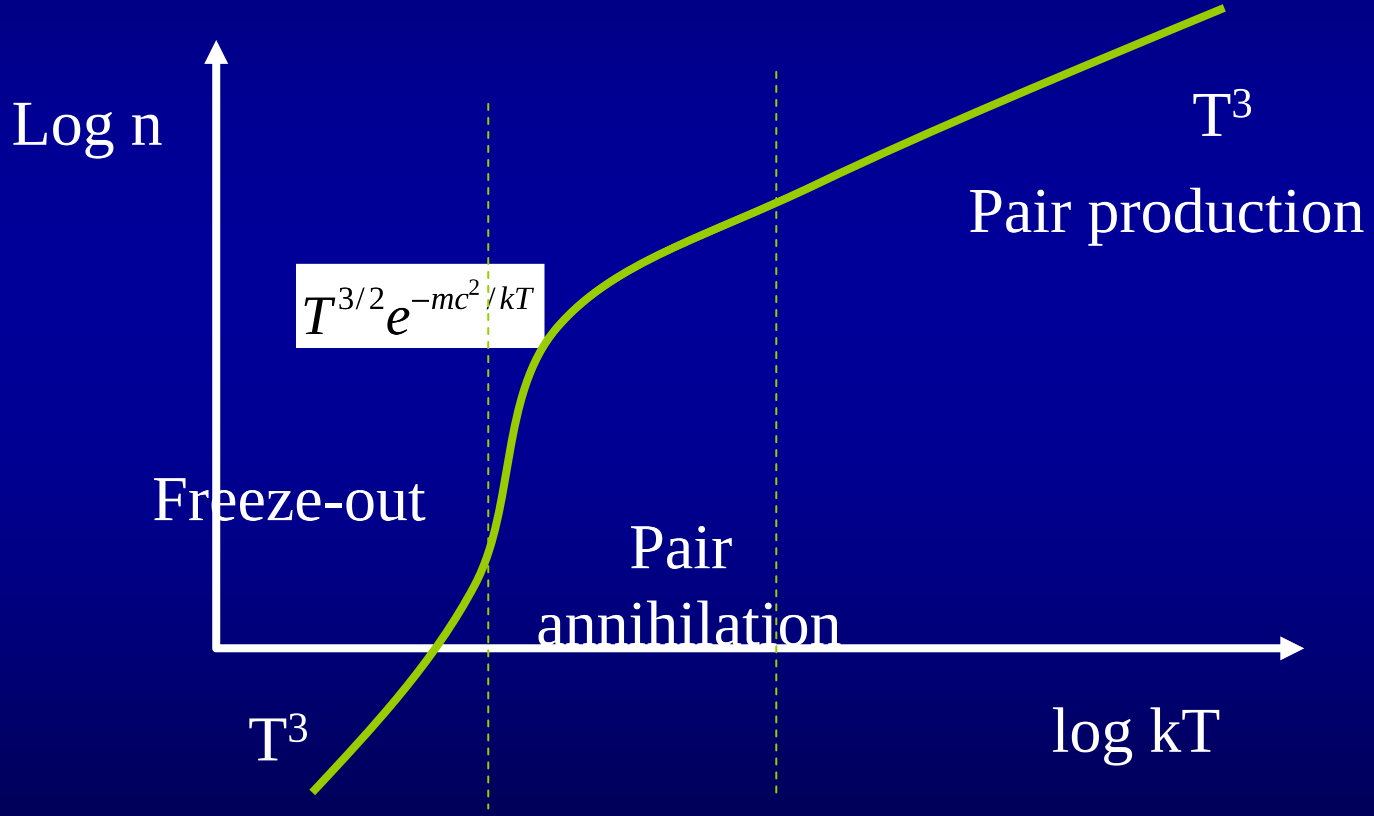
Loss due to annhilation
$\tau_{int} \sim 1/(\langle \sigma v \rangle n) \sim R^3$

- Timescales:



Freeze-out and Relic Particles

- Creation, annihilation and freeze-out of particle relics:



Freeze-out and Relic Particles

- Electrons-Positrons annihilation and neutrino decoupling:

- What happens to the energy released by $e^- e^+ \rightarrow \gamma\gamma$?

- At early times only have photons, neutrinos and $e^- e^+$ pairs in equilibrium.

- At T = 5x10^9K (3 seconds) e-e+ pairs annihilate.

$$e^- e^+ \rightarrow \gamma\gamma$$
$$\not\rightarrow \nu\bar{\nu}$$

As weak force decoupled at T=10^{10}K.

Freeze-out and Relic Particles

- So radiation is boosted above neutrino temperature by neutrino decay.

- Before $n_\nu \sim n_\gamma$, after $n_\nu < n_\gamma$ and $T_\nu < T_\gamma$.

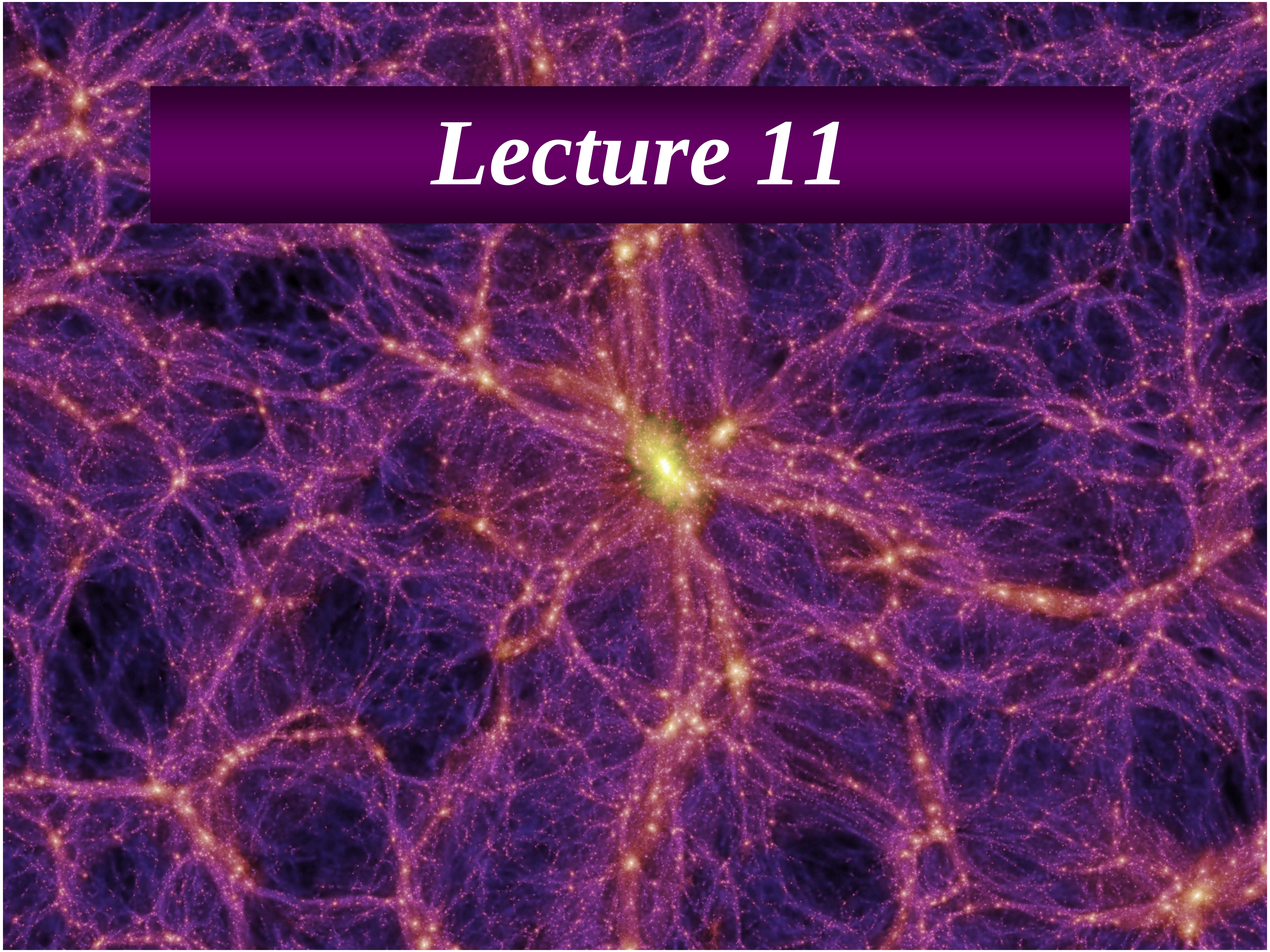
- But recall entropy is conserved $\dot{s} = 0$

$$\Rightarrow s \propto g_* T^3$$

where

$$g_* = \sum_{Bosons} g_B + \frac{7}{8} \sum_{Fermions} g_F$$

Lecture 11



Freeze-out and Relic Particles

- How much is photon temperature boosted ?

- Entropy:

  $g_e=2$

  $g_\gamma=2$

$$\Rightarrow s \propto g_* T^3$$

$$\begin{aligned} s_{after}(\gamma) &= s_{before}(\gamma + e^+ + e^-) \\ &= \left(1 + \frac{7}{8}\left[\frac{g_{e^+} + g_{e^-}}{g_\gamma}\right]\right) s_{before}(\gamma) \\ &= \frac{11}{4} s_{before}(\nu) \end{aligned}$$

- Neutrino Temperature:

$$T_\nu = \left(\frac{4}{11}\right)^{1/3} T_\gamma (= 2/73K) = 1.95K$$

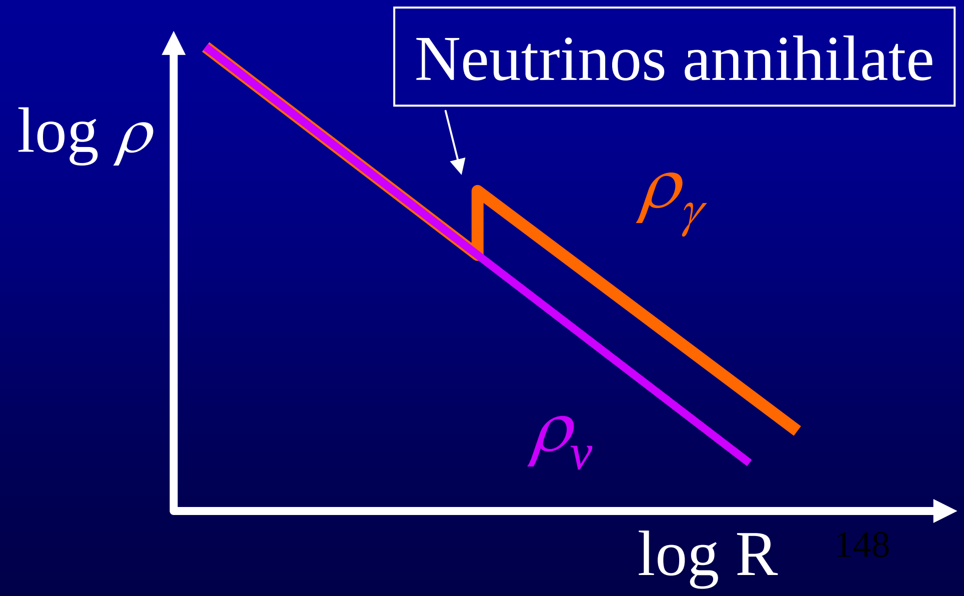
Freeze-out and Relic Particles

- How much is radiation energy boosted ?
- Neutrino energy:

$$u_\nu = \frac{7}{8}\left(\frac{4}{11}\right)^{4/3} u_\gamma = 0.227\ u_\gamma$$

- Enhances ρ_r by factor of 1.68 for 3 neutrino species:

$$\rho_r = (u_\gamma + 3u_\nu)c^2$$
$$= 1.68\ \rho_\gamma$$



Relic massive neutrinos

- Can we put cosmological constraints on the mass of neutrinos ?

- 1960's: Particle physics models with $m_\nu = 0$.

- 1970's: Particle physics models with massive neutrinos.

- 1990's: Non-zero mass detected (Superkamiokande,
  and Sudbury Neutrino Observatory (SNO)
  confirms Solar model).

Relic massive neutrinos

- Can we put cosmological constraints on the mass of neutrinos ?

- Number-density of cosmological neutrinos:

$$n_F(\nu + \bar{\nu}) = \tfrac{3}{4} n_B(\gamma \,|\, T_\gamma = 1.95K)$$
$$= 113 \quad \nu / cm^3 / species$$

- Mass-density:

$$\rho_\nu = m_\nu n_\nu = \frac{3\Omega_\nu H^2}{8\pi G}$$

Relic massive neutrinos

- Can we put cosmological constraints on the mass of neutrinos ?

- Density-parameter of cosmological neutrinos:

$$\Omega_\nu = \frac{1}{93.5 h^2 eV} \sum_{i=1}^{N_\nu} m_{\nu_i}$$

- Re-arrange:

$$\overline{m}_\nu = \frac{1}{3} \sum_{i=1}^{N_\nu} m_{\nu_i} \leq 4.6 eV \left(\frac{N_\nu}{3} \right)^{-1} \left(\frac{\Omega_\nu}{0.3} \right) \left(\frac{h}{0.7} \right)^2$$

- Compare with lab: $m_{\nu e}$<15eV, $m_{\nu\mu}$<0.17MeV, $m_{\nu\tau}$<24MeV.

$\Delta m^2 = m_i^2 - m_j^2 = [7\text{x}10^{-3}\ \text{eV}]^2$.

Big-Bang Nucleosynthesis (BBN)

- As R tends to zero and T increases, eventually reach nuclear burning temperatures.
- 1940's: George Gamow suggests nuclear reactions in early Universe led to Helium.
  - Prediction of a radiation background (CMB)
  - Predicted 25% helium by mass, as found in stars.
- 1960's: Details worked out by Hoyle, Burbidge and Fowler.
- First need free protons and neutrons to form.

Big-Bang Nucleosynthesis (BBN)

- So first need proton & neutron freeze-out.

- Recall:
  - Below $T<10^{13}$K ($M_p \sim M_n \sim 10^3$ MeV):

$$p\bar{p} \rightarrow \gamma\gamma$$
$$n\bar{n} \rightarrow \gamma\gamma$$

- Annihilation leaves a residual $\Delta p/p \sim \Delta n/n \sim 10^{-9}$.

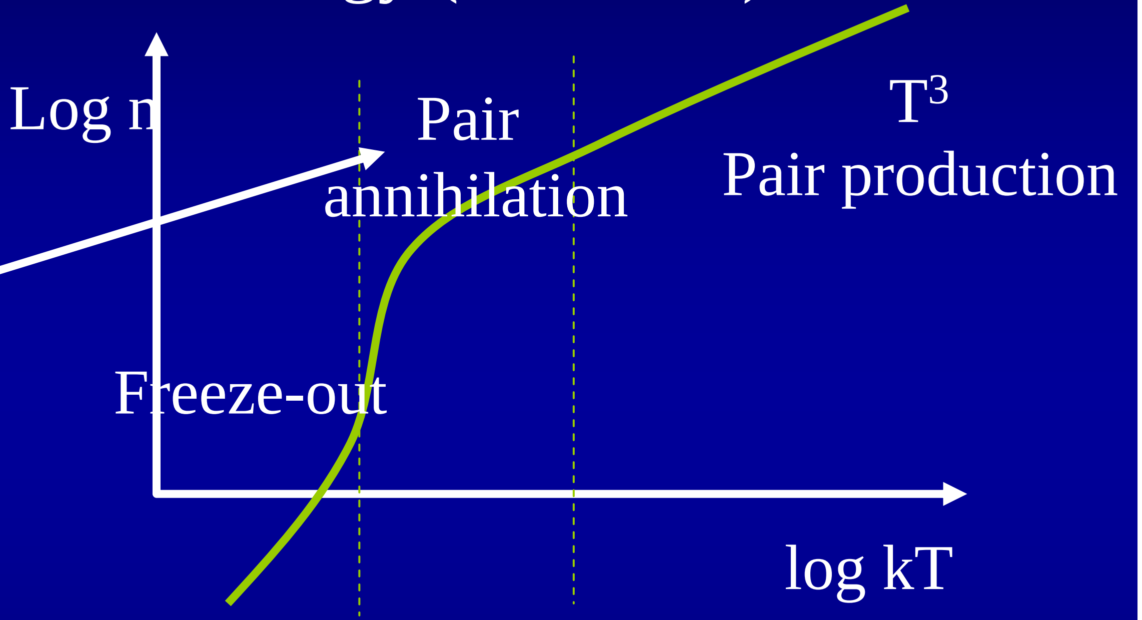
- Protons and neutrons undergo Weak Interactions:

$$p + e^- \leftrightarrow n + \nu$$
$$n + e^+ \leftrightarrow p + \bar{\nu}$$

Big-Bang Nucleosynthesis (BBN)

- Assume equilibrium and low energy ($kT \ll mc^2$) limit:

$$n \propto T^{3/2} e^{-mc^2/kT}$$



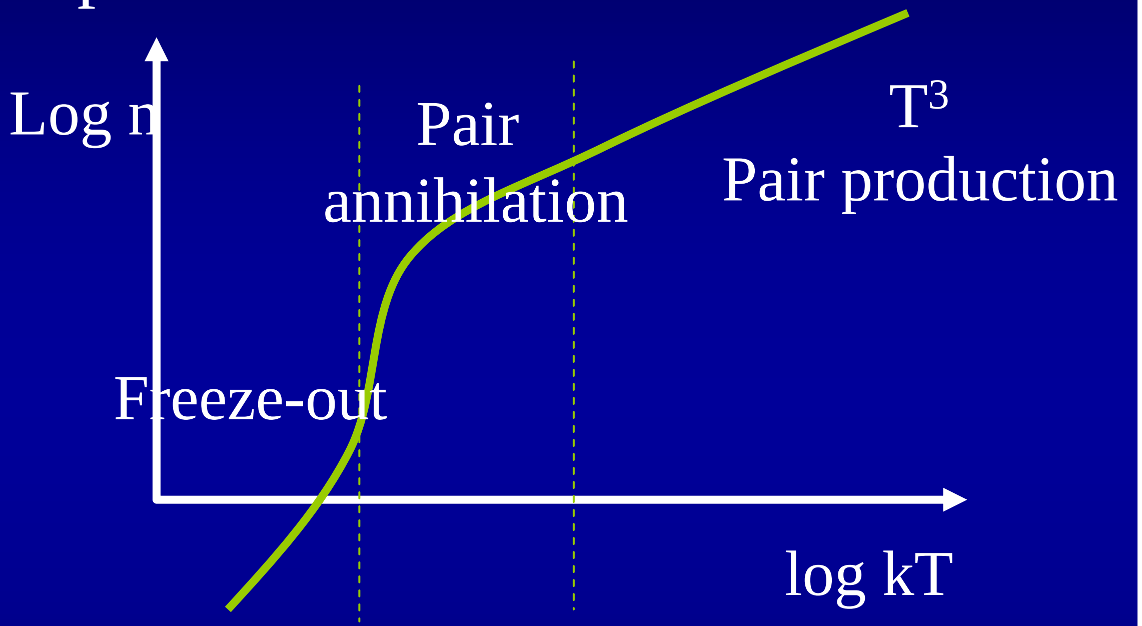
- Ratio of neutrons to protons at temp T ($\Delta m = m_n - m_p = 1.3$MeV):

$$\frac{n_n}{n_p} = e^{-\Delta mc^2/kT} \approx e^{-1.5\times10^{10}\,K/T}$$

Big-Bang Nucleosynthesis (BBN)

- Annihilations stop when p & n freeze-out occurs:

Reaction time	>	expansion time
$\tau_{int} \sim 1/\sigma v n$		$\tau_{exp} \sim 1/H$



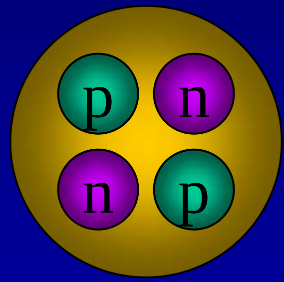
- So ratio is frozen in at $T_{freezeout}$

$$\frac{n_n}{n_p} = e^{-\Delta mc^2 / kT_{freezeout}} \approx e^{-1.5\times10^{10} K / T_{freezeout}}$$

Big-Bang Nucleosynthesis (BBN)

- What is the observed neutron-proton ratio?

- Most He is in the form of ^{4}He:



- He fraction by mass is: $$Y = \frac{(4 \times n_n / 2)m}{(n_n + n_p)m} = \frac{2}{1 + n_p / n_n}$$

- Observe Y=0.25 for stars.

- So $n_p/n_n = 2/Y - 1 = 7$, or: $$\frac{n_n}{n_p} \approx \frac{1}{7} \approx 0.14$$

Big-Bang Nucleosynthesis (BBN)

- At what time, then, does neutron freeze-out happen?

- Need to know weak interation rates: <σv>$_{weak}$
- This was calculated by Enrico Fermi in 1930's.
- Find

$$T_{freezeout}(n) \approx 1.4\times10^{10}\,K$$

- So expected neutron-proton ratio is:

$$\frac{n_n}{n_p} \approx e^{-1.5\times10^{10}\,K/T_{freezeout}} \approx 0.34$$

Big-Bang Nucleosynthesis (BBN)

- Expect $n_n/n_p=0.34$.
- But we said $\frac{n_n}{n_p} = 0.14$ from observed stellar abundances.
- Close, but a bit big.
- But:
  1. We have assumed $kT_{freezeout} >> m_e c^2$ but really $kT_{freezeout} \sim m_e c^2$
  2. Neutrons decay. $\tau_n = 887 +/- 2$ seconds for free neutrons. Need to be locked away in a few seconds:

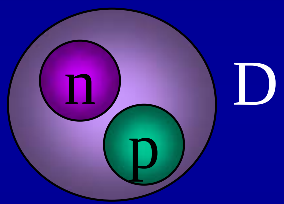
$$\frac{n_n}{n_p} \approx 0.34 \rightarrow 0.14 \quad \Rightarrow \quad Y = 0.25$$

Big-Bang Nucleosynthesis (BBN)

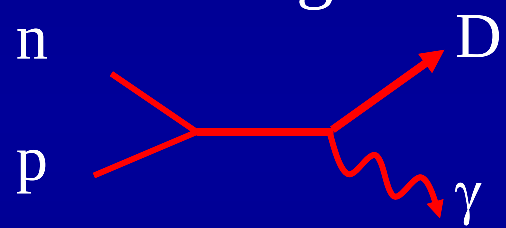
- The onset of Nuclear Reactions.

- At the same time nuclear reactions become important.

•Neutrons get locked up in Deuteron via the strong interaction



$$n + p \rightarrow D + \gamma$$



- Happens at deuteron binding energy kT~2.2MeV

- Dominant when T(D formation)=8×10^{8}K, or at a time t=3 minutes.

Big-Bang Nucleosynthesis (BBN)

- The formation of Helium.
- ^{4}He is preferred over H or D on thermodynamic grounds.
  Binding energies: E(He)= 7 MeV
  E(D) = 1.1 MeV
- After Deuteron forms:

$$D + D \rightarrow {}^3He + n$$
$$D + D \rightarrow T + p$$

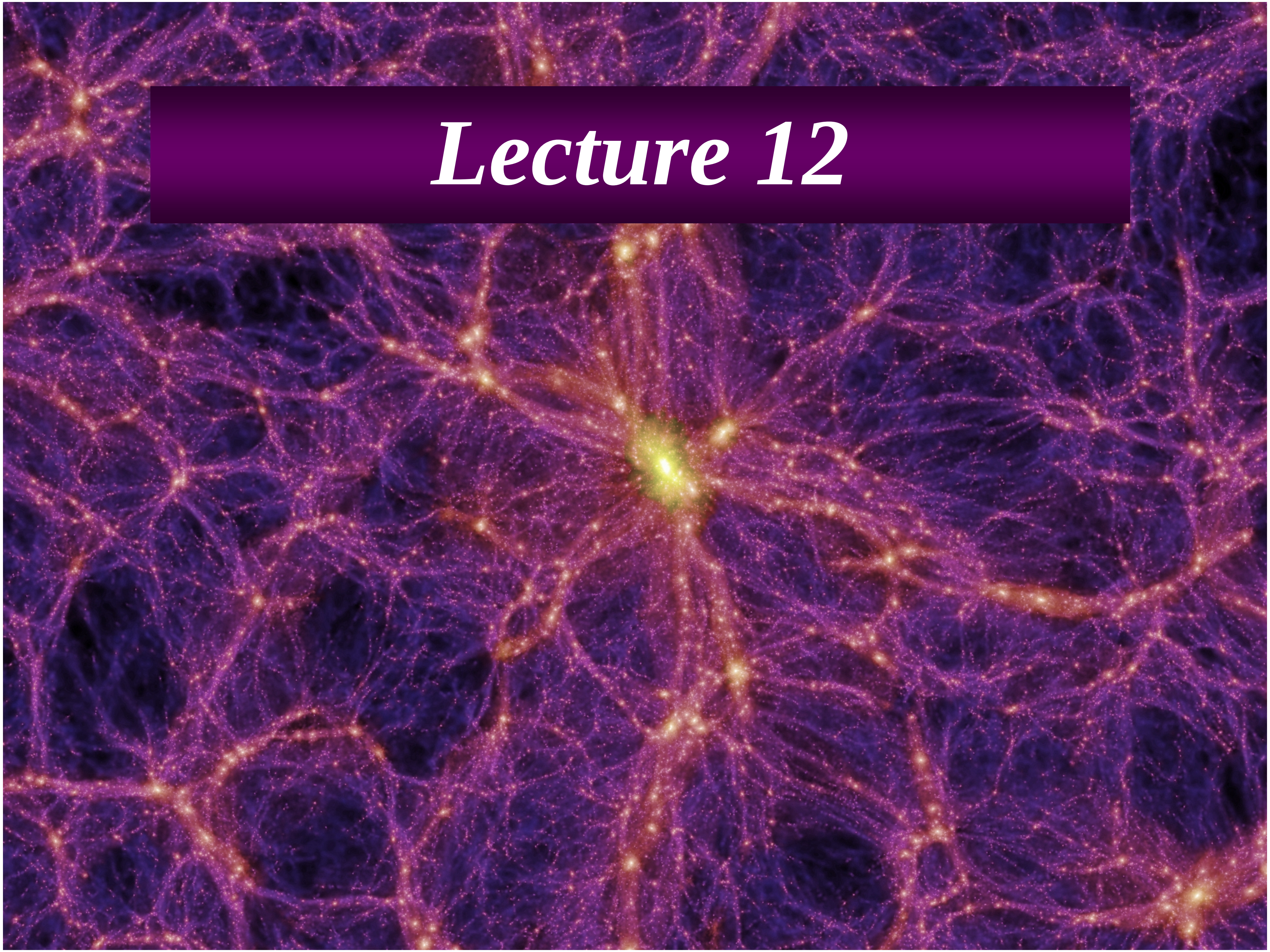
- Then

$$D + D \rightarrow {}^4He$$
$$T + p \rightarrow {}^4He$$
$$D + {}^3He \rightarrow {}^4He + p$$
$$T + D \rightarrow {}^4He + n$$

T gets too low and reactions stop at Li & Be.

BBN starts at 10^{10}K, t=1s. Ends at 10^9K, t=3mins.

Lecture 12



Big-Bang Nucleosynthesis (BBN)

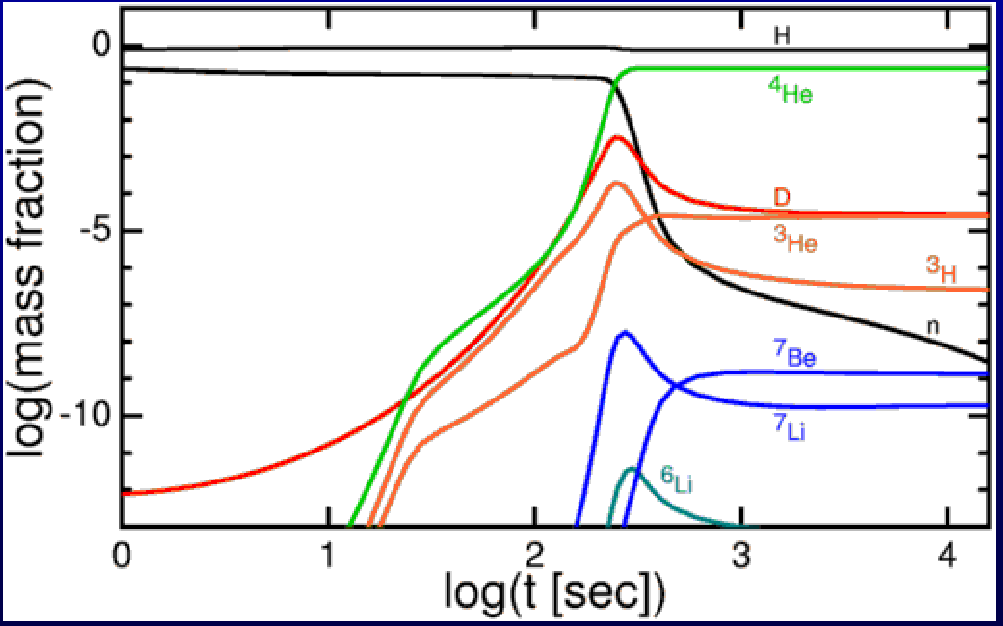
- Summary of BBN:

- $T=10^{13}$K, t~0.1s: Neutron & proton annihilation ($\Delta p/p \sim \Delta n/n \sim 10^{-9}$).

- $T=10^{10}$K, t=1s: Neutron freeze-out (n_n/n_p=0.34.)
  Neutrons decay (n_n/n_p=0.14.)
  Nuclear reactions start.

$$n + p \rightarrow D + \gamma$$

- $T=10^{9}$K, t=3mins:
  Formation of Helium.
  Peak of D formation.
  End of nuclear reactions
  H, D, ^{3}He, ^{4}He, ^{7}Li, ^{7}Be



Big-Bang Nucleosynthesis (BBN)

- The number of neutrino generations:
- The ratio of neutrons to protons is:

$$\frac{n_n}{n_p} = f_{Decay} e^{-\Delta mc^2 / kT_{Freezeout}} \approx 0.163(\Omega_B h^2)^{0.04} \left(\frac{N_\nu}{3} \right)^{0.2} \approx 0.14$$

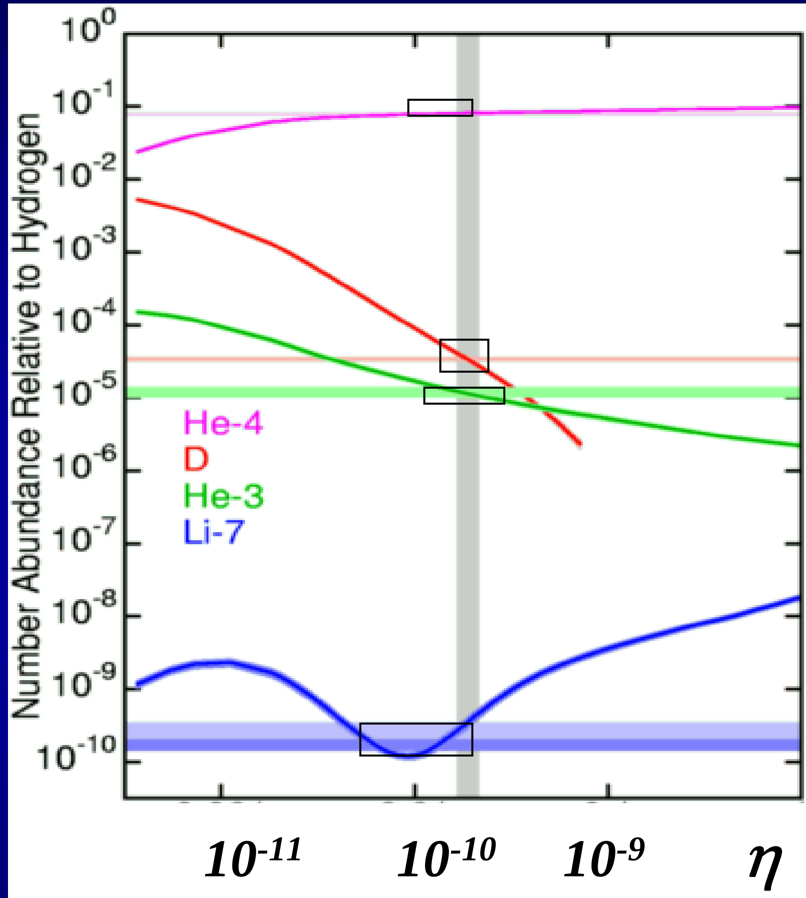
- Depends weakly on ρ_B: higher baryon density means closer packed, so n locked up in nucleons (D) faster.
- Depends on N_ν: More neutrinos, more ρ_r (g_*), so Hubble rate increases, neutron freezeout happens sooner, so more n.
- Cosmological constraint that $N_\nu<4$.
- In 1990's LEP at CERN sets $N_\nu=3$.

Big-Bang Nucleosynthesis (BBN)

- Testing BBN:
- The abundance of elements is sensitive to density of baryons:

- η is number density of baryons per unit entropy.

$$\eta = \frac{n_n + n_p}{n_\gamma} \approx 3 \times 10^{-10}$$



- This gives us the matter-antimatter difference of $\Delta p/p \sim 10^{-9}$

Agreement between BBN theory and observation is a spectacular confirmation of the Big-Bang model !

Big-Bang Nucleosynthesis (BBN)

- Using BBN to weigh the baryons:

- The abundance of elements is sensitive to the density of baryons.
- The photon density scales as T^{-3}, so:

$$\eta = \frac{n_n + n_p}{n_\gamma} = 2.74\times10^{-8}(\Omega_B h^2)\left(\frac{T}{2.73K}\right)^{-3} \approx 3\times10^{-10}$$

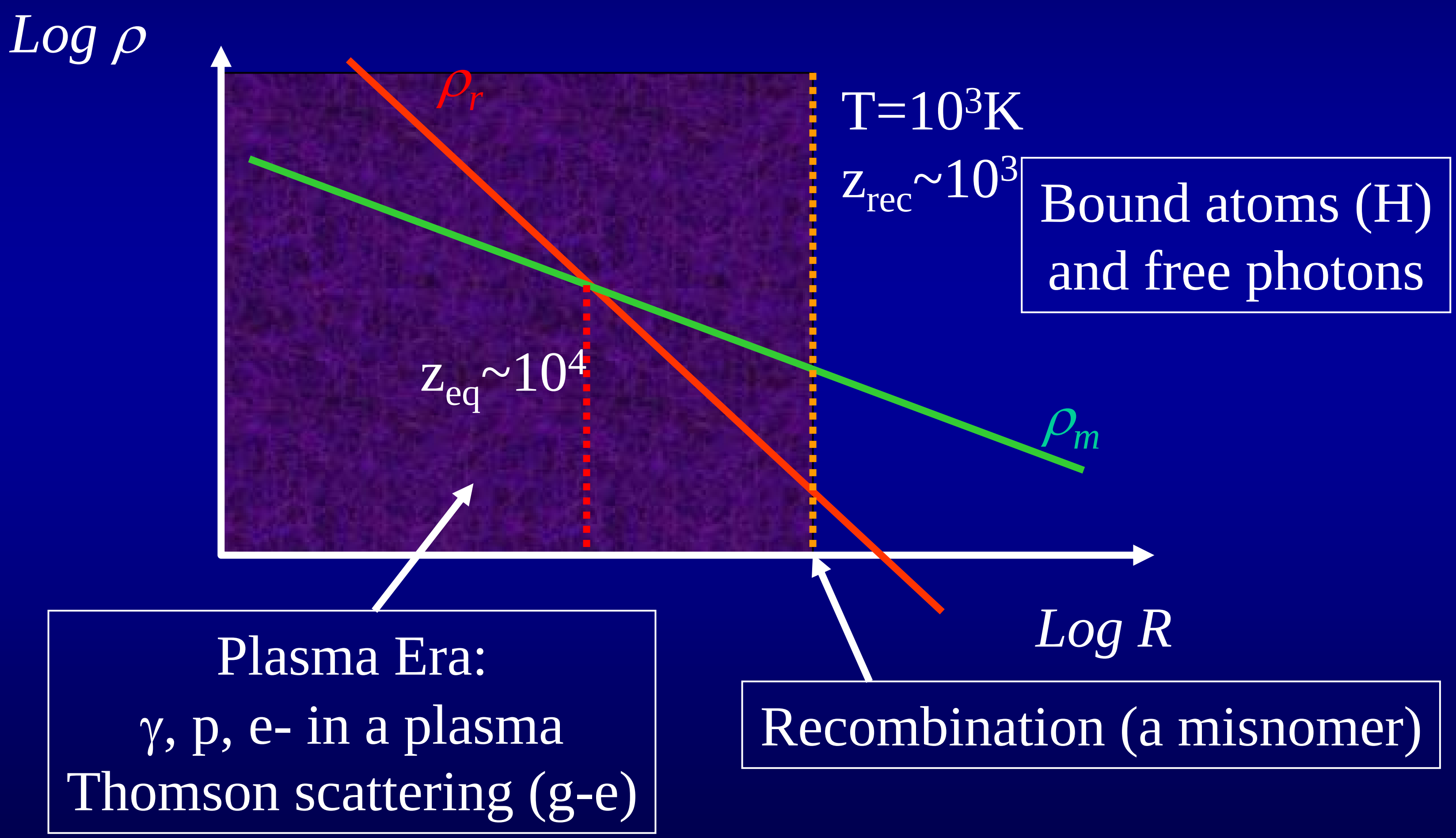
- This yields $\Omega_B h^2 = 0.02 \pm 0.002 \quad \Rightarrow \quad \Omega_B = 0.04\left(\frac{h}{0.7}\right)^{-2}$

- But: $\Omega_m \approx 0.3 >> \Omega_B$

So most of the matter in the Universe cannot be made of Baryons !

Recombination of the Universe

- Energy-density of the Universe:



Recombination of the Universe

- Ionization of a plasma:
- Assume thermal equilibrium.
- Use Saha equation for ionization fraction, *x*:

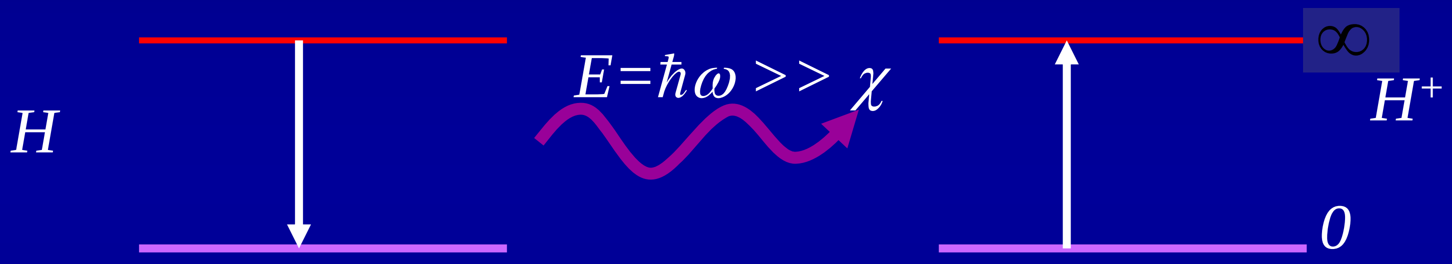
$$x = \frac{H^+}{H^+ + H}$$

$$\frac{x^2}{1 - x} = \frac{(2\pi mkT)^{3/2}}{n(2\pi\hbar)^3} e^{-\chi/kT}$$

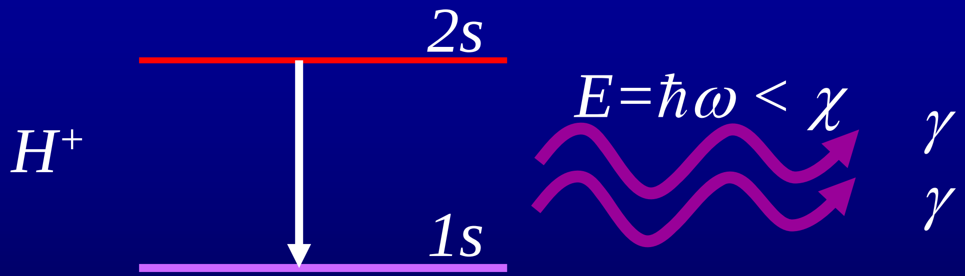
$\chi = 13.6$ eV – H binding energy.

Recombination of the Universe

- Ionization of a plasma:
- But equilibrium rapidly ceases to be valid. Interactions are too fast, and photons cannot escape.



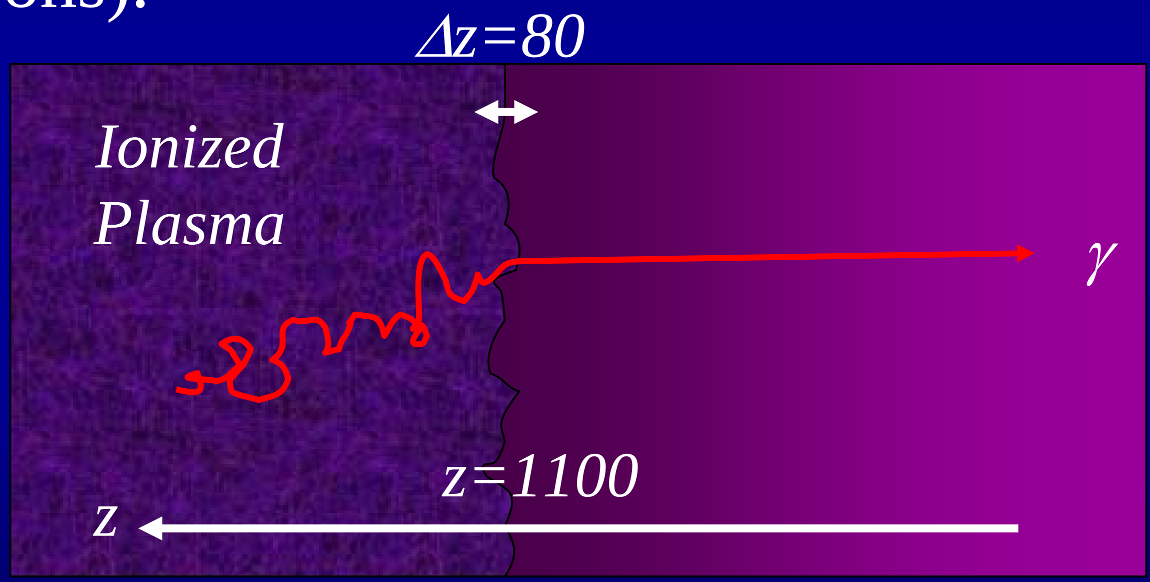
- Escape bottleneck with 2-photon emission:



- This means the ionization fraction is higher than predicted by the Saha equation.

Recombination of the Universe

- The surface of last scattering:
- In plasma photons random walk (Thomson scattering off electrons).



- After recombination photons travel freely and atoms form.
- The last scattering surface forms a photosphere (like sun), The Cosmic Microwave Background.

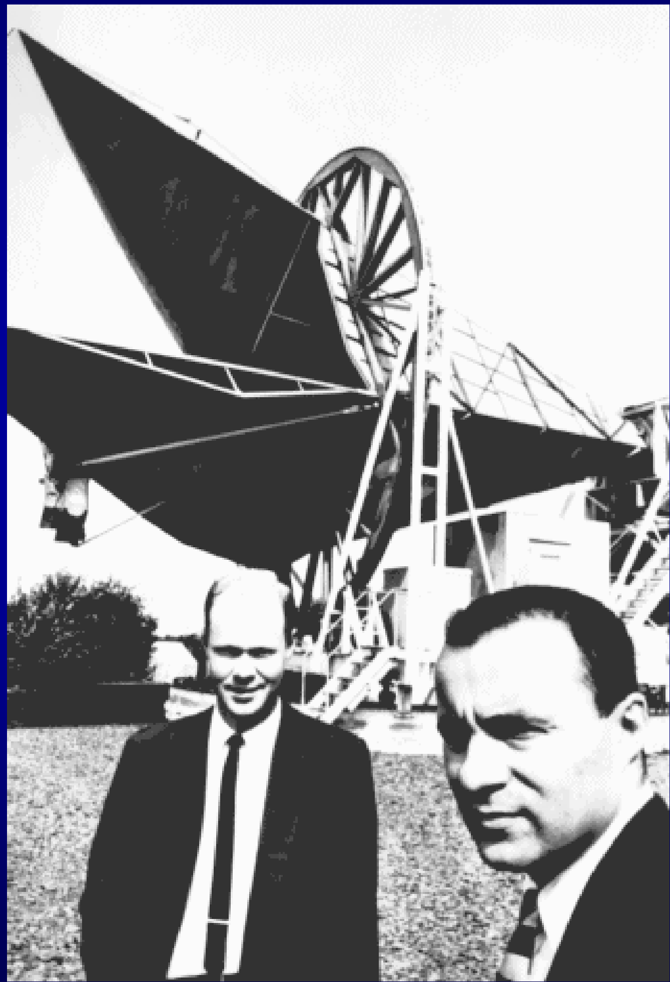
The Cosmic Microwave Background

- The CMB spectrum:

- The CMB was discovered by accident in 1965 by Arno Penzias and Bob Wilson, two researchers at Bell Labs, New Jersey.

- This confirmed the Big-Bang model, and ruled out the competing Steady-State model of Hoyle.

- They received the 1978 Nobel Prize for Physics.

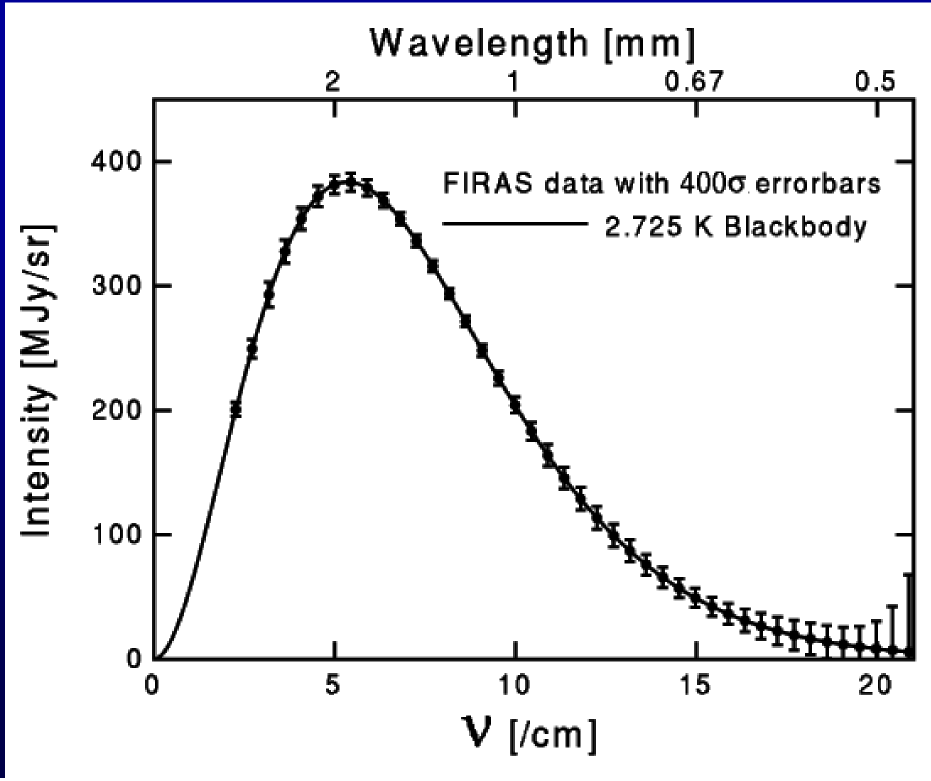


The Cosmic Microwave Background

- The CMB spectrum:

- The Big Bang model predicts a thermal black-body spectrum (thermalized early on and adiabatic expansion).

- The observed CMB is an almost perfect BB spectrum:



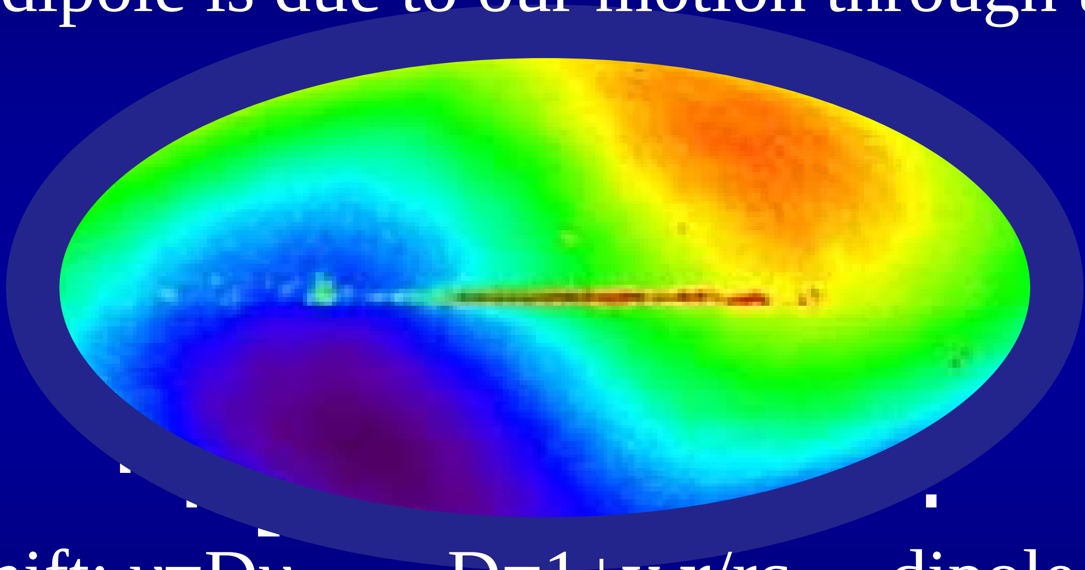
$$T = 2.725 \pm 0.002 K$$

- Accuracy limited by reference BB source.

- CMB contributes to 1% of TV noise.

The Cosmic Microwave Background

- The CMB dipole:
- The CMB dipole is due to our motion through universe.



- Doppler Shift: $\nu = D\nu_0$, $D = 1 + \mathbf{v.r}/rc$ - dipole.

$$n_\gamma(\nu) = \left(e^{h\nu/kT} - 1\right)^{-1} \rightarrow \left(e^{h\nu/DkT_0} - 1\right)^{-1}$$

- Same as temperature shift: $T = (1 + \mathbf{v.r}/rc)T_0$

The Cosmic Microwave Background

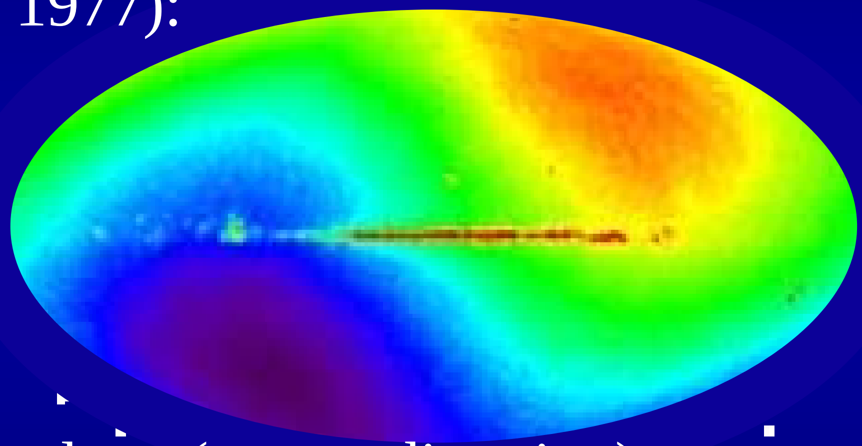
- The CMB dipole:

- This gives us the absolute motion of the Earth (measured by George Smoot in 1977):

  V_{Earth} = 371+/- 1 km/s,

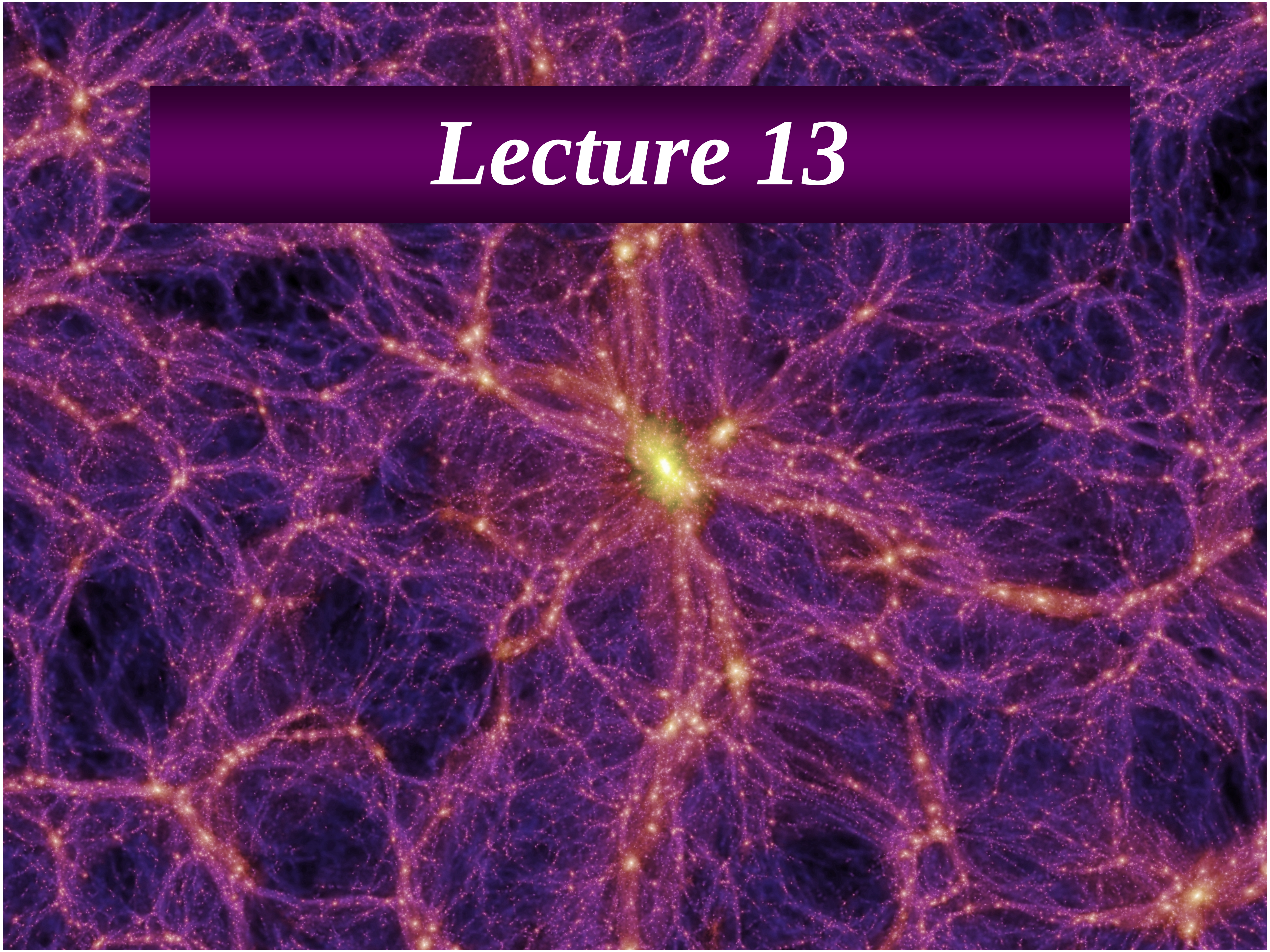
  (l,b) = (264°,48°)

  assuming no intrinsic dipole.



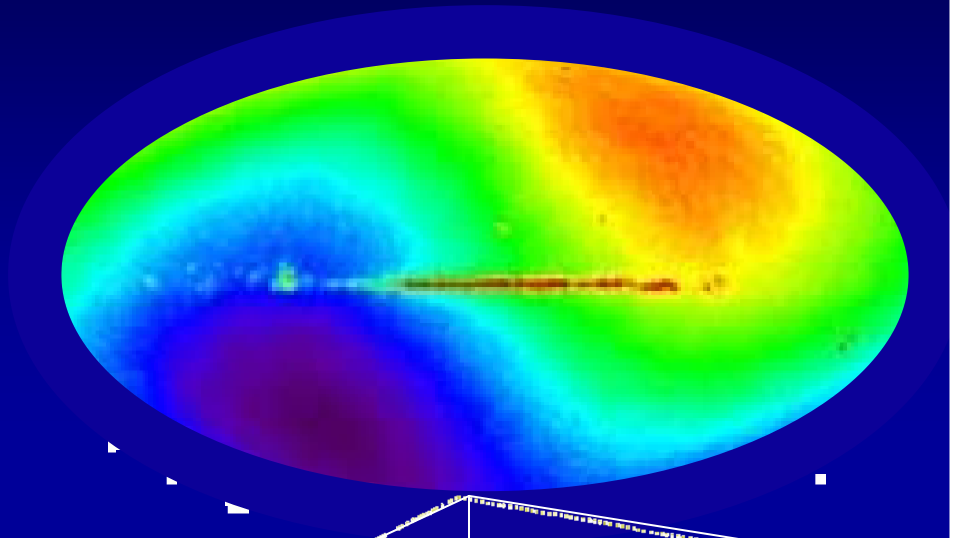
- What is its origin?
  - Not due to rotation of sun around galaxy (wrong direction). v=300km/s, (l,b)=(90°,0°).
  - Motion of the Local Group?
  - Implies V_{LG}=600km/s (l,b)=(270°,30°).

Lecture 13

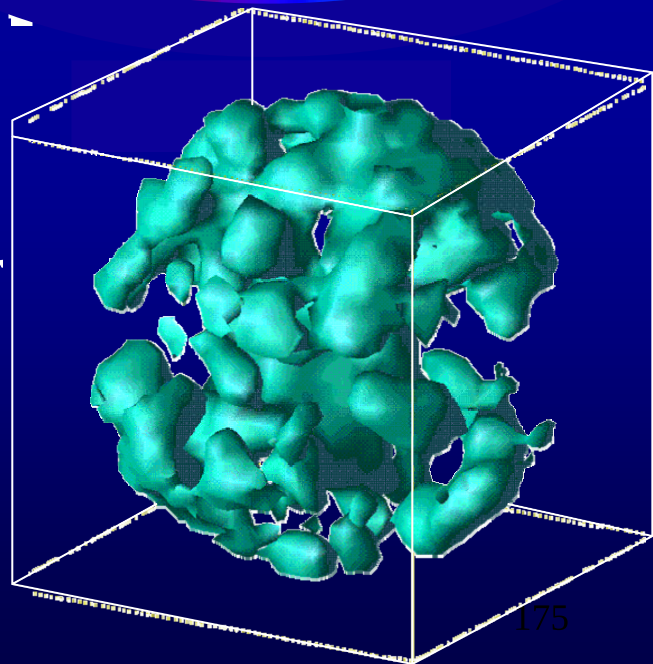


The Cosmic Microwave Background

- The CMB dipole:
- What is its origin of the dipole?
  - Motion of the Local Group.
  - V_{LG}=600km/s (l,b)=(270^o,30^o).



- Motion due to gravitational attraction of large-scale structure:
  - LG is falling into the Virgo Supercluster (~10Mpcs away)
  - Which is being pulled by the Hercules Supercluster (the Great Attractor, ~150Mpc away).



Dark Matter

- Recall from globular cluster ages, supernova and BBN:

$$\Omega_m \approx 0.3 >> \Omega_B = 0.04$$

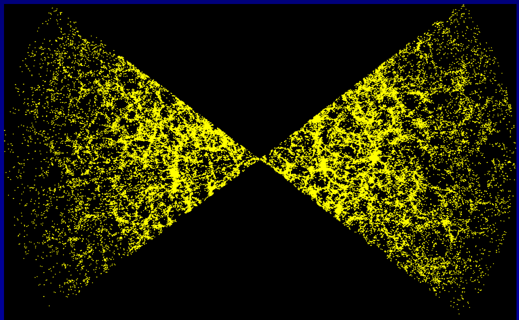
- So we infer most of the matter in the Universe is non-baryonic.

- How secure is the density parameter measurement?

- If it's wrong and lower, could all just be baryons.

Dark Matter

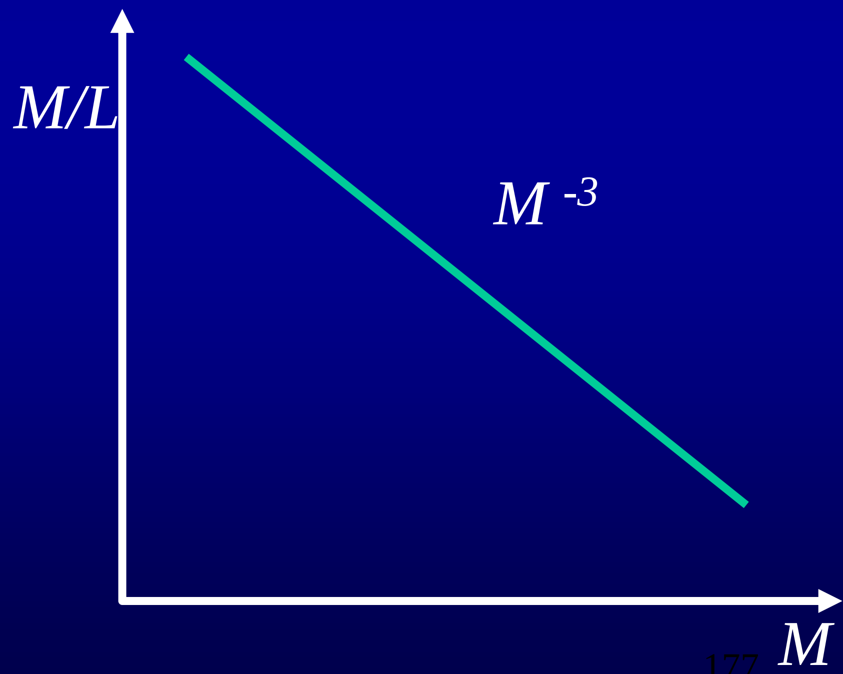
- Mass-to-light ratio of galaxies:



$$\rho_m = \left(\frac{M}{L}\right)\rho_L$$

← *luminosity density from galaxy surveys*

- We can expect M/L=F(M)



Comets: $\frac{M}{L} \approx 10^{12} \frac{M_{Sun}}{L_{Sun}}$

Low-Mass stars: $\frac{M}{L} \approx 10 \frac{M_{Sun}}{L_{Sun}}$

Galaxy stars: $\frac{M}{L} \approx 1-10 \frac{M_{Sun}}{L_{Sun}}$

Dark Matter

- In blue star-light:

$$\rho_L = (2.0 \pm 0.7) \times 10^8 h L_{Sun} Mpc^{-3}$$

$$\rho_{crit} = 2.78 \times 10^{11} \Omega_m h^2 M_{Sun} Mpc^{-3}$$

- So we find:

$$\left(\frac{M}{L}\right)_{Blue} \approx (300 \pm 100)\left(\frac{\Omega_m}{0.3}\right)\left(\frac{h}{0.7}\right)\frac{M_{Sun}}{L_{Sun}}$$

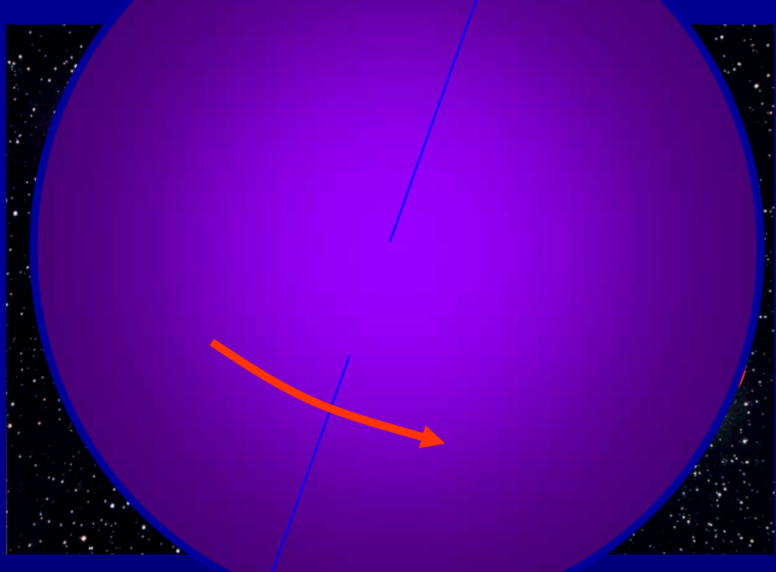
This is way above the M/L=10 we see in stars.

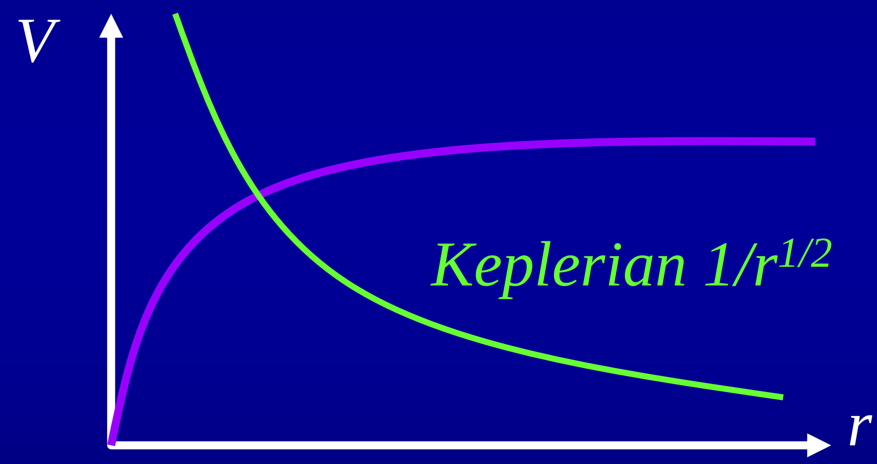
- So not enough luminous baryons in stars.
- In fact not enough baryons in stars to make Ω_B=0.04, so there must be baryonic dark matter too.

Dark Matter

- Dark Matter in Galaxy Halos:
  - In 1970's Vera Rubin found galaxies rotate like solid spheres, not Keplerian.

$$V^2 = \frac{GM(<r)}{r}$$





- For V=const, need M(<r)~r, so

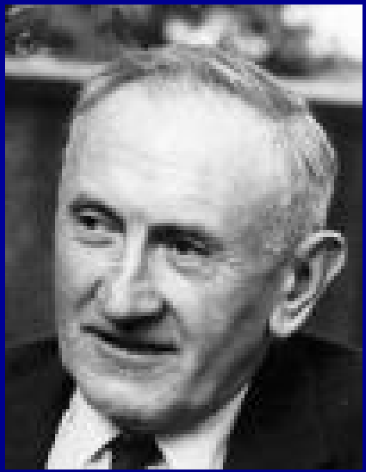
$$\rho \propto \frac{M}{r^3} \propto \frac{1}{r^2}$$

- Density profile of Isothermal Sphere.
- Yields dark matter = 5 x stellar mass

Dark Matter in Galaxy Clusters

- In 1933 Fritz Zwicky found the Doppler motion of galaxies in the Coma cluster were moving too fast to be gravitationally bound.
- First detection of dark matter.

Zwicky (1898-1974)



- Assume hydrostatic equilibrium:

$$F = -\frac{GM(<r)}{r^2} = \frac{1}{\rho}\frac{\partial}{\partial r}P = \frac{1}{\rho}\frac{\partial}{\partial r}\rho\sigma_V^2$$

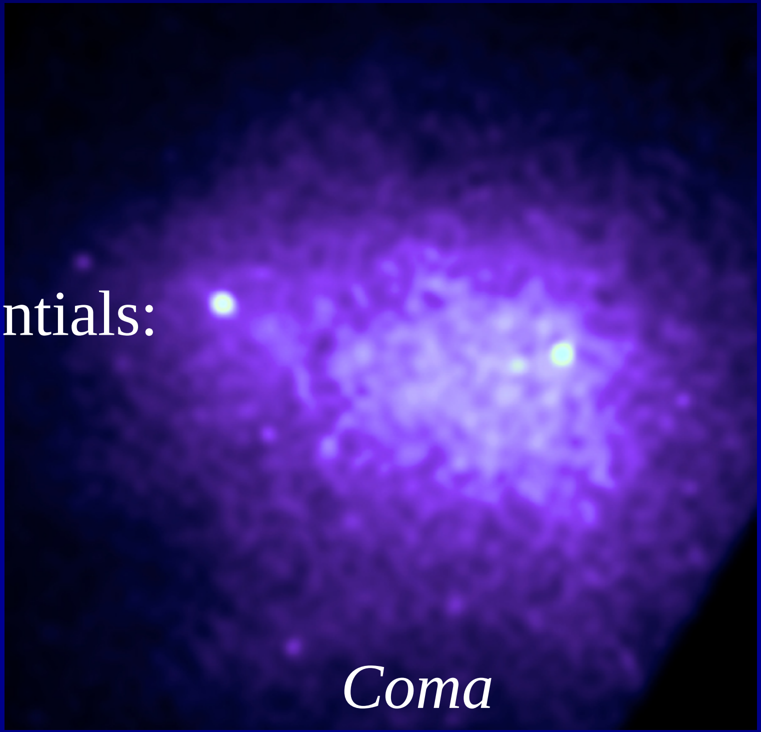
Velocity dispersion

- So need 10 – 100 x stellar mass.

Dark Matter in Galaxy Clusters

- X-ray emission from galaxy clusters.
- Hot gas emits X-rays.
- Assume hydrostatic equilibrium.
- Equate gravitational and thermal potentials:

$$\frac{GM(<r)}{r} = -\frac{kT(r)}{\mu m_p}\left(\frac{d\ln T}{d\ln r} + \frac{d\ln \rho_{gas}}{d\ln r}\right)$$



- Get both total mass, and baryonic (gas) mass.

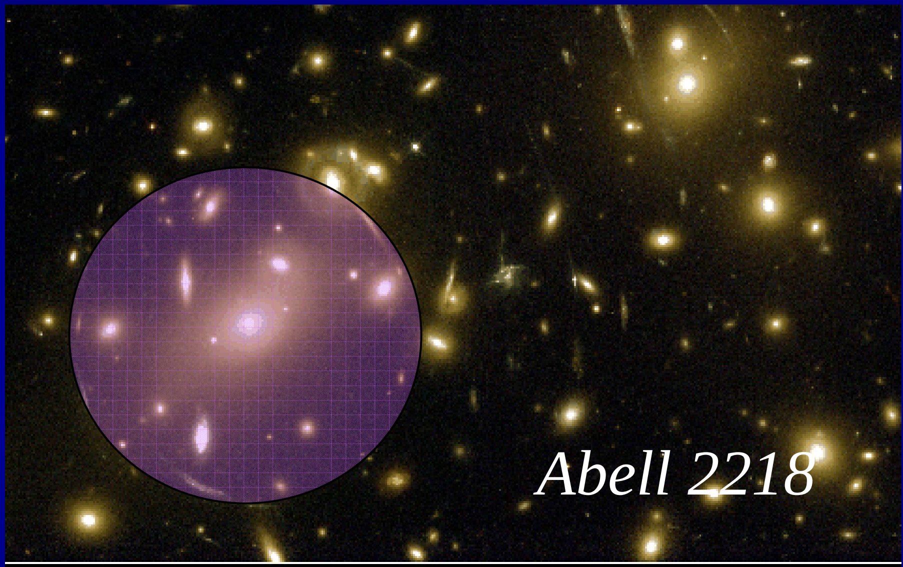
$$\frac{M_B}{M_{Tot}} = \frac{M_{gas} + M_{stars}}{M_{gas} + M_{stars} + M_{DM}} = 0.01 + 0.09\left(\frac{h}{0.7}\right)^{-3/2} \approx 0.1$$

- So $M_{DM} = 10 M_B$

Dark Matter in Galaxy Clusters

- Gravitational lensing by clusters of galaxies.
- Use giant arcs around clusters to measure projected mass.
- Strongest distortion at the Einstein radius:

$$\theta_E \propto \sqrt{M(<\theta_E)}$$



- Independent of state of cluster (equilibrium).
- Find again $M_{Tot} = 10 - 100\ M_{stars}$

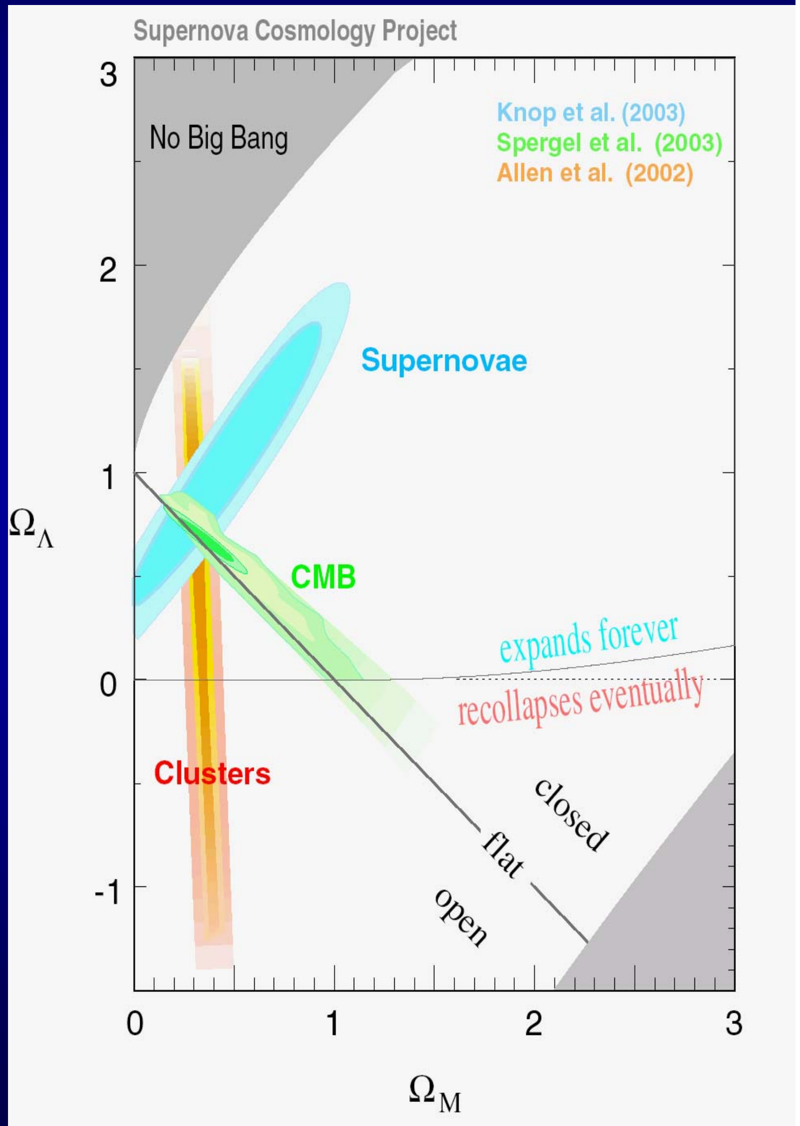
Dark Matter and Ω_m

- So independent methods show in galaxy clusters:

  $M_{DM} \sim 10\ M_{gas} \sim 100\ M_{stars}$

- Can estimate mass-density of Universe from clusters:

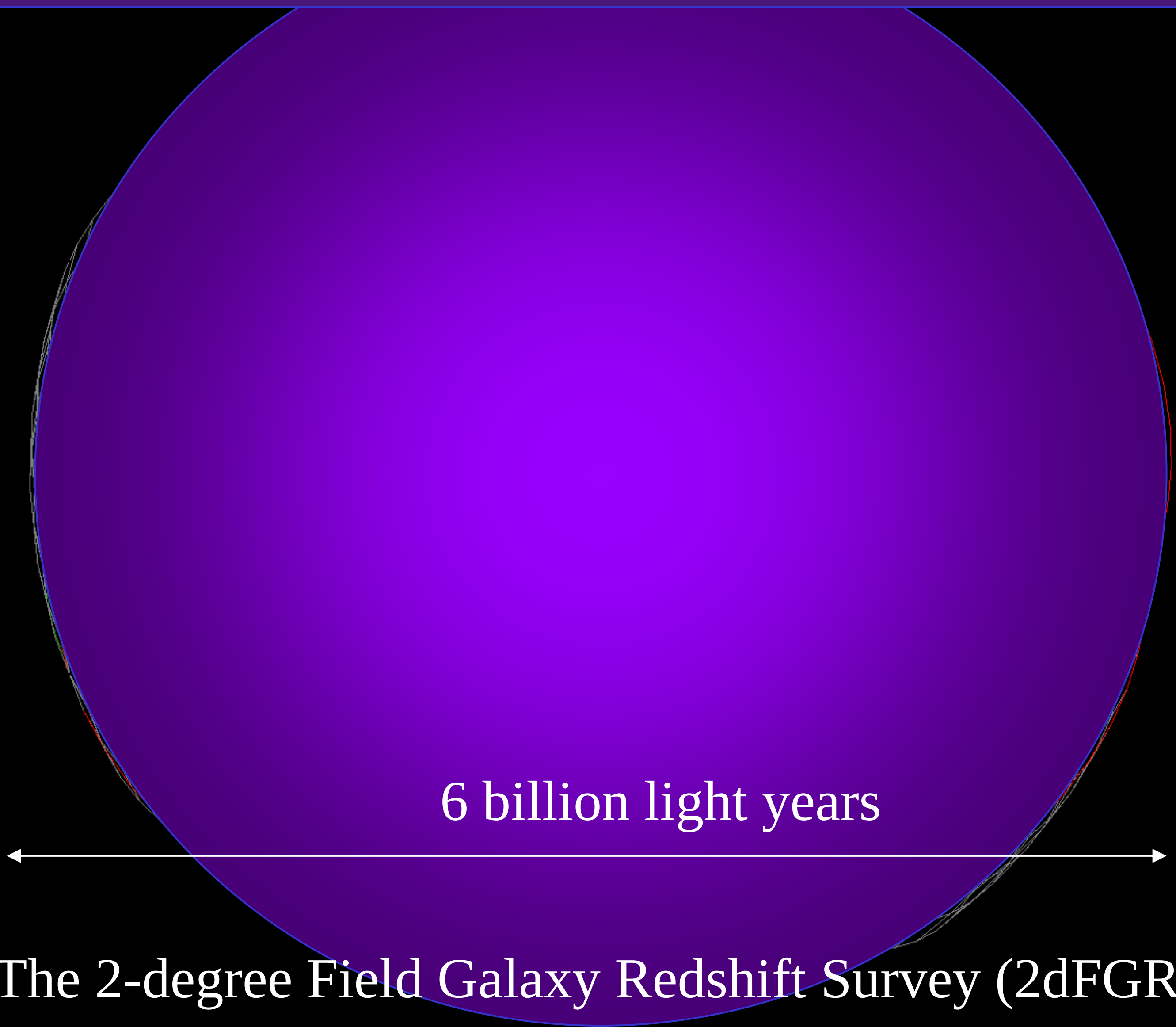
$$\rho_m = M_{cluster} n_{cluster} \approx 10^{14} M_{Sun} 10^{-3} Mpc^{-3}$$
$$\rho_{crit} = 2.78 \times 10^{11} M_{Sun} Mpc^{-3}$$
$$\Omega_m \approx 0.3$$



Large-scale structure in the Universe

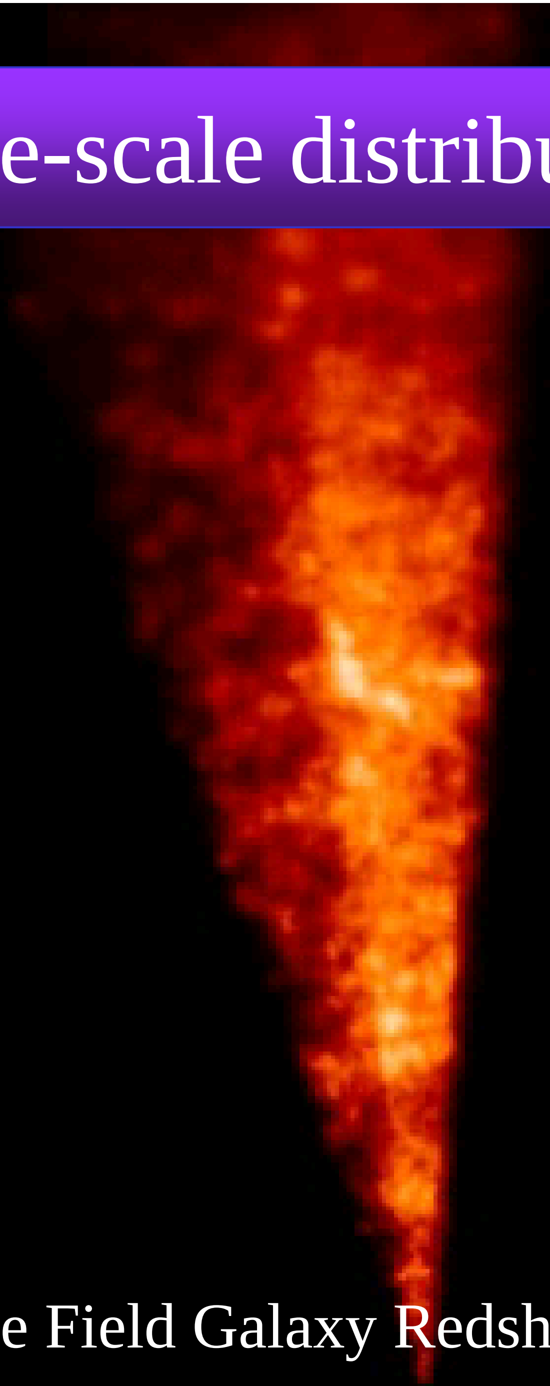
- The distribution of matter in the Universe is not uniform.
- There exists galaxies, stars, planets, complex life etc.
- Where does all this structure come from?
- Is there a fossil remnant from when it was formed?
- How do we reconcile this structure with the Cosmological Principle & Friedmann model ?

The large-scale distribution of galaxies



The 2-degree Field Galaxy Redshift Survey (2dFGRS)

The large-scale distribution of galaxies

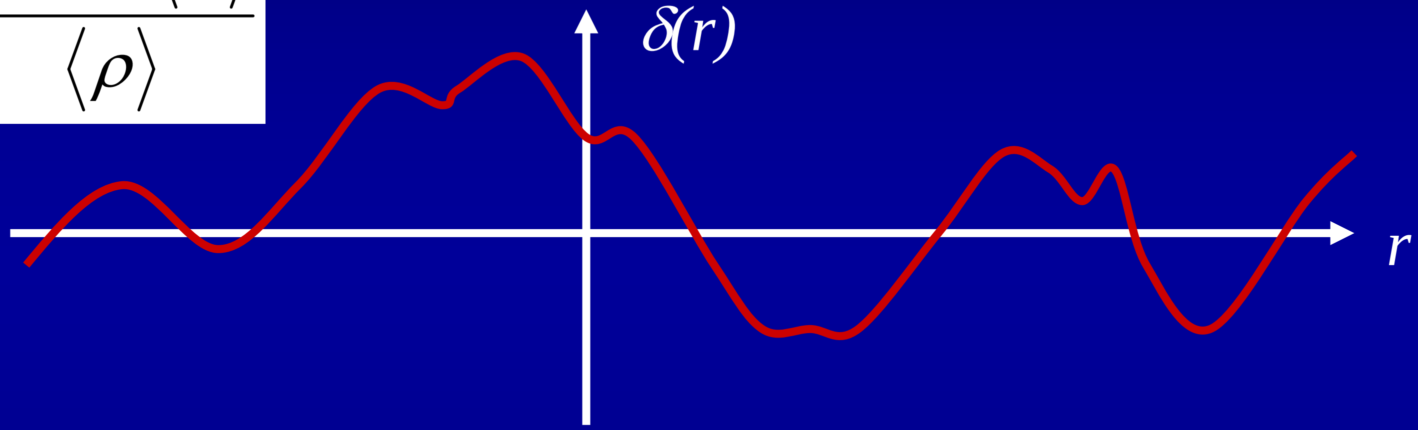


The 2-degree Field Galaxy Redshift Survey (2dFGRS)

Large-scale structure in the Universe

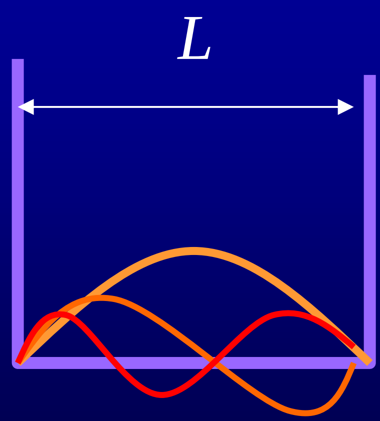
- The matter density perturbation:

$$\delta(r) = \frac{\rho(r) - \langle\rho\rangle}{\langle\rho\rangle}$$



- Fourier decomposition:

$$\delta(r) = \sum_k \delta_k e^{-i\mathbf{k.r}}$$



$k_x = n\pi/L, \quad n = 1, 2\ldots$

Large-scale structure in the Universe

- The statistical properties:
  - The Ergodic Theorem: $$\langle \ldots \rangle = \frac{1}{V}\int d^3r \ldots$$

    Volume averages are equal to ensemble averages.
- Moments of the density field:

$$\langle \delta \rangle = 0, \qquad \langle \delta^2 \rangle = \frac{1}{V}\int d^3r\, \delta^2(r) = \frac{1}{V}\int d^3r \left| \sum_k \delta_k e^{-i\mathbf{k.r}} \right|^2 = \sum_k |\delta_k|^2$$

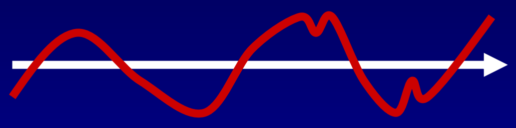
- Define the power spectrum:

$$P(k) = |\delta_k|^2$$

$$\Delta^2(k) = \frac{k^3 P(k)}{2\pi^2} = \frac{d\langle \delta^2 \rangle}{d\ln k} \approx \langle \delta^2 \rangle (R \approx 2\pi / k)$$

Large-scale structure in the Universe

- The statistical properties:

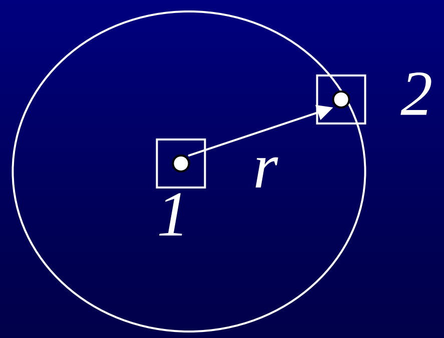


- The correlation function:

$$\xi(r) = \langle \delta(x)\delta(x+r) \rangle = \frac{1}{V}\int d^3x\, \delta(x)\delta(x+r) = \frac{1}{V}\int d^3x \sum_k \delta_k e^{-ik.x} \sum_{k'} \delta_{k'} e^{-ik.(x+r)} = \sum_k |\delta_k|^2 e^{-ik.r}$$

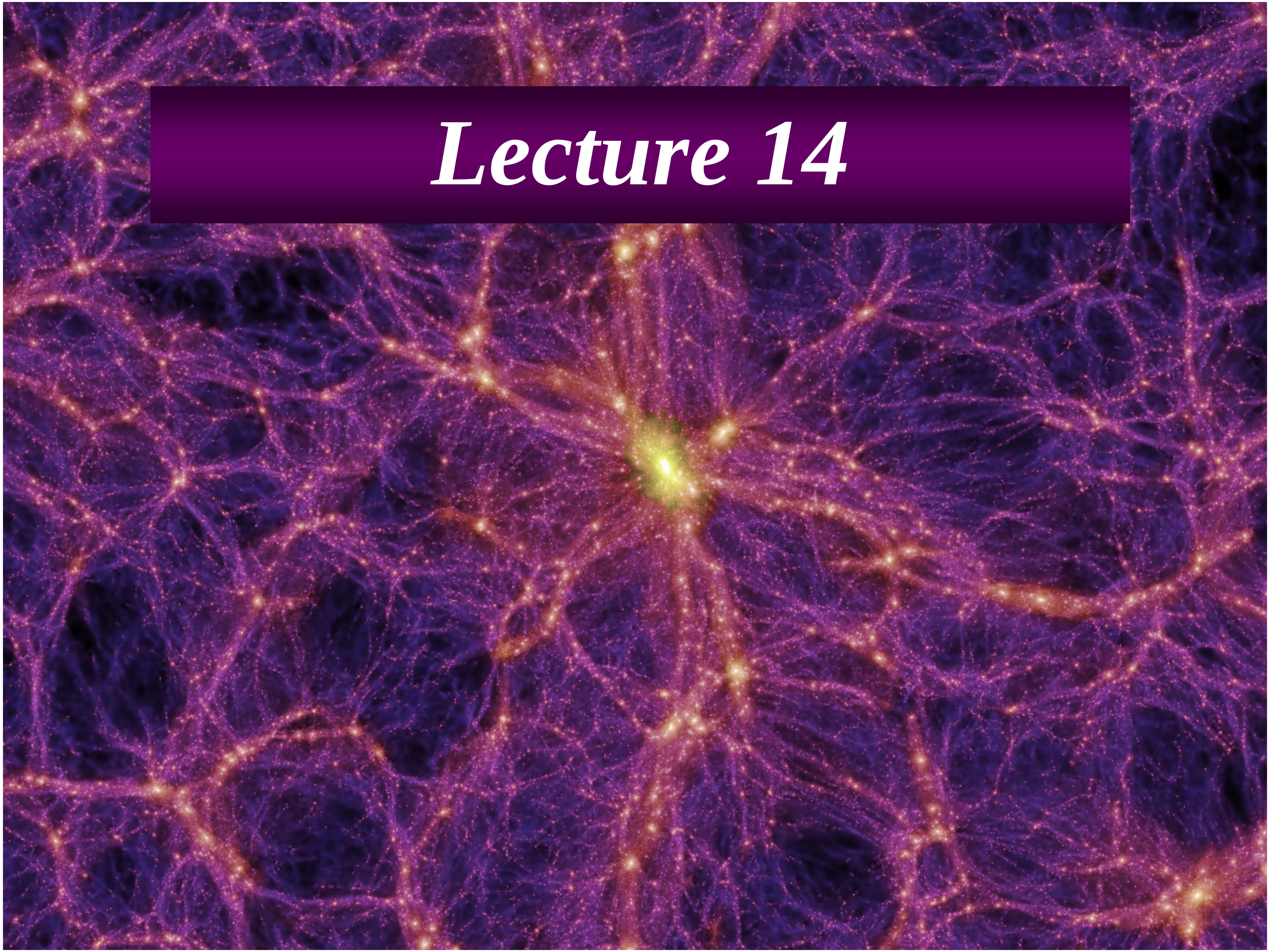
- So correlation function is the Fourier transform of the power spectrum, P(k).

- For point processes, correlation function is the excess probability of finding a point at 2 given a point at 1:



$$dP(2|1) = n^2(1+\xi(r))dV_1 dV_2$$

Lecture 14

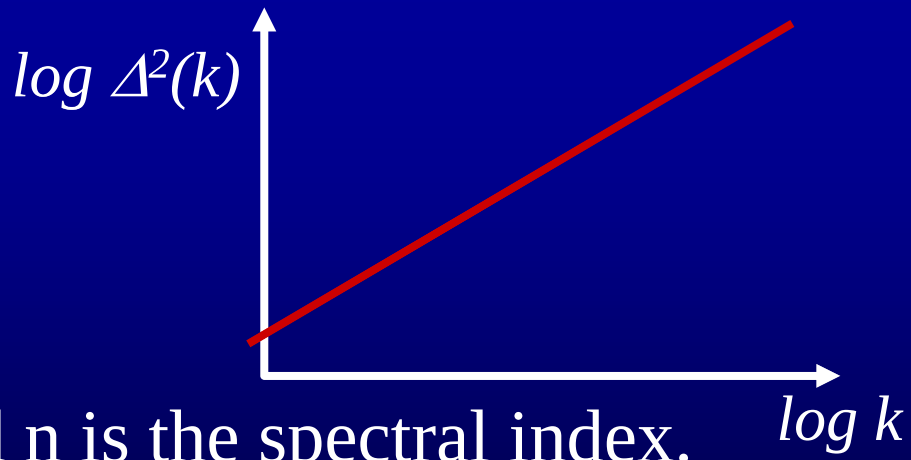


Large-scale structure in the Universe

- The Matter Power Spectrum:
- So 2-point statistics can be found from P(k).
- What is the form of P(k)?
- For simplicity let's assume for now it's a power-law:

$$P(k) = Ak^n$$

$$\Delta^2(k) = \frac{Ak^{n+3}}{2\pi^2}$$



where A is an amplitude and n is the spectral index.

Large-scale structure in the Universe

- The Potential Power Spectrum:
- Can we put limits on spectral index, n?
- Consider the potential field, Φ.
- So far we have assumed Φ<<1 (so metric is Freidmann).
- Poisson equation:

$$\nabla^2\Phi = 4\pi G\rho_0\delta$$
$$-k^2\Phi_k = 4\pi G\rho_0\delta_k$$
$$\Phi_k = -4\pi G\rho_0\delta_k k^{-2}$$

- So

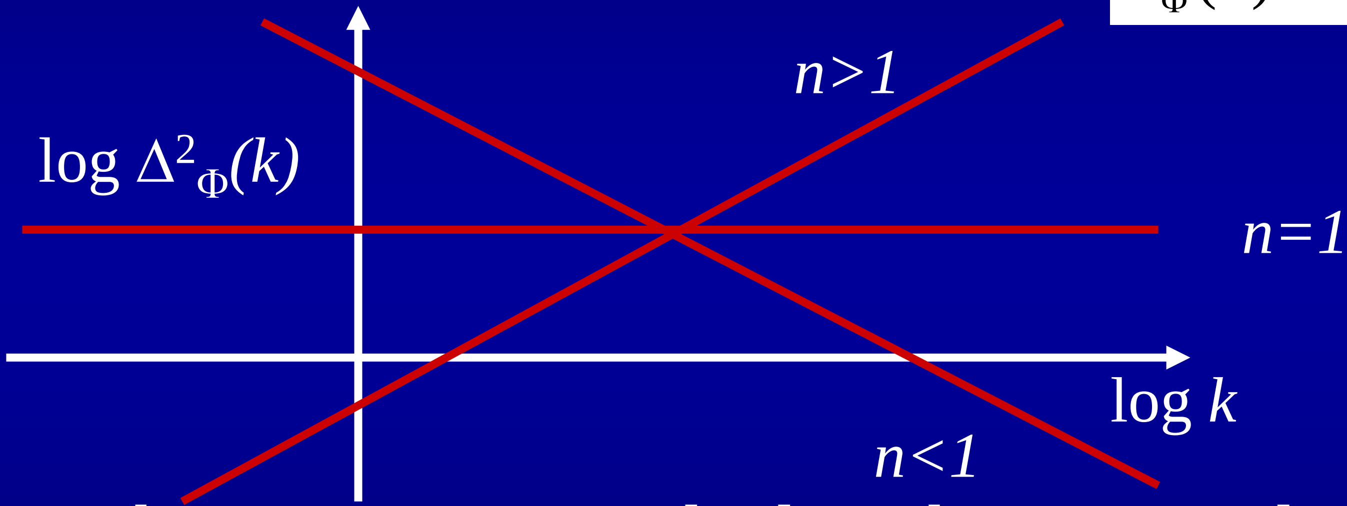
$$P_\Phi(k) = \left\langle\left|\Phi_k\right|^2\right\rangle \propto k^{n-4}$$
$$\Delta^2_\Phi(k) \propto k^{n-1}$$

Large-scale structure in the Universe

- The Potential Power Spectrum:
- Can we put limits on spectral index, n?

$$P_\Phi(k) = \langle |\Phi_k|^2 \rangle \propto k^{n-4}$$

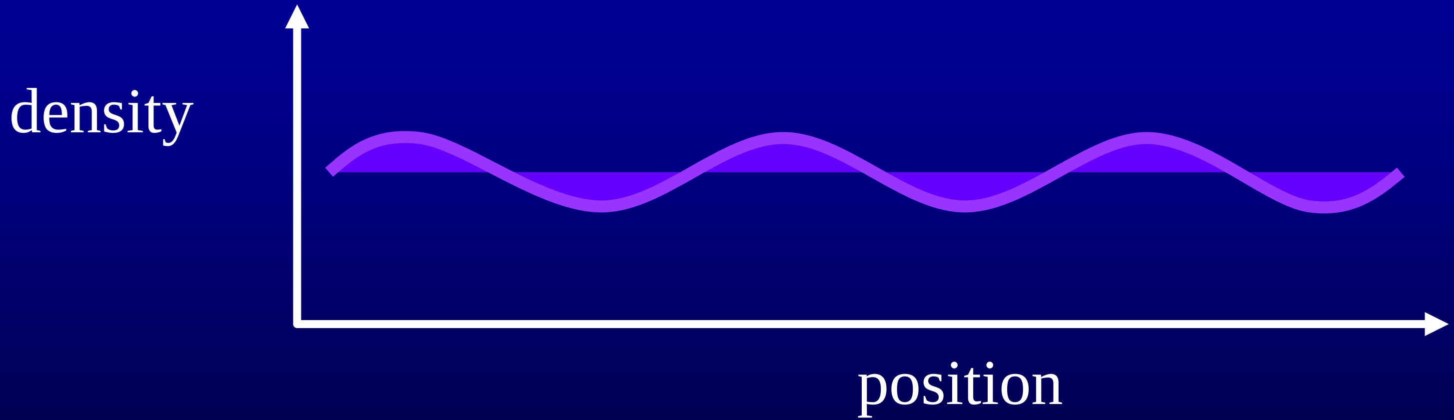
$$\Delta^2_\Phi(k) \propto k^{n-1}$$



- To keep homogeneity, need n less than or equal to 1.
- To avoid black holes, need n greater or equal to 1.
- So must have n=1, with Δ^2_Φ=const~10^{-10} (from CMB).
- n=1 is scale invariant (fractal) in the potential field.

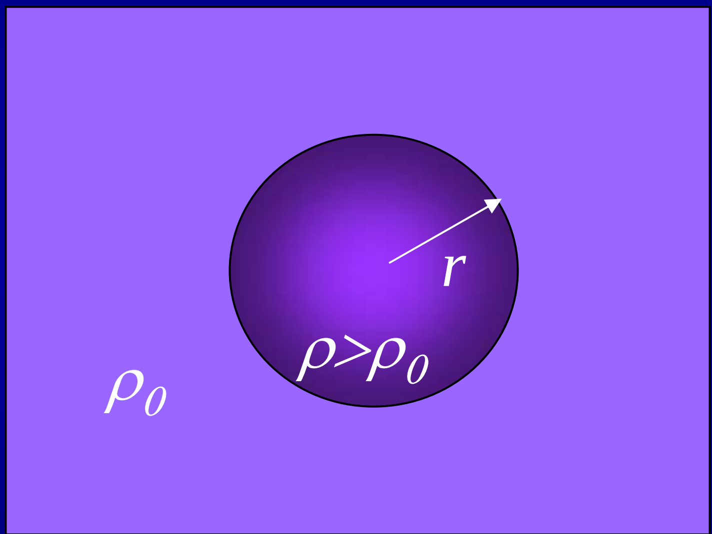
Structure in the Universe

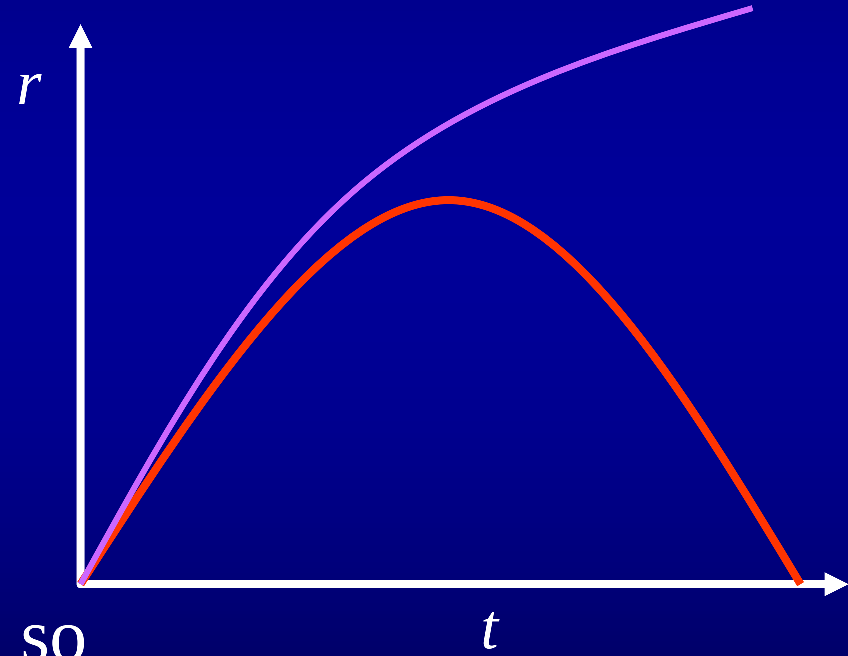
- Where did this structure come from ?
- In 1946 Russian physicist Evgenii Lifshitz suggested small variations in density in the Early Universe grow due to gravitational instability.



Dynamics of structure formation

- Consider the gravitational collapse of a sphere:
- Assume Einstein-de Sitter (Ω_m=1, p_m=0).





- Behaves like a mini-universe, so

$$r = A(1-\cos\theta)$$
$$t = B(\theta-\sin\theta)$$

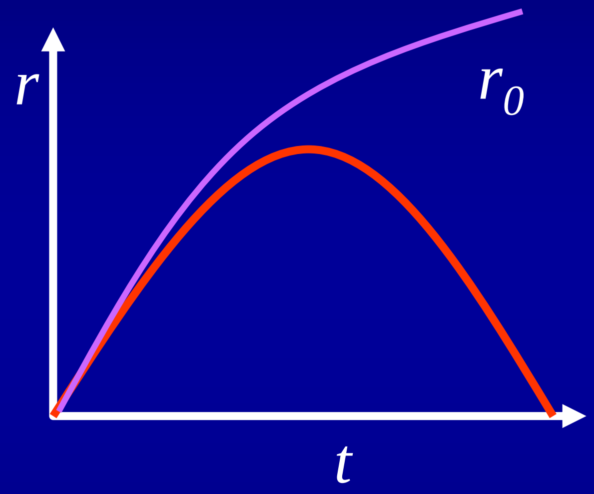
Dynamics of structure formation

Since $\ddot{r} = -\dfrac{GM}{r^2}$ we find $A^3 = GMB^2$

- Linear theory growth:

$$r = A(1-\cos\theta) \approx A\tfrac{1}{2}\theta^2(1-\tfrac{1}{12}\theta^2)$$

$$t = B(\theta-\sin\theta) \approx B\tfrac{1}{6}\theta^3(1-\tfrac{1}{20}\theta^2)$$

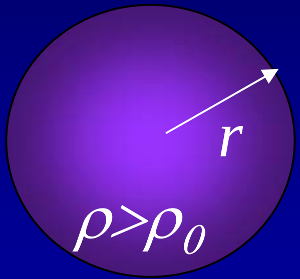


- 0th order: $r_0 = \dfrac{A}{2}\left(\dfrac{6t}{B}\right)^{2/3} \propto a(t)$ - expansion of universe.

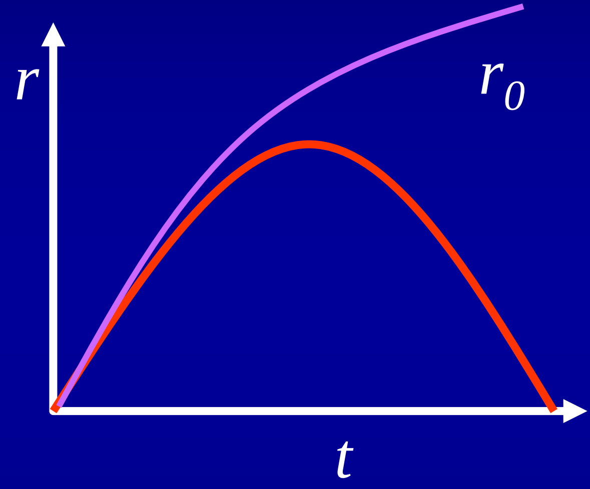
- 1st order: $r = \dfrac{A}{2}\left(\dfrac{6t}{B}\right)^{2/3}\left(1-\dfrac{1}{20}\left(\dfrac{6t}{B}\right)^{2/3}\right)$

Dynamics of structure formation

- Linear growth of over-densities:



$$r = \frac{A}{2}\left(\frac{6t}{B}\right)^{2/3}\left(1-\frac{1}{20}\left(\frac{6t}{B}\right)^{2/3}\right)$$



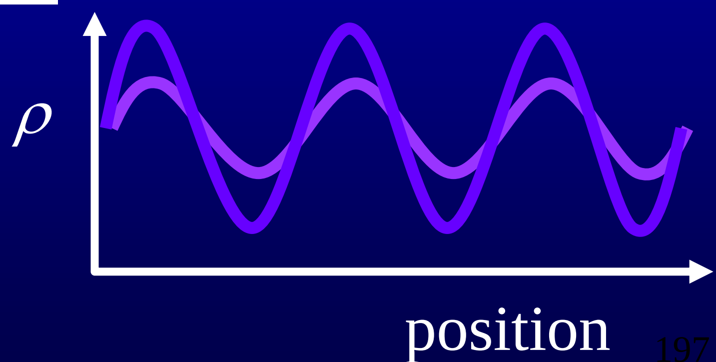
- Density ~ 1/Vol:

$$\frac{\rho}{\rho_0} = 1+\delta = \left(\frac{r}{r_0}\right)^3$$

$$r = r_0(1+\delta)^{-1/3} \approx r_0(1-\tfrac{1}{3}\delta)$$

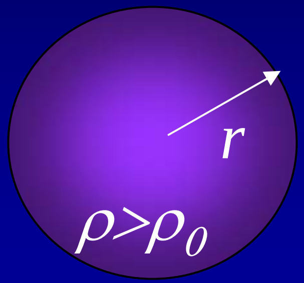
- Linear growth in E-dS:

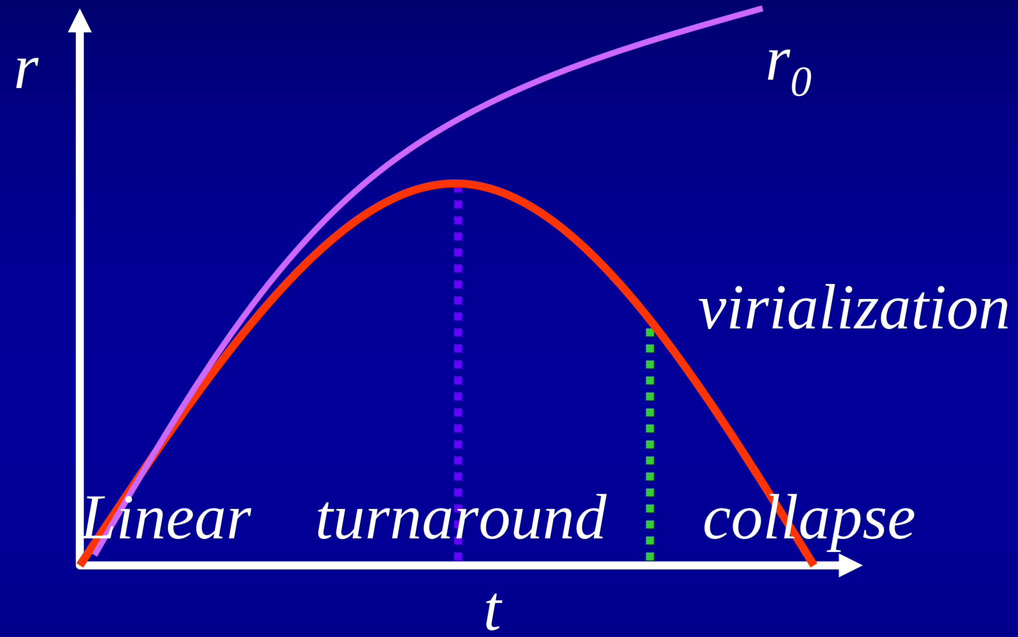
$$\delta = \frac{3}{20}\left(\frac{6t}{B}\right)^{2/3} \propto a(t)$$



Dynamics of structure formation

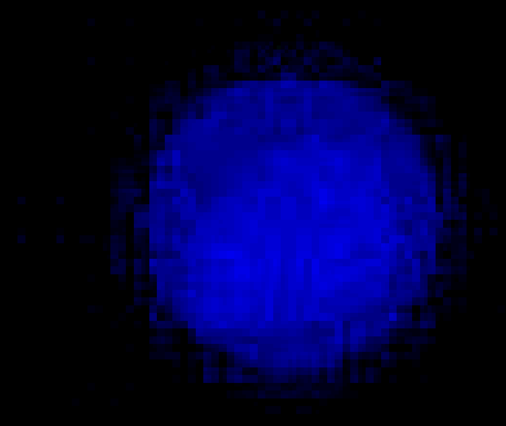
- Nonlinear growth of over-densities:





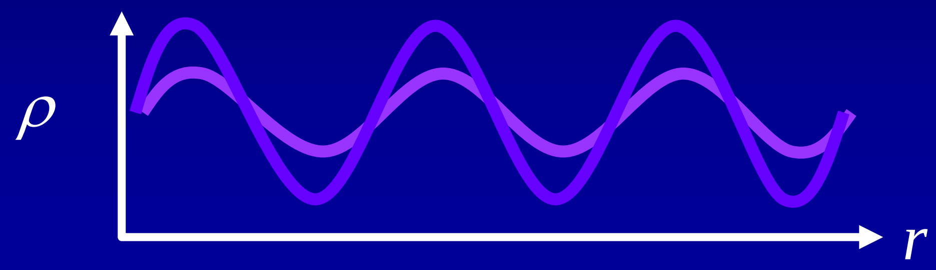
- Linear growth: $\delta \propto a$
- Turnaround: $\theta = \pi, \quad t = B\pi, \quad \delta = 5.55$
- Collapse: $\theta = 2\pi, \ \mathrm{r} = 0, \quad \delta = \infty$
- Virialization: $V = -2K, \quad \mathrm{r} = \frac{1}{2} r_{\max}, \quad \theta = \frac{3}{2\pi}, \quad \delta = 177, \quad \delta_{lin} = 1.69$

Formation of a galaxy cluster

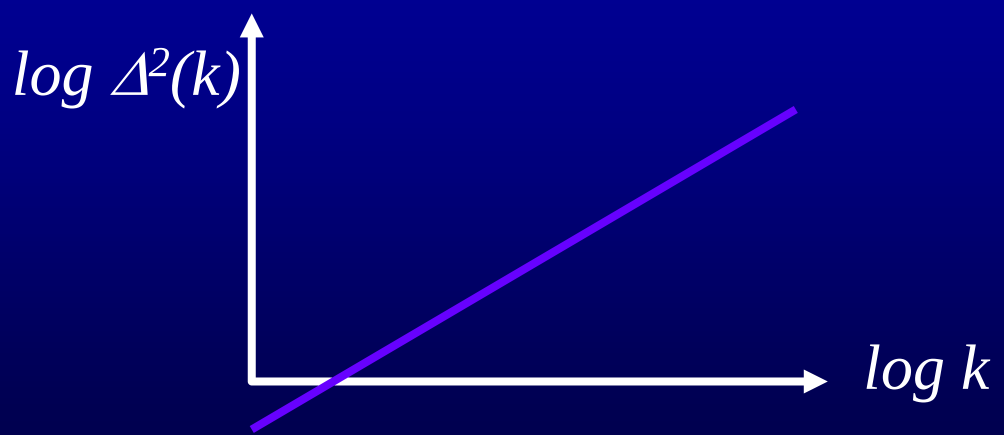


Dark Matter and the Power Spectrum

- Since δ ~ a on all linear scales, the matter power spectrum preserves its shape in the linear regime.



- Linear regime, δ<<1, valid at early times and on large scales.
- If n>-3 initial shape will be preserved on large scales.



- Power spectra is shaped by dark matter, so leaves imprint.

Dark Matter and the Power Spectrum

- Have already seen we need non-baryonic dark matter
- (Ω_B=0.04 < Ω_m=0.3, from BBN and clusters, SN, ages…).
- But what can the dark matter be?
  - Massive Neutrinos?

    Now know to have mass, so possibly.
  - Black holes?

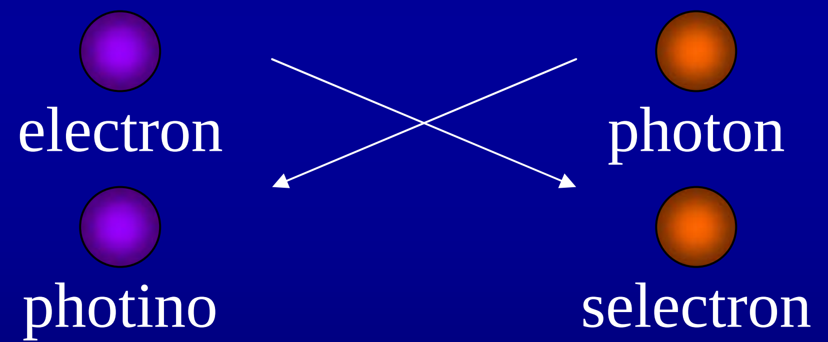
    Also now know to exist at centre of all galaxies. But if too large, disrupt galactic disk & lens stars in LMC and galactic bugle (MACHO and OGLE surveys). Too small and will over-produce Hawking radiation emission.
  - A frozen-out particle relic from the early universe:

    Weakly Interacting Massive Particles (WIMPS).

What is Dark Matter ?

- Must be weakly interacting to avoid detection so far.
- A promising idea in particle physics is Supersymmetry:

Matter Particles (fermions) Force Particles (boson)



- The lightest supersymmetric particle (the neutralino = gravitino+photino+zino) could be detected in 2007 at Europe's CERN Large Hadron Collider (LHC).

Dark Matter and the Power Spectrum

- It is convenient to divide dark matter candidates into 3 types:

1. Hot Dark Matter (HDM):

   Relativistic at freeze-out (e.g. neutrinos), $kT \gg mc^2$.

$$n_{HDM} \approx n_{\gamma}, \qquad m_{HDM} \approx eV$$

2. Warm Dark Matter (WDM):

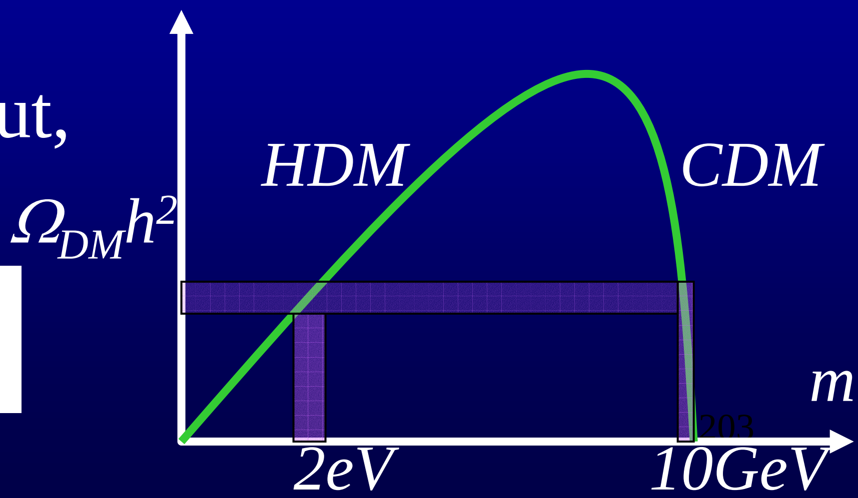
   Some momentum at freeze-out, $kT \sim mc^2$.

$$n_{WDM} < n_{\gamma}, \qquad m_{WDM} \approx 1-10 keV$$

3. Cold Dark Matter (CDM):

   No momentum at freeze-out, $kT \ll mc^2$.

$$\rho = mn \propto me^{-m/MeV}$$

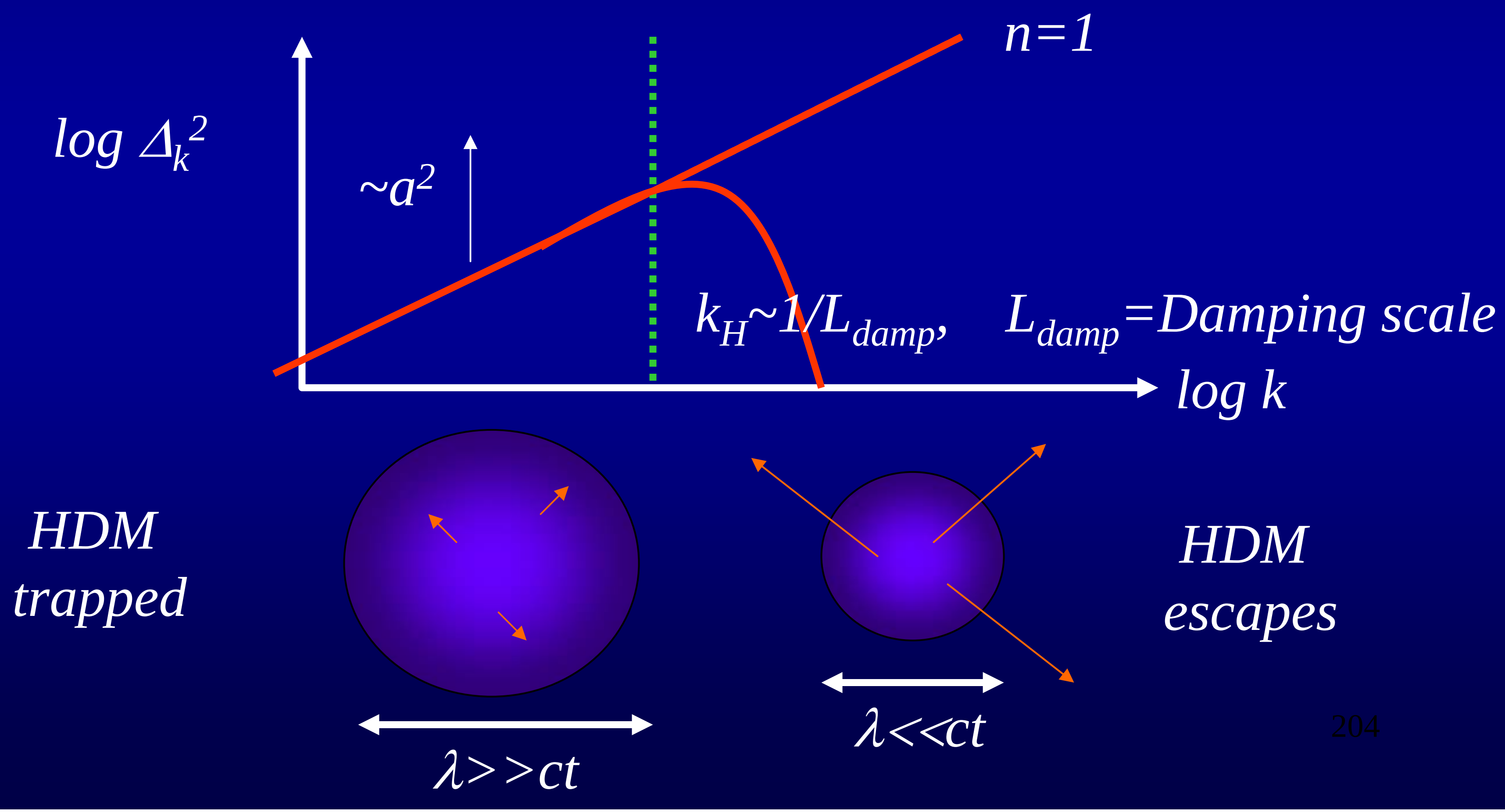


Dark Matter and the Power Spectrum

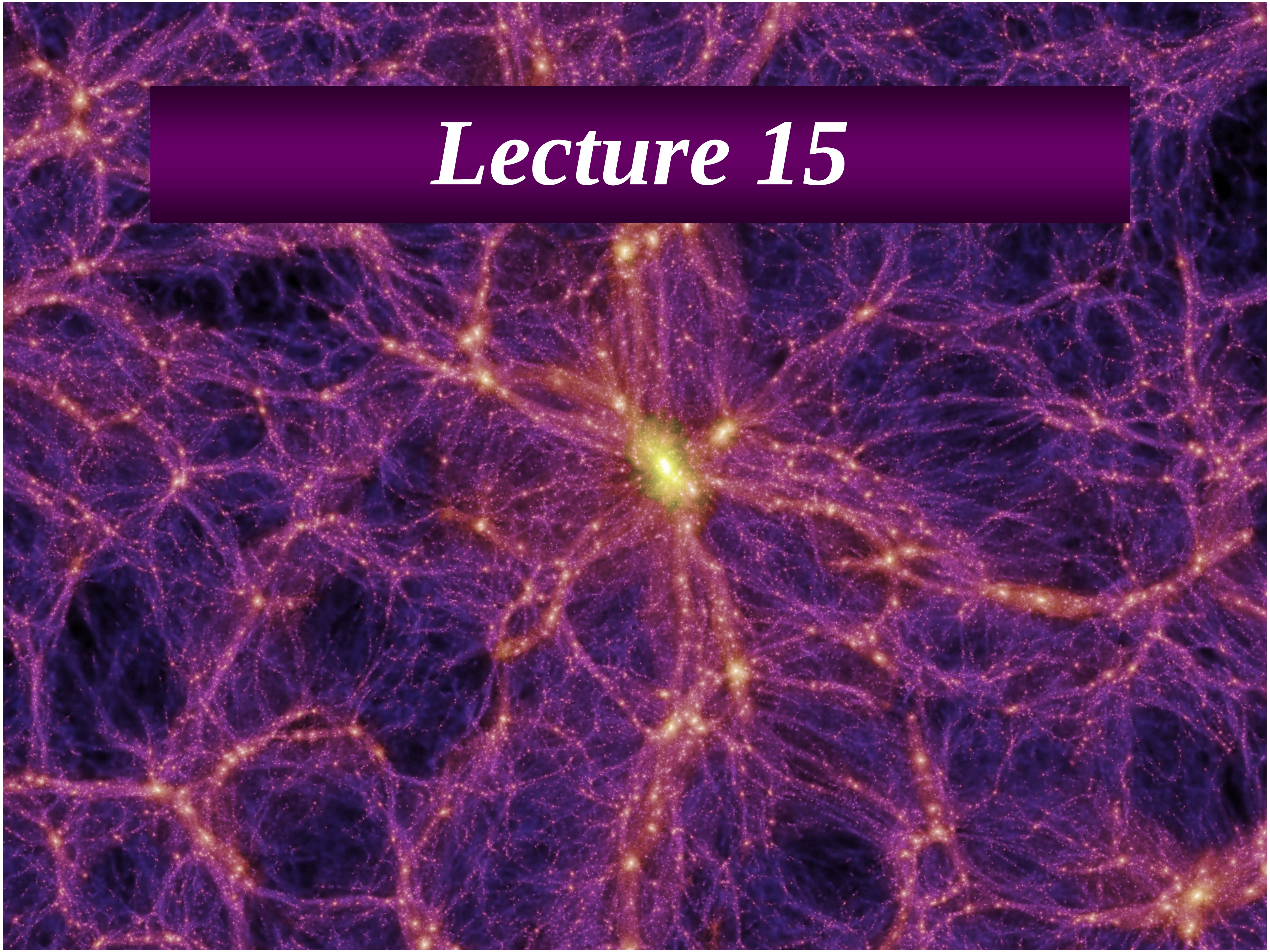
- The matter Transfer Functions:

Dark matter affects the matter power spectrum of density perturbations.

- **HDM: Free-streaming and damping:** HDM freezes-out relativistically.

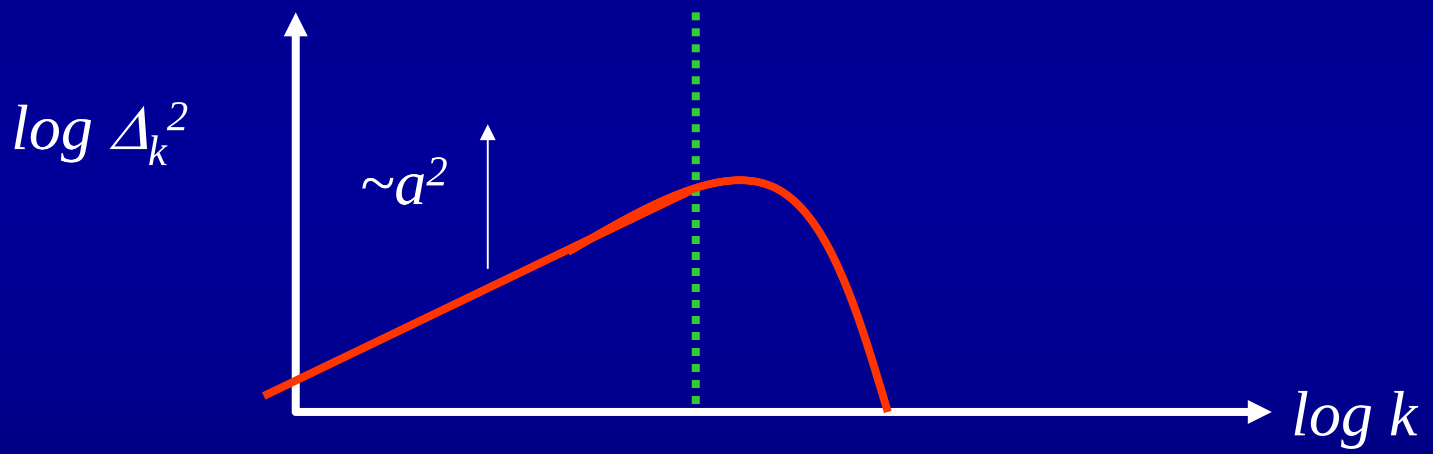


Lecture 15



Dark Matter and the Power Spectrum

- **HDM: Free-streaming and damping:**
  - HDM freezes-out relativistically, v~c, so can free-stream out of density perturbations in matter-dominated regime.



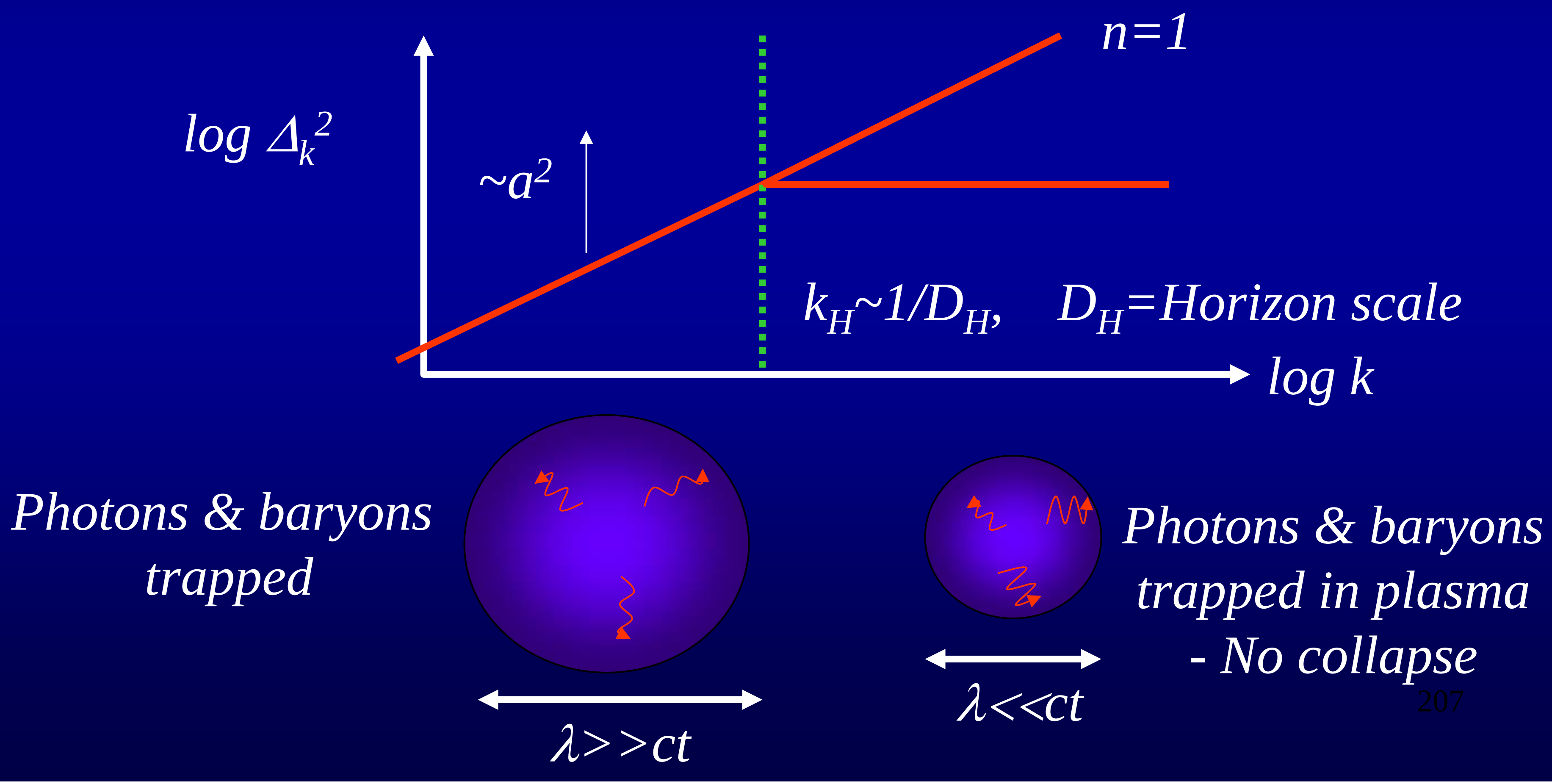
$$L_{damp} \approx ct(kT = mc^2) \approx D_H(z_{eq}) \propto \left(\Omega_m z_{eq}\right)^{-1/2}, \qquad z_{eq} = 23{,}900(\Omega_m h^2)$$

$$L_{damp}(neutrino) \approx 16(\Omega_\nu h^2)^{-1} Mpc$$

- So if HDM, expect no structure (galaxies) on small-scales today!.
- This rules out an HDM-dominated universe.

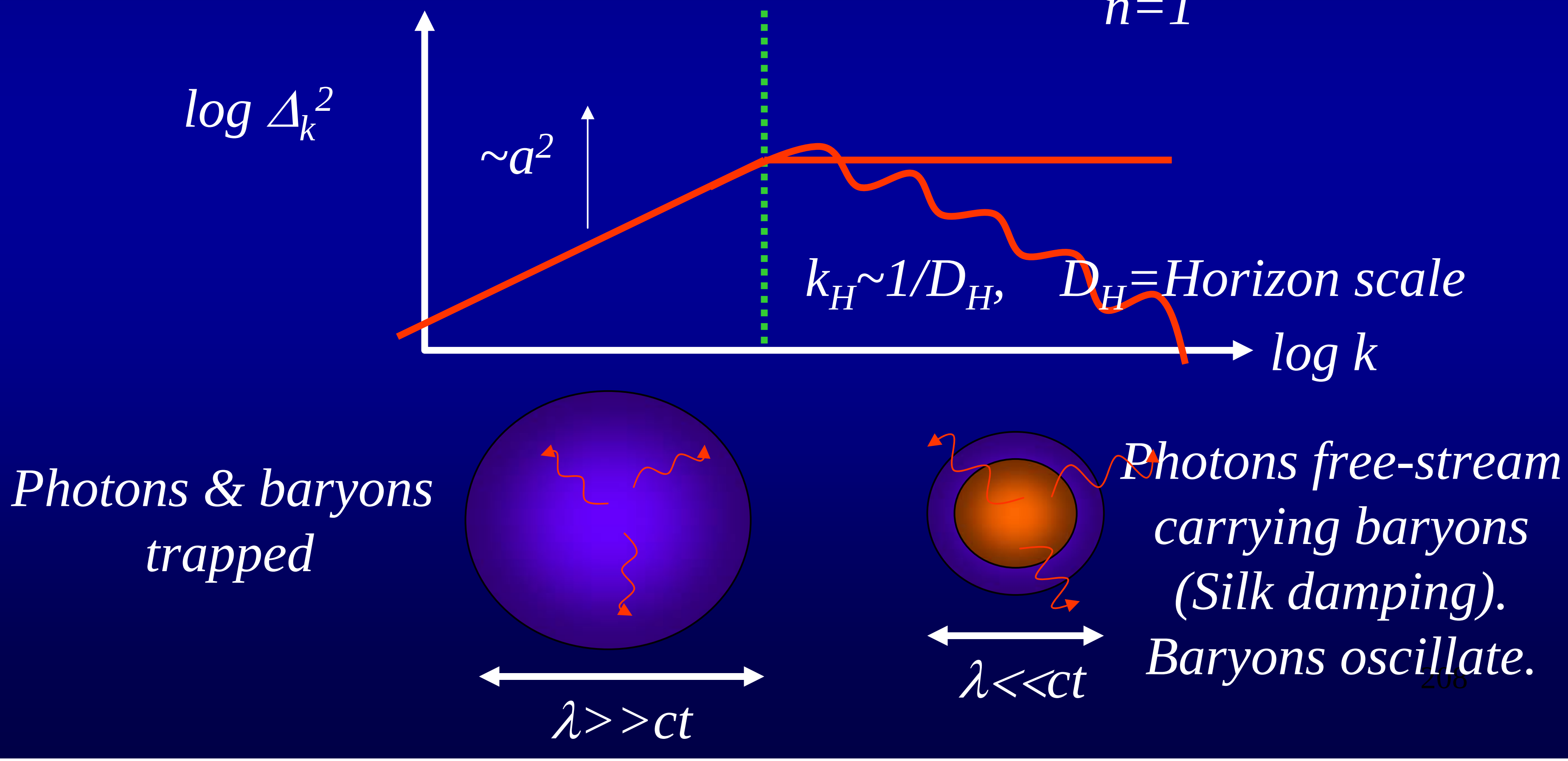
Dark Matter and the Power Spectrum

- **Baryons + photons: Baryon Oscillations and Silk damping.**
  - $t<t_{rec}$: Baryon-photon plasma



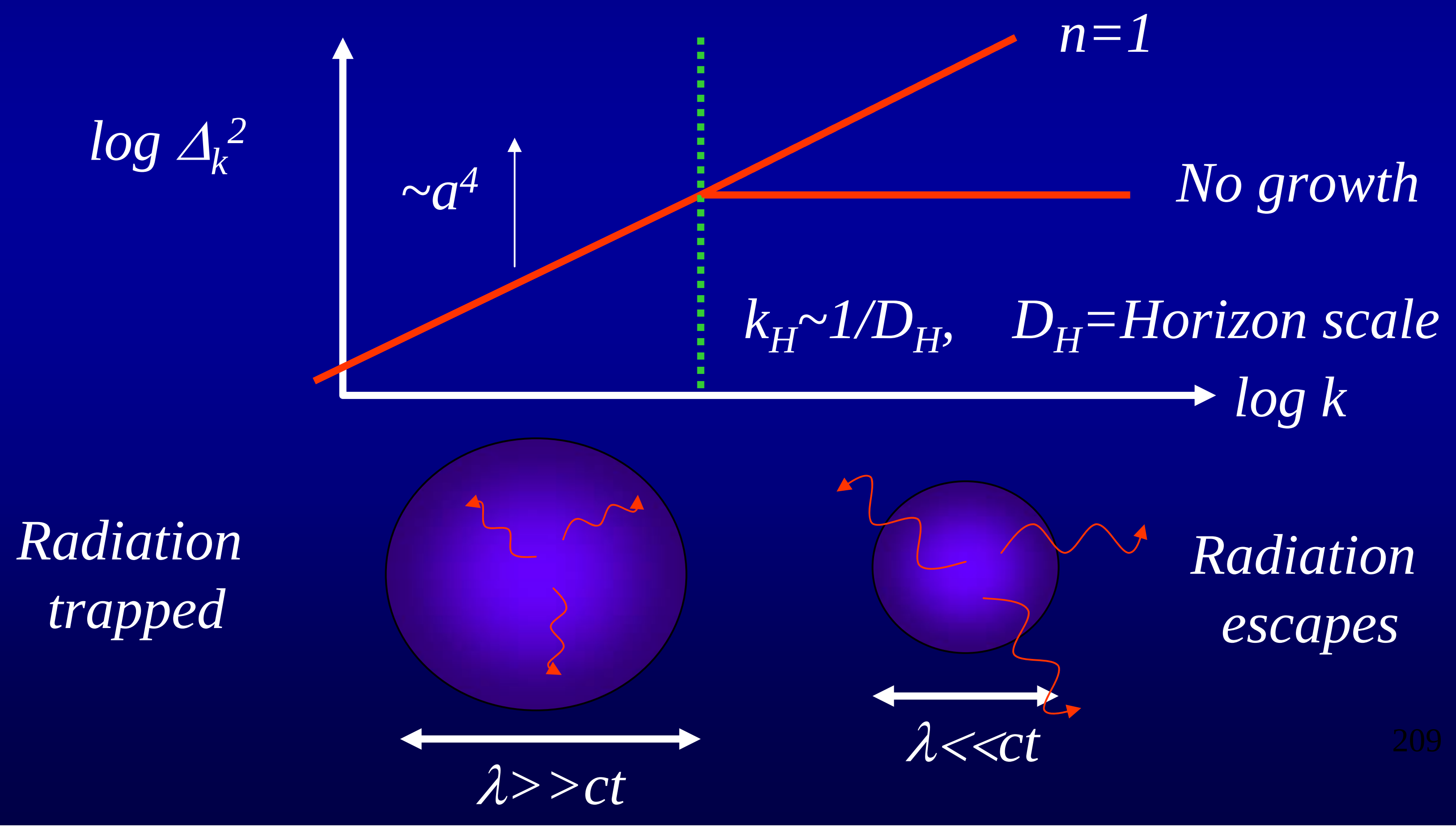
Dark Matter and the Power Spectrum

- **Baryons + photons: Baryon Oscillations and Silk damping.**
  - $t > t_{rec}$: Baryon and photons free. Baryons oscillate.



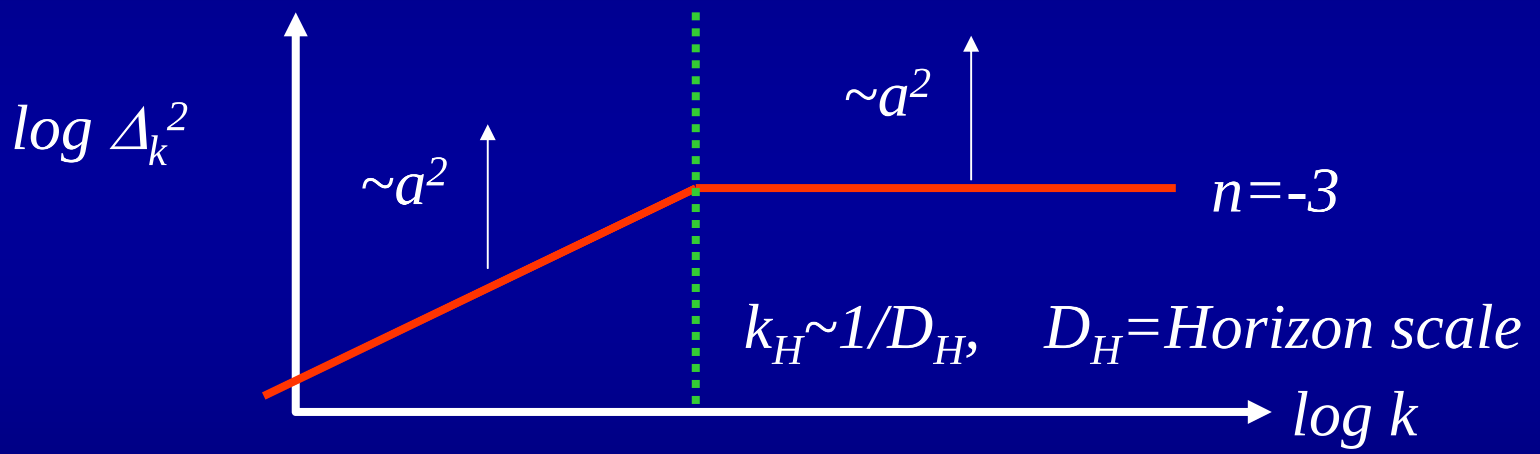
Dark Matter and the Power Spectrum

- **CDM + photons: The Meszaros Effect.**
  - Recall at early times $\rho_\gamma >> \rho_m$



Dark Matter and the Power Spectrum

- **CDM + photons: The Meszaros Effect.**
  - After matter-radiation equality, all scales grow the same.



  - Produces a break in the matter power spectrum at comoving horizon scale at z_{eq}=23,900$\Omega_m h^2$.

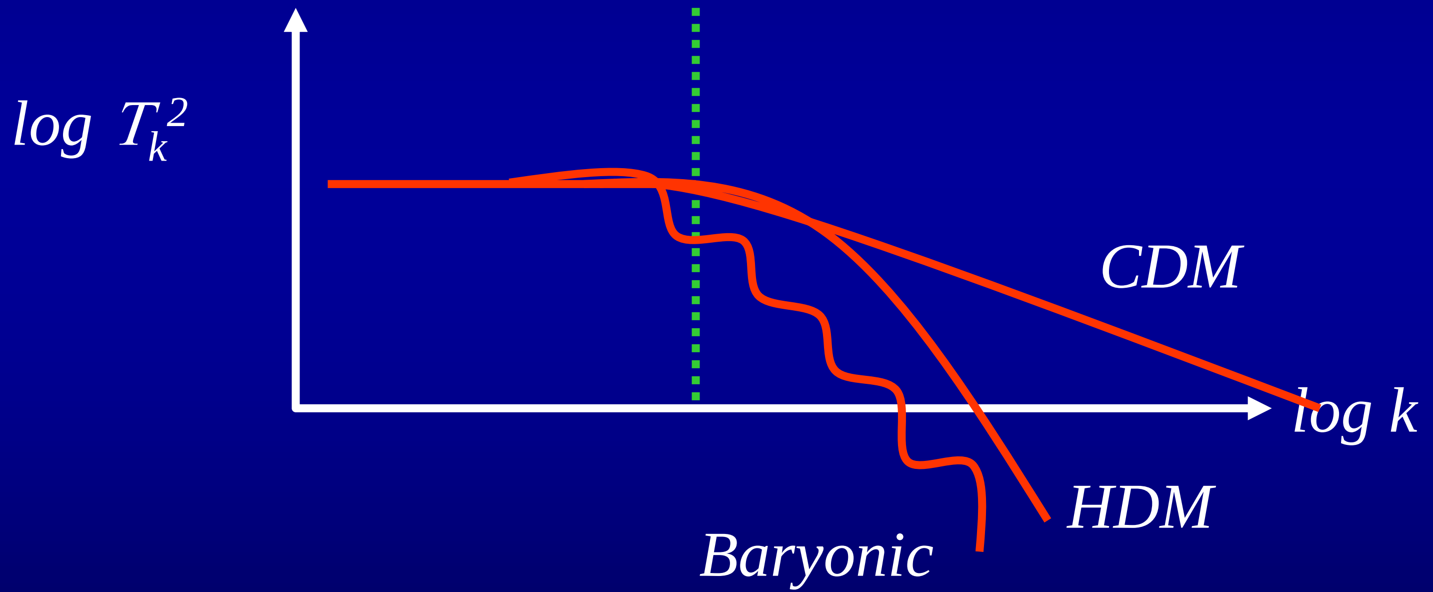
$$D_H(z_{eq}) = R_0 r_H(z_{eq}) \approx 16(\Omega_m h^2)^{-1} Mpc$$

Predicts hierarchical sequence of structure formation (smallest first).

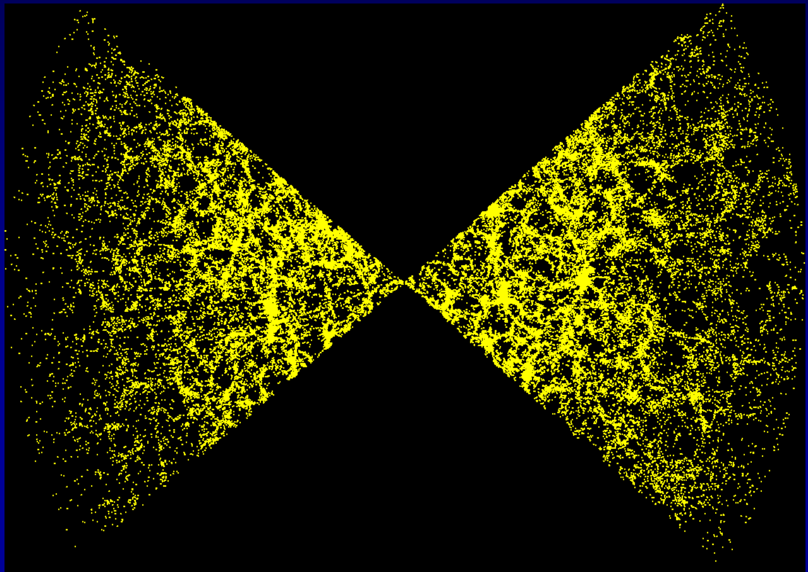
Dark Matter and the Power Spectrum

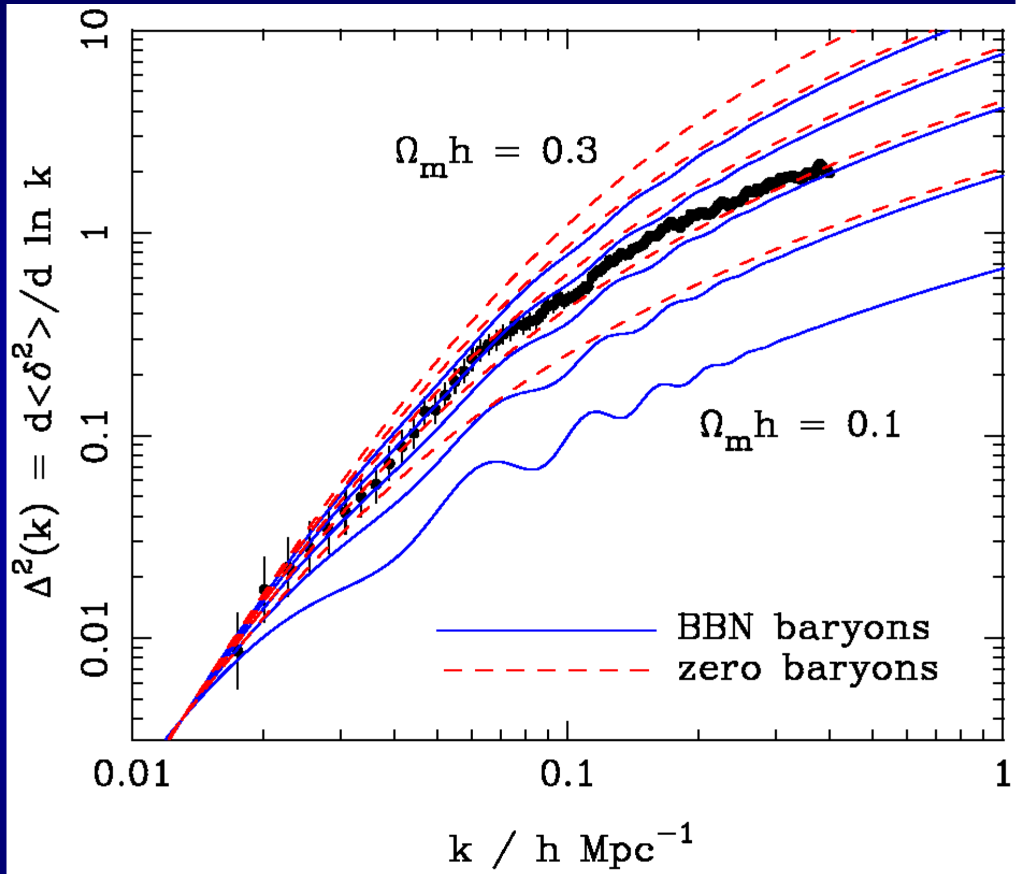
- **Transfer Functions:**
  - Can quantify all this with the transfer function, T(k):

$$\Delta_k^2(z=0) \propto T_k^2 \Delta_k^2(z=\infty)$$



Observations: 2dFGRS Power-Spectrum





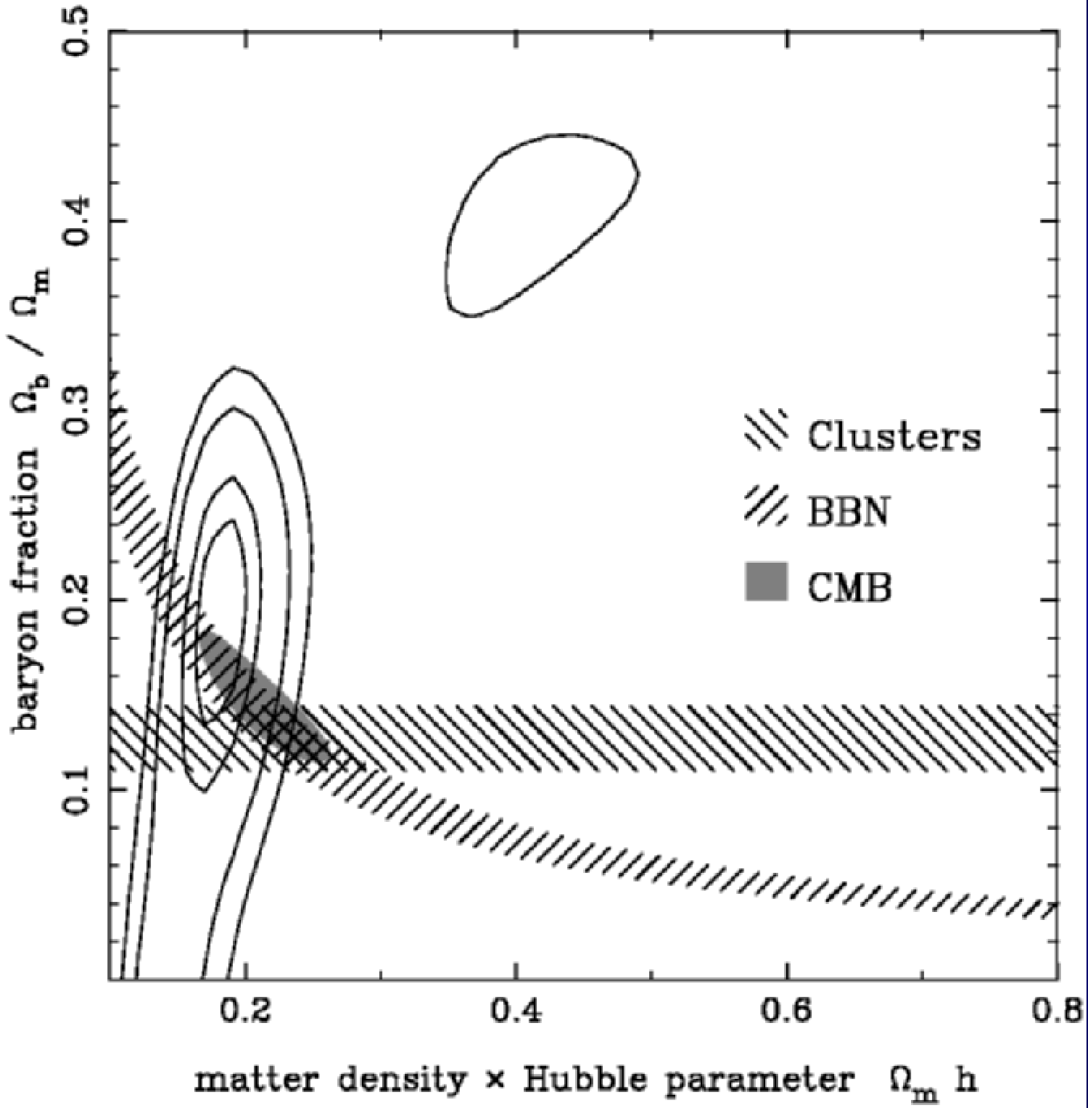
• No large oscillations or damping.

• Rules out a pure baryonic or pure HDM universe.

• Smooth power – expected for CDM-dominated universe.

• Detection of baryon oscillations – trace baryons.

Cosmological Parameters from 2dFGRS



Likelihood contours from the shape of the power spectrum:

Break scale:

Matter density:

$\Omega_m h = 0.19 \pm 0.02$

Baryon oscillations:

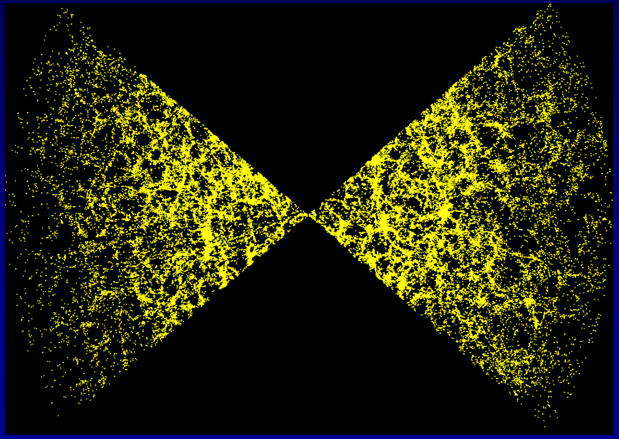
Baryon fraction

$= 0.18 \pm 0.06$ (if n = 1)

So $\Omega_m = 0.27\ (h/0.7)^{-1}$

$\Omega_B = 0.04\ (h/0.7)^{-1}$

Observations: 2dFGRS Power-Spectrum

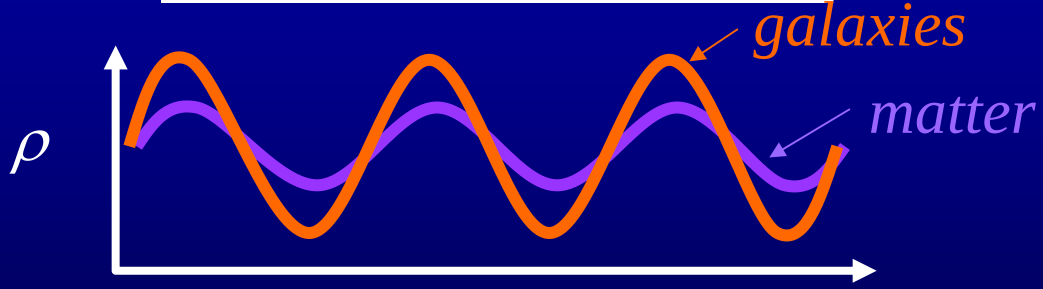


- Information about the amplitude of the power spectrum is confused, as we are looking a galaxies, not matter.

- We usually assume a linear relation between matter and density perturbations:

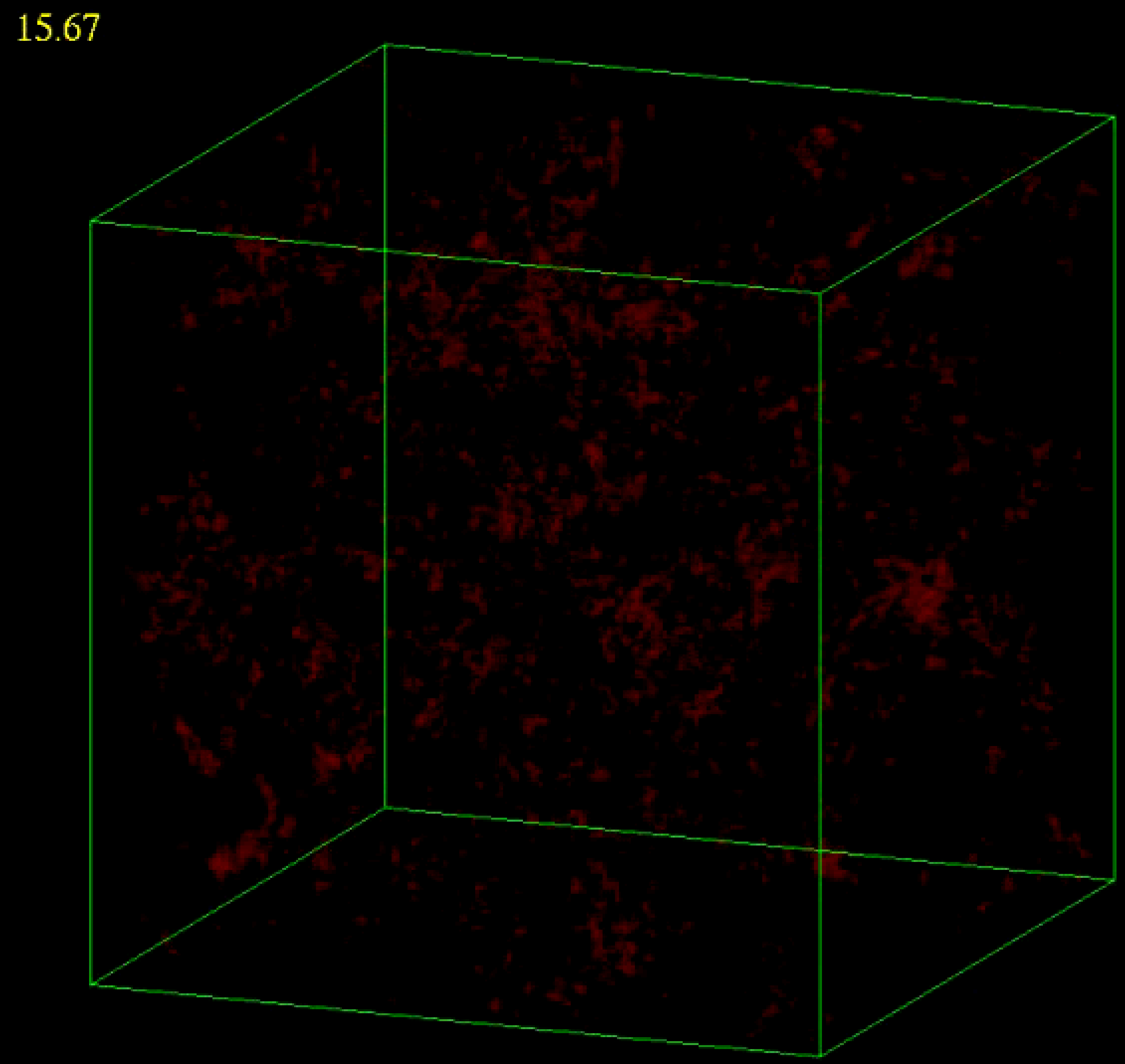
$$\delta_{galaxies} = b\delta_{matter}$$

b-bias parameter



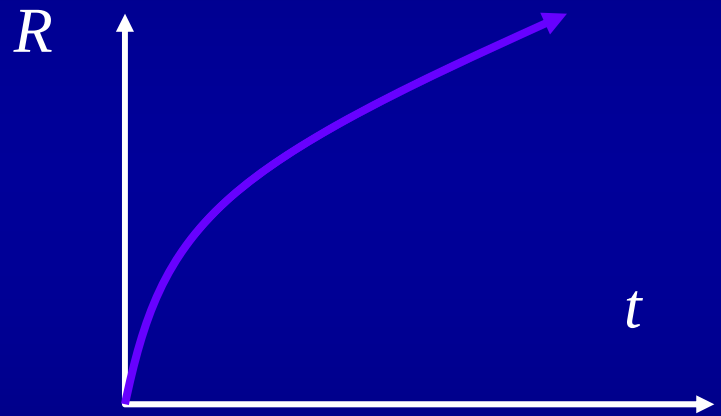
So amplitude of galaxy clustering mixes primordial power and process of galaxy formation.

Structure Formation in a CDM Universe

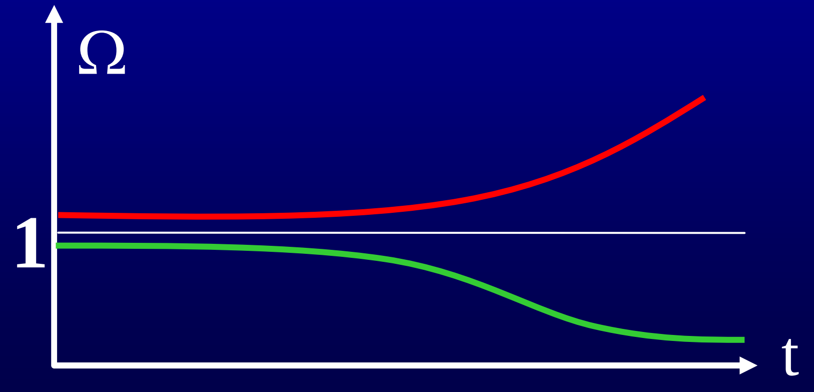


Cosmological Inflation

- Standard Model of Cosmology explains a lot (expansion, BBN, CMB, evolution of structure) but does not explain:
  - Origin of the Expansion:
    Why is the Universe expanding at t=0?

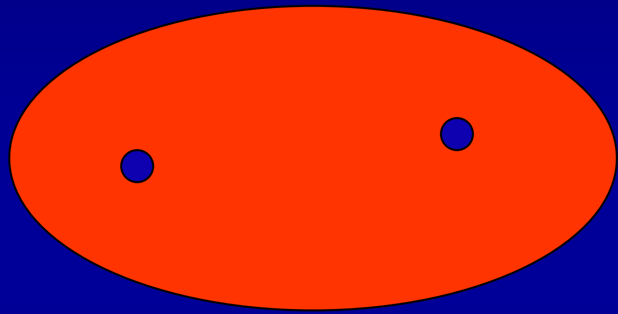


  - Flatness Problem:
    Why is Ω~1?

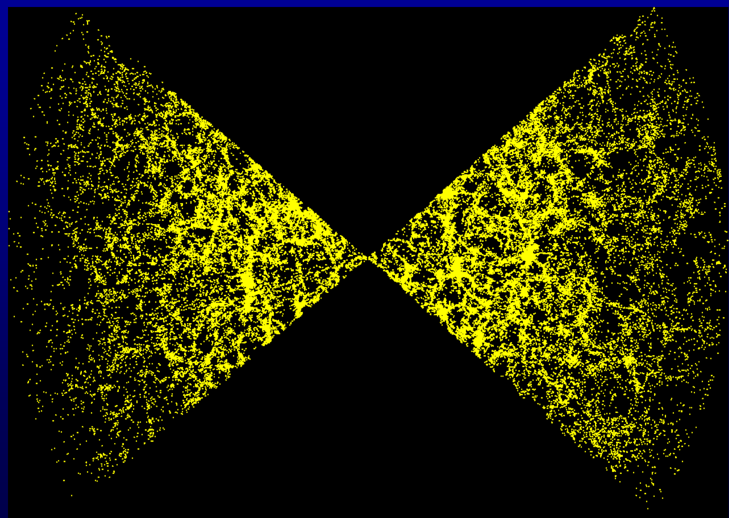


Cosmological Inflation

- Standard Model of Cosmology explains a lot (expansion, BBN, CMB, evolution of structure) but does not explain:
  - Horizon Problem:
    Why is the CMB so uniform over large angles, when the causal horizon is 1 degree?



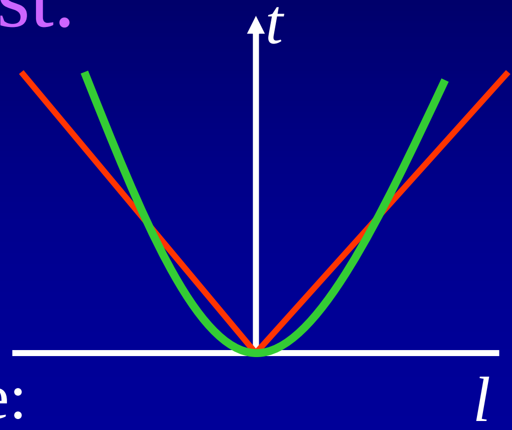
  - Structure Problem:
    What is the origin of the structure?



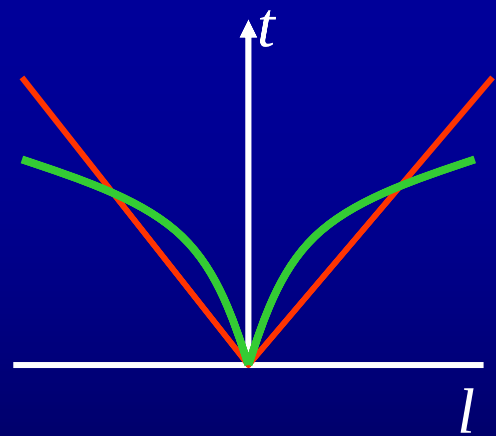
Cosmological Inflation

- Lets tackle the horizon problem first.

- Recall for $R \sim t^{1/2}$ we have a particle horizon:



- But if $R \sim t^{\alpha}$, $\alpha > 1$, can causally connect universe:



- More generally $\ddot{R} > 0$

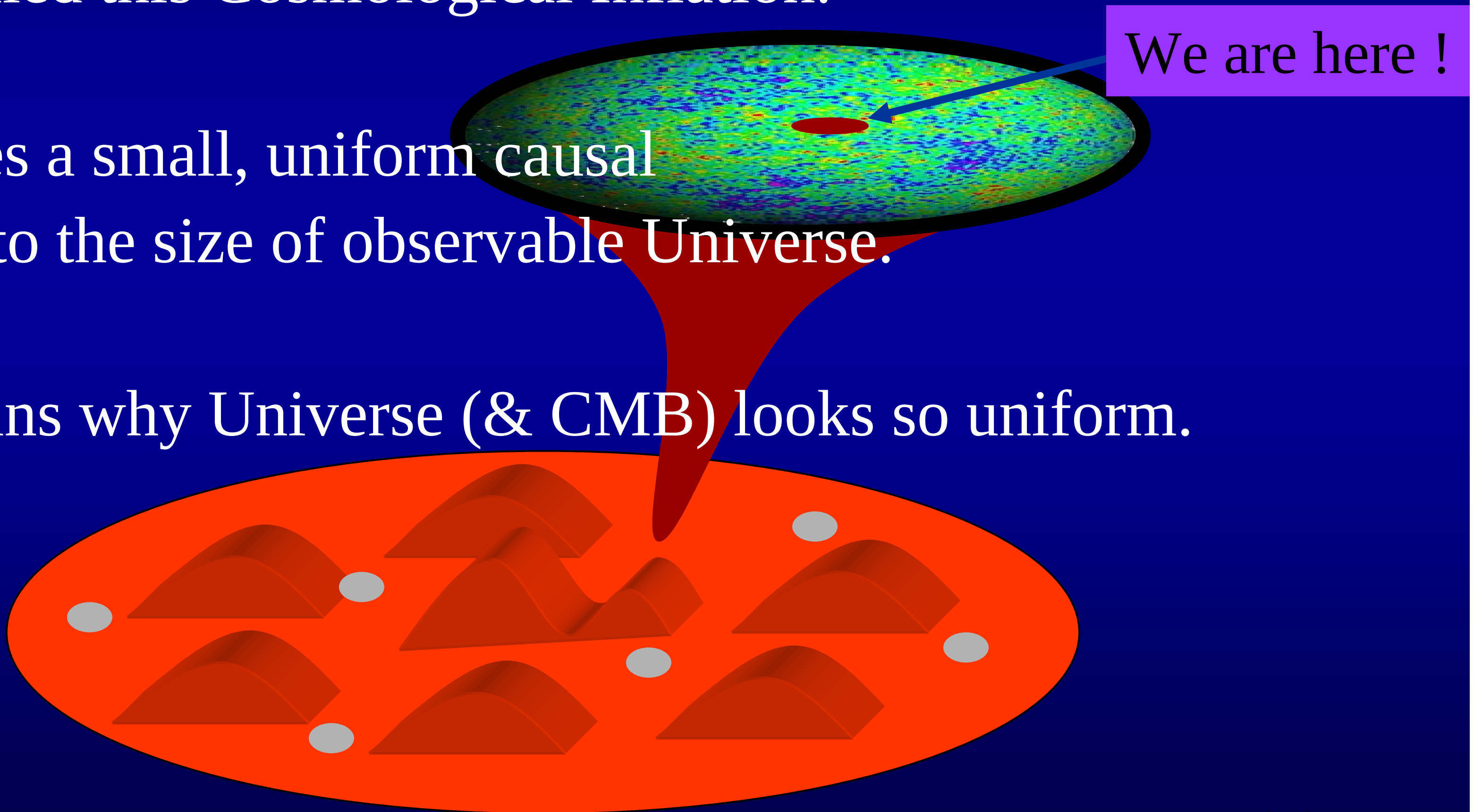
- This happens when:

$$\ddot{R} = -\frac{4\pi G}{3} R\left(\rho + 3p / c^2\right) > 0$$

which we get from Vacuum Energy, $p_V = -\rho_V c^2$

Cosmological Inflation

- In 1980 Alan Guth proposed that the Early Universe had undergone acceleration, driven by vacuum energy.
- He called this Cosmological Inflation.
- Inflates a small, uniform causal patch to the size of observable Universe.
- Explains why Universe (& CMB) looks so uniform.



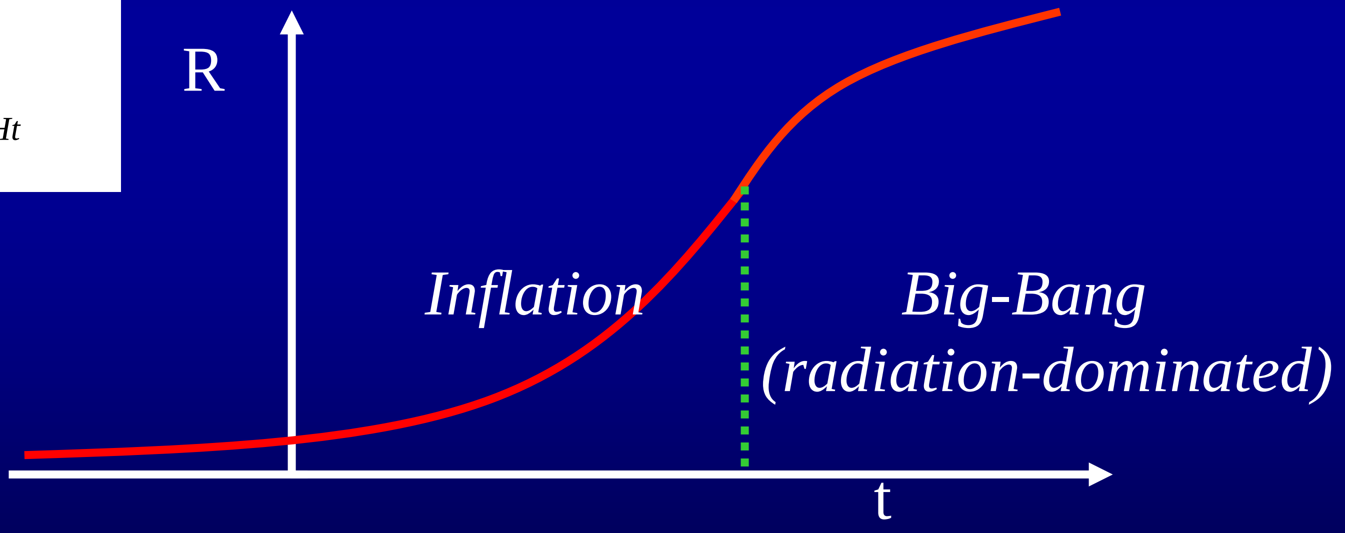
Cosmological Inflation

- Expansion Problem:
- Vacuum energy leads to acceleration of the Early Universe
- This powers the expansion (recall Eddington).

$$H^2 \propto \rho_V = \text{const}$$

$$\dot{R} = HR$$

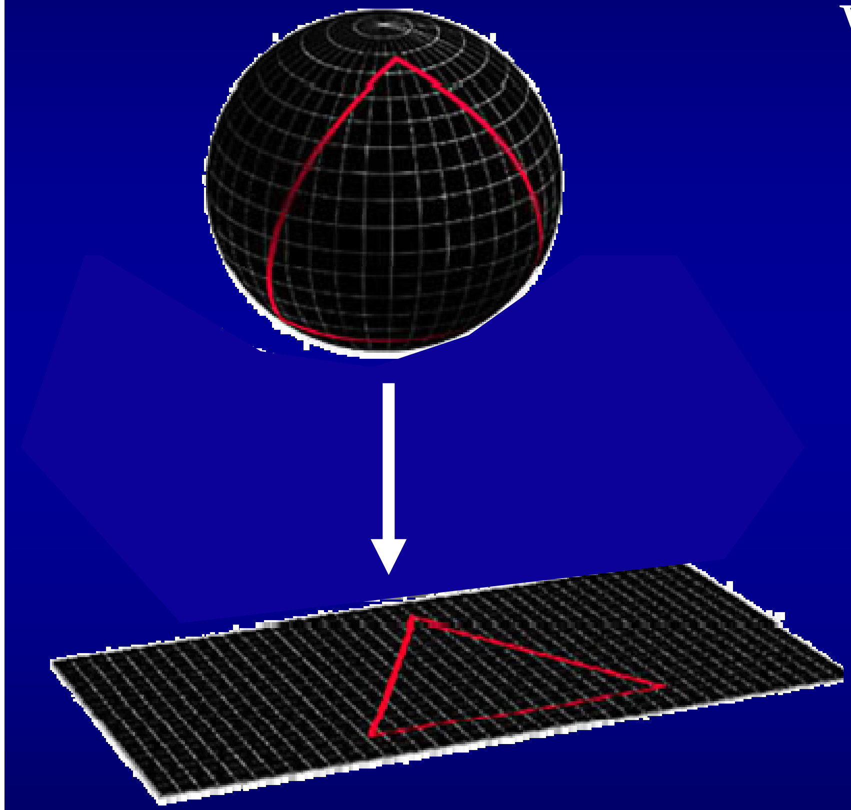
$$R = R_0 e^{Ht}$$



- Need Inflation to end to start BB phase.

Cosmological Inflation

- The Flatness Problem: Recall that if we expand a model with curvature, it looks locally flat:



$$H^2 = \frac{8\pi G\rho_V}{3} - \frac{c^2 k}{R^2}$$

$$R \to \infty$$

$$H^2 = \frac{8\pi G\rho_V}{3}$$

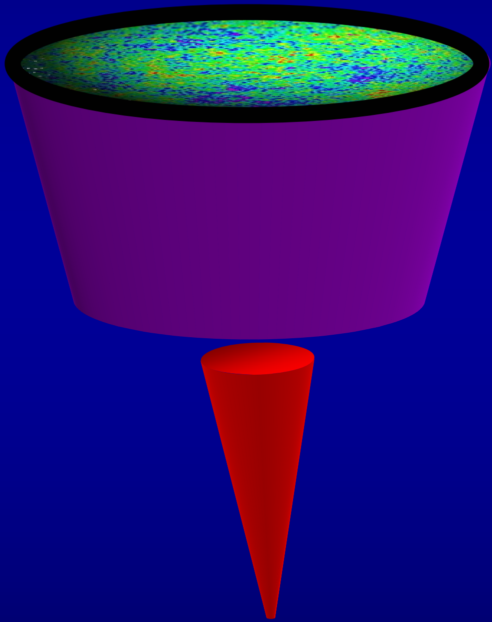
$$k = 0$$

- So Inflation predict $\Omega=\Omega_m+\Omega_v=1$ to high accuracy.
- Compare with SN & galaxy clustering results.

Cosmological Inflation

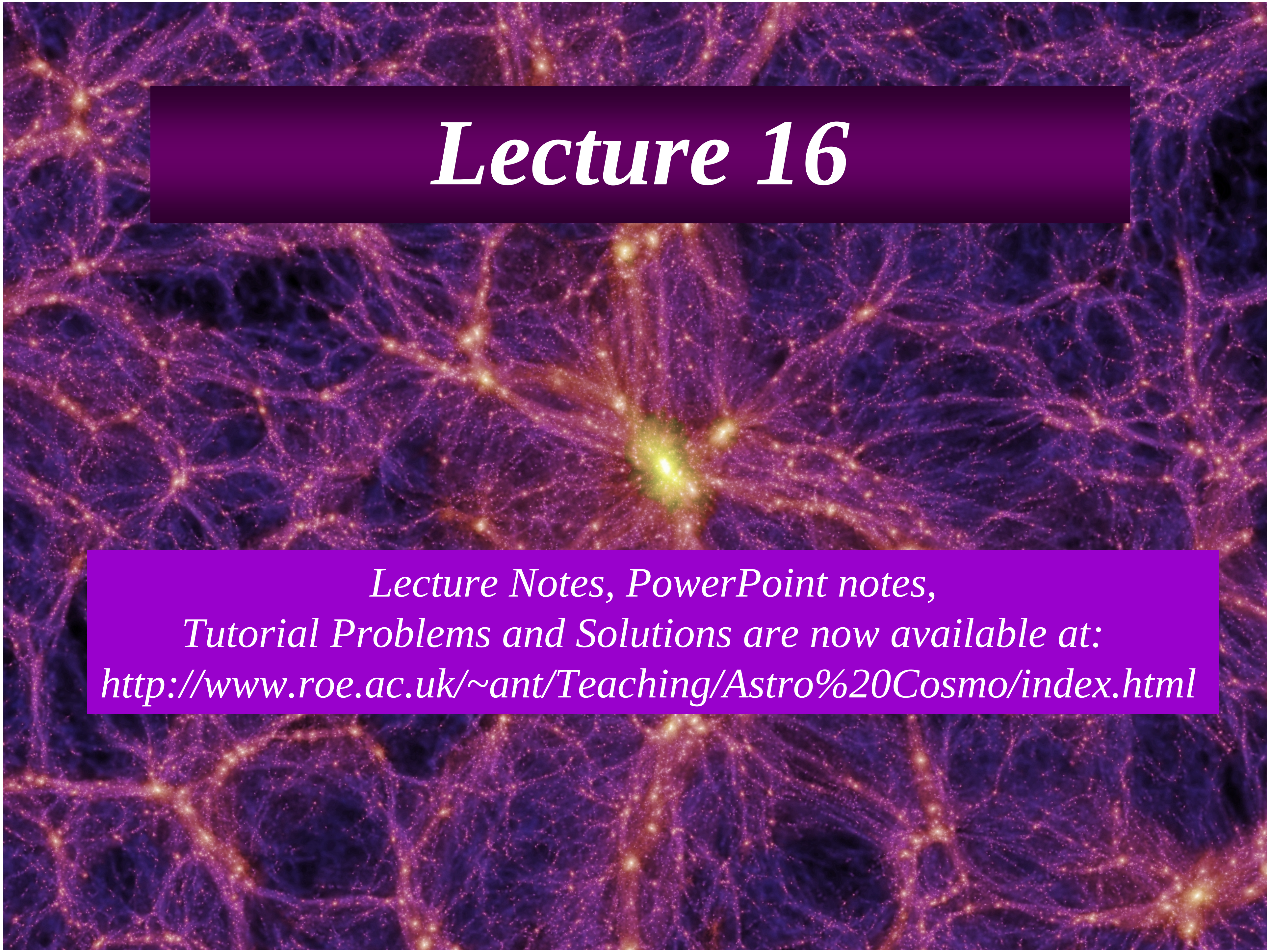
- How much Inflation do we need?
- Usually assume Inflation happens at GUT era, E_{GUT}~10^{15}GeV.
- So how large is the current horizon at the GUT era?

$$d_H(today) = 6000\,h^{-1}Mpc$$
$$d_H(GUT) = d_H(today)/(1+z_{GUT}),$$
$$1+z_{GUT} = \frac{E_{GUT}}{E_{CMB}} = \frac{10^{15}\,GeV}{2.5\times10^{-13}\,GeV} \approx 10^{27}$$
$$d_H(GUT) \approx 10^{-24}\,Mpc \approx 10^{-2}\,m$$



- But causal horizon at GUT era is just $d_{GUT}=ct_{GUT}=3x10^{-27}$m.
- So need to stretch GUT horizon by factor $a_{Infl}=10^{29} \sim e^{60}$

Lecture 16



Lecture Notes, PowerPoint notes,
Tutorial Problems and Solutions are now available at:
http://www.roe.ac.uk/~ant/Teaching/Astro%20Cosmo/index.html

Dynamics of Inflation

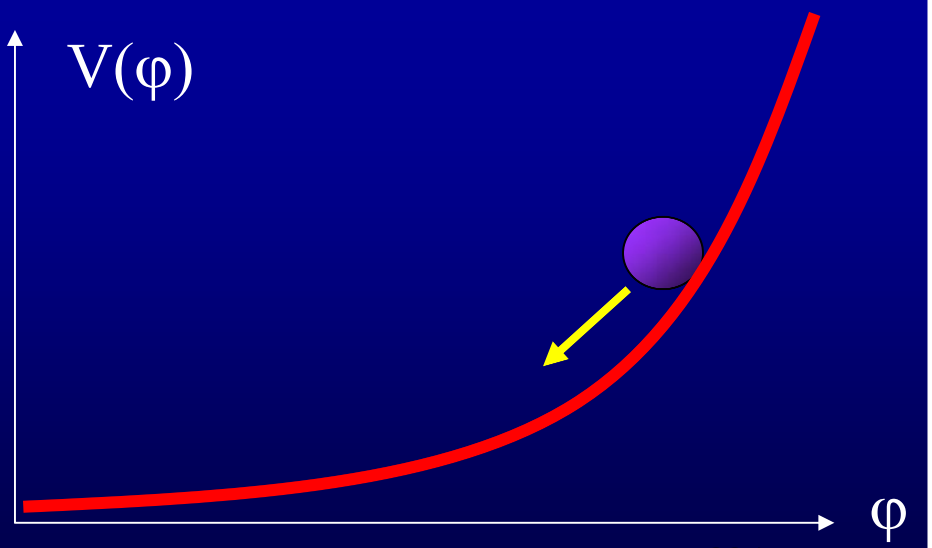
- We need a dynamical process to switch off inflation.
- Simplest models are based on scalar fields (spin-0), φ, the inflaton (e.g. π-mesons, Higgs bosons). No idea what φ is…
- Must obey energy equation: $E^2 = p^2c^2 + m^2c^4$
- Quantize to get Klein-Gordon equation: $p = -i\hbar\nabla, \quad E = i\hbar\partial_t$

$$\ddot{\varphi} - c^2\nabla^2\varphi = -(m^2c^4\hbar^{-2})\varphi$$

- Assume φ is uniform and add expansion term, 3H:

$$\ddot{\varphi} + 3H\dot{\varphi} = -V'(\varphi), \quad V' = {}^{dV}\!/_{d\varphi}$$

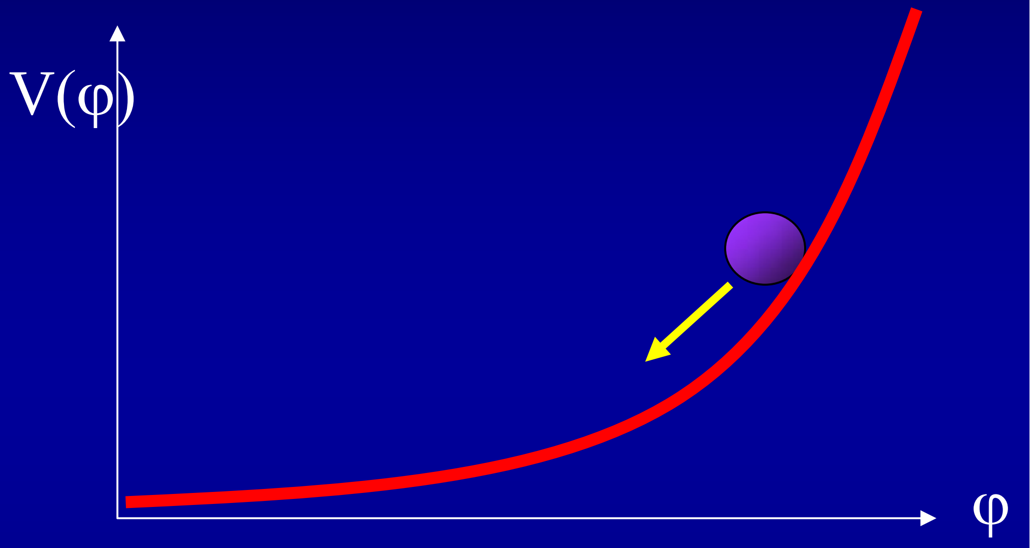
- Equation of a particle in a potential $V=(m^2c^4\hbar^{-2})\varphi^2/2$.



Dynamics of Inflation

- Evolution of a scalar field, φ , in a potential V(φ).
- Energy of scalar field:

$$\rho = \frac{1}{2}\dot{\varphi}^2 + V(\varphi)$$



- Assume Slow Roll:

$$\dot{\varphi}^2 << V$$

$$\rho \approx V \approx const.$$

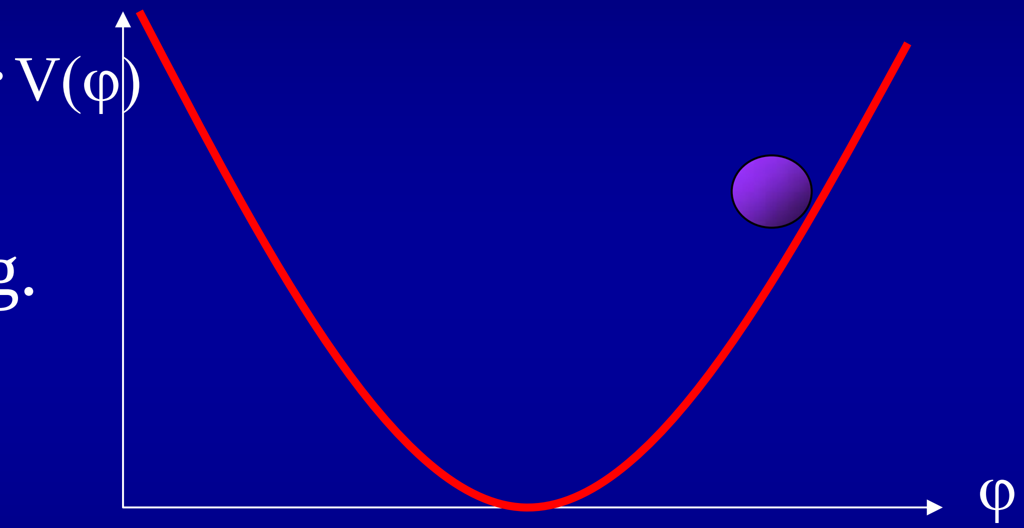
- So like a vacuum energy with pressure

$$p = -\rho c^2$$

- Drives acceleration of expansion.

Dynamics of Inflation

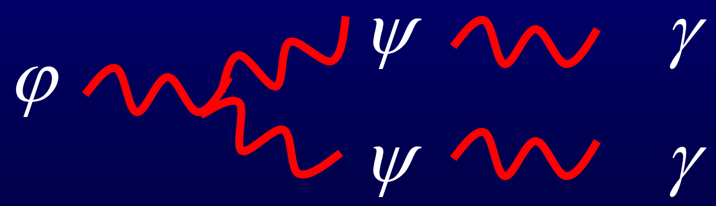
- The End of Inflation.
- Eventually the inflaton will reach the bottom of the potential and oscillate.



- No longer slow-rolling.

$$\dot{\varphi}^2 >> V$$

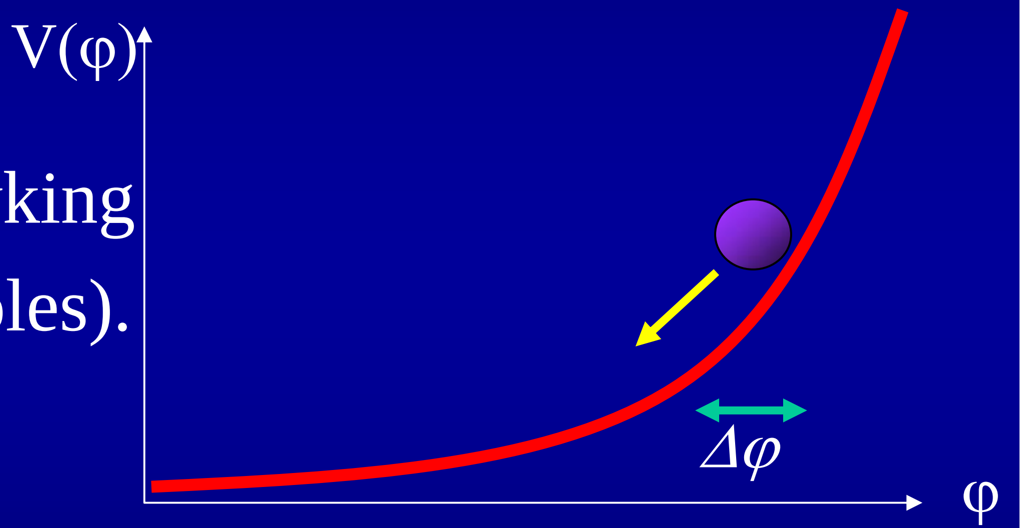
- The Inflaton can then decay into other particles and radiation, re-heating Universe for radiation-domination.



Dynamics of Inflation

- The Origin of Structure in Inflation.
- The evolution of the inflaton has quantum fluctuations

$$\varphi = \varphi_{classical} + \delta\varphi_{quantum}$$



- Fluctuations due to Hawking Radiation (c.f. Black Holes).

$$\delta\varphi = \frac{\hbar H}{2\pi}$$

- So the universe expands at different rates, leading to density perturbations:

$$\delta = \frac{\delta\rho}{\rho} = -3\frac{\delta R}{R} = -3H\delta t = -3H\frac{\delta\varphi}{\dot{\varphi}}, \qquad \delta t = \frac{\delta\varphi}{\dot{\varphi}}$$

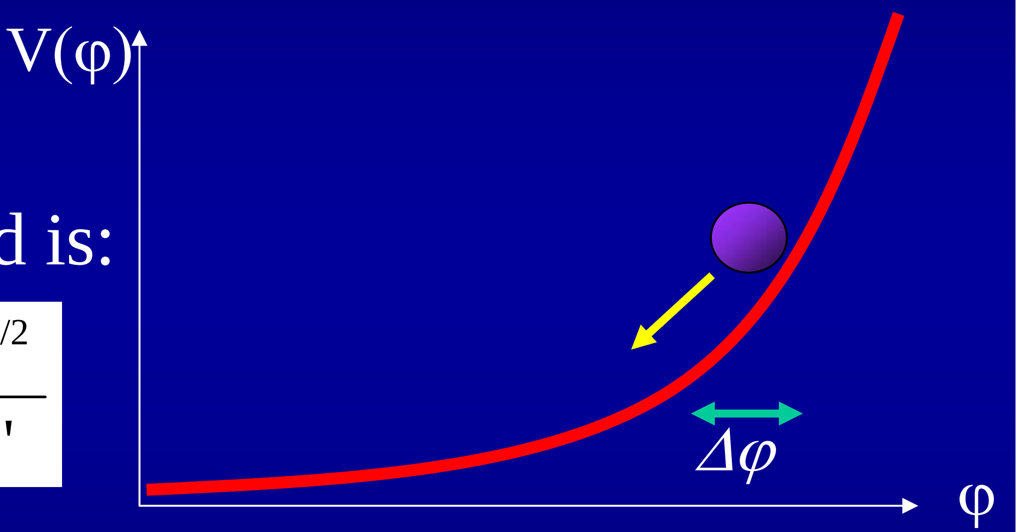
Dynamics of Inflation

- The Origin of Structure in Inflation.
- During Slow-Roll $|\ddot{\varphi}| < |V'|$ equation of motion simplifies:

$$3H\dot{\varphi} = -V', \Rightarrow \dot{\varphi} = -\frac{V'}{3H}$$

- The induced density field is:

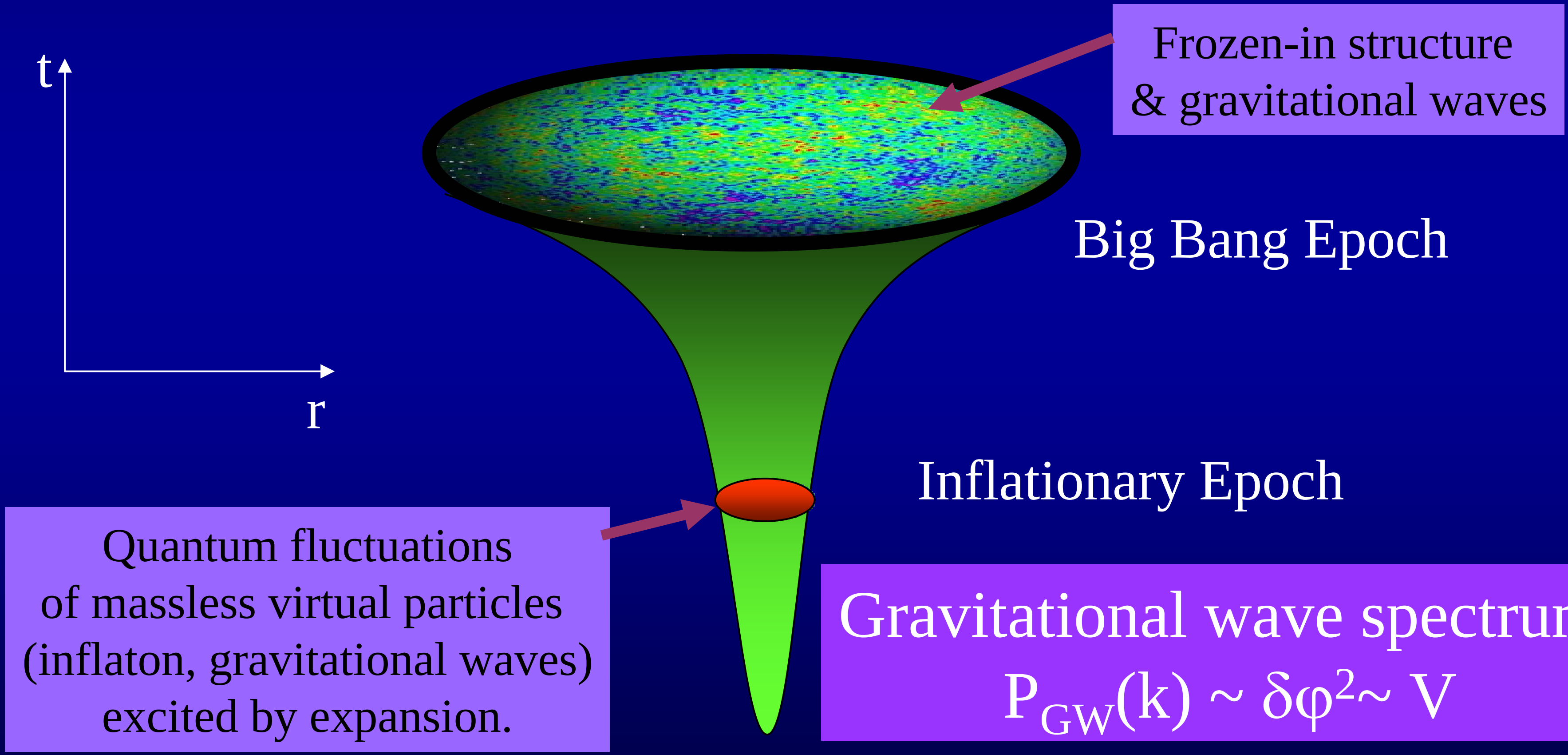
$$\delta = -3H\frac{\delta\varphi}{\dot{\varphi}} = \frac{9\hbar H^3}{2\pi V'} \propto \frac{V^{3/2}}{V'}$$



- The density power spectrum is $P(k) \propto \langle \delta^2 \rangle \propto \frac{V^3}{V'^2}$
- Exponential expansion generates a fractal in the potential field: so spectral index is n=1.

A Gravitational Wave Background from Inflation

- Structure created by freezing in quantum fluctuations during an inflationary epoch. Leaves imprint in structure.



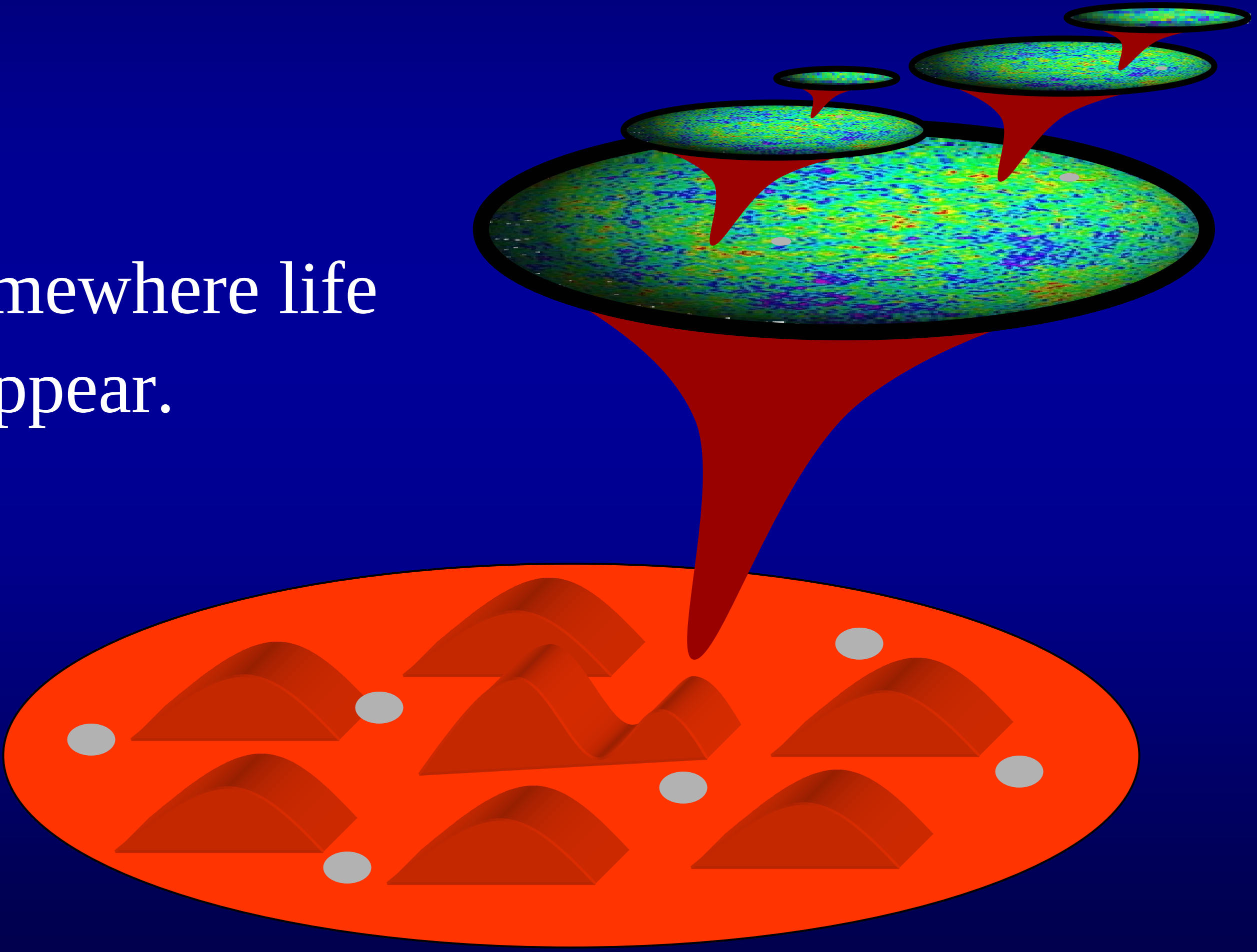
Gravitational wave spectrum:

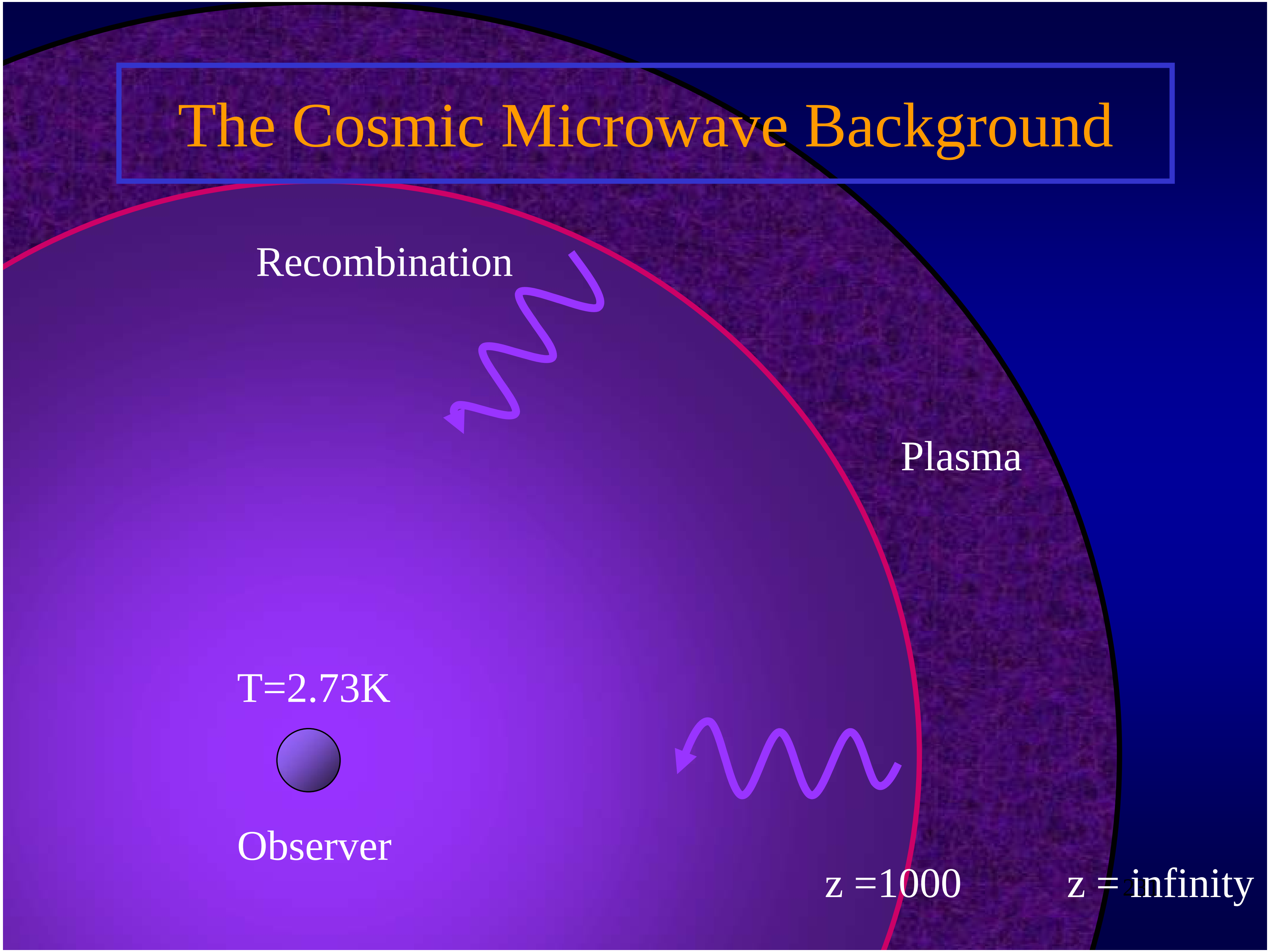
$$P_{GW}(k) \sim \delta\varphi^2 \sim V$$

The Multiverse in Inflation

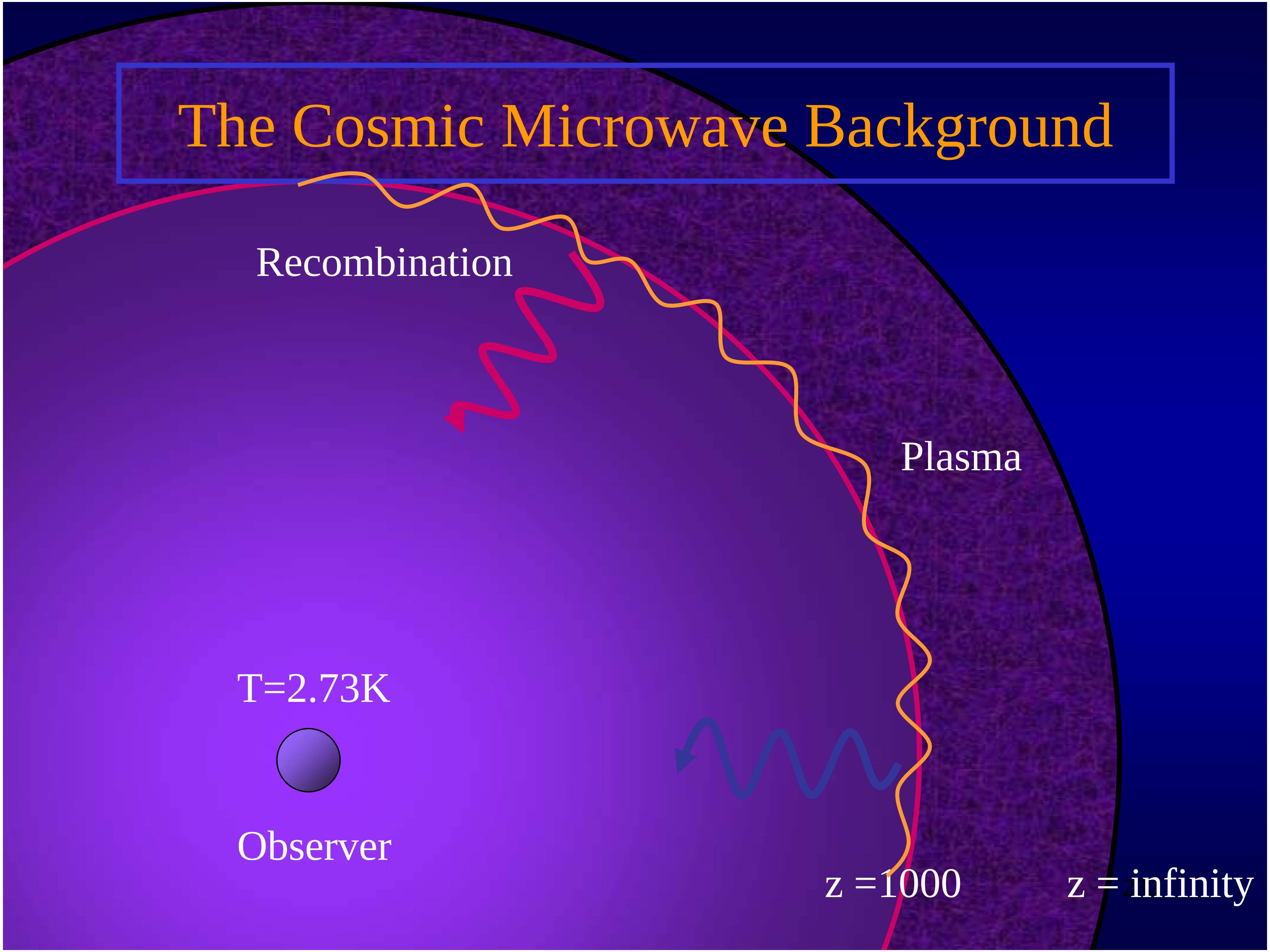
- Inflation can happen lots of times, producing a Multiverse.

- So somewhere life will appear.

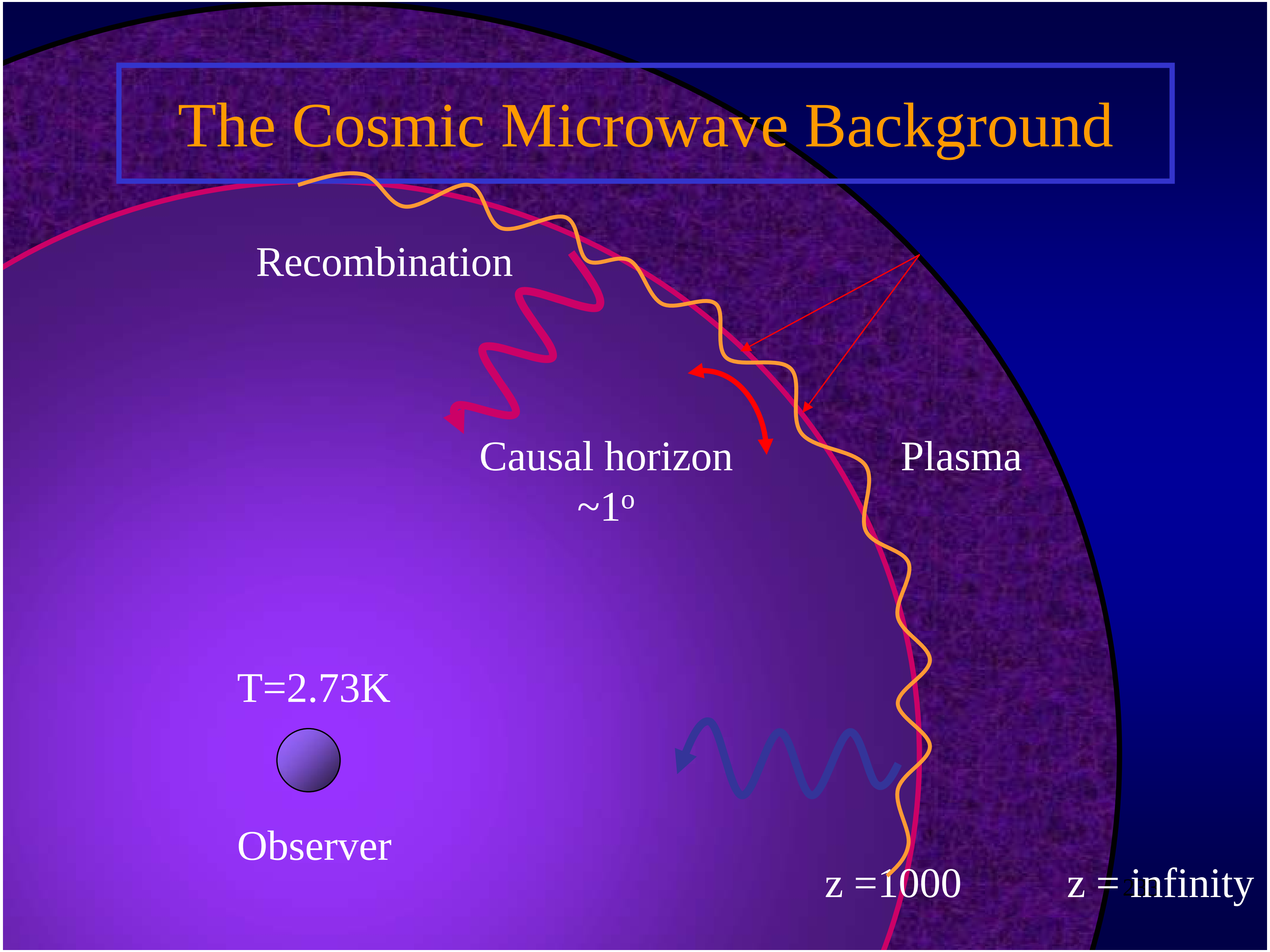



The Cosmic Microwave Background
Recombination
Plasma
T=2.73K
Observer
z =1000
z = infinity

The Cosmic Microwave Background

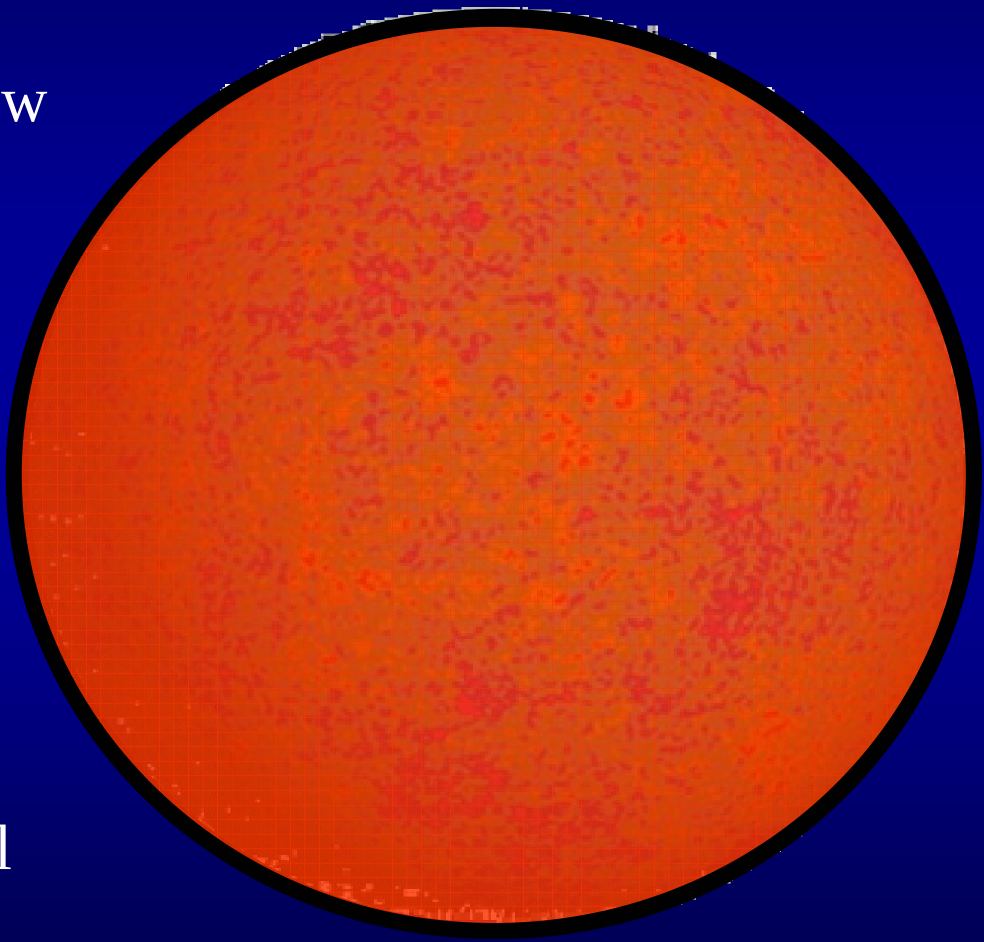


The Cosmic Microwave Background



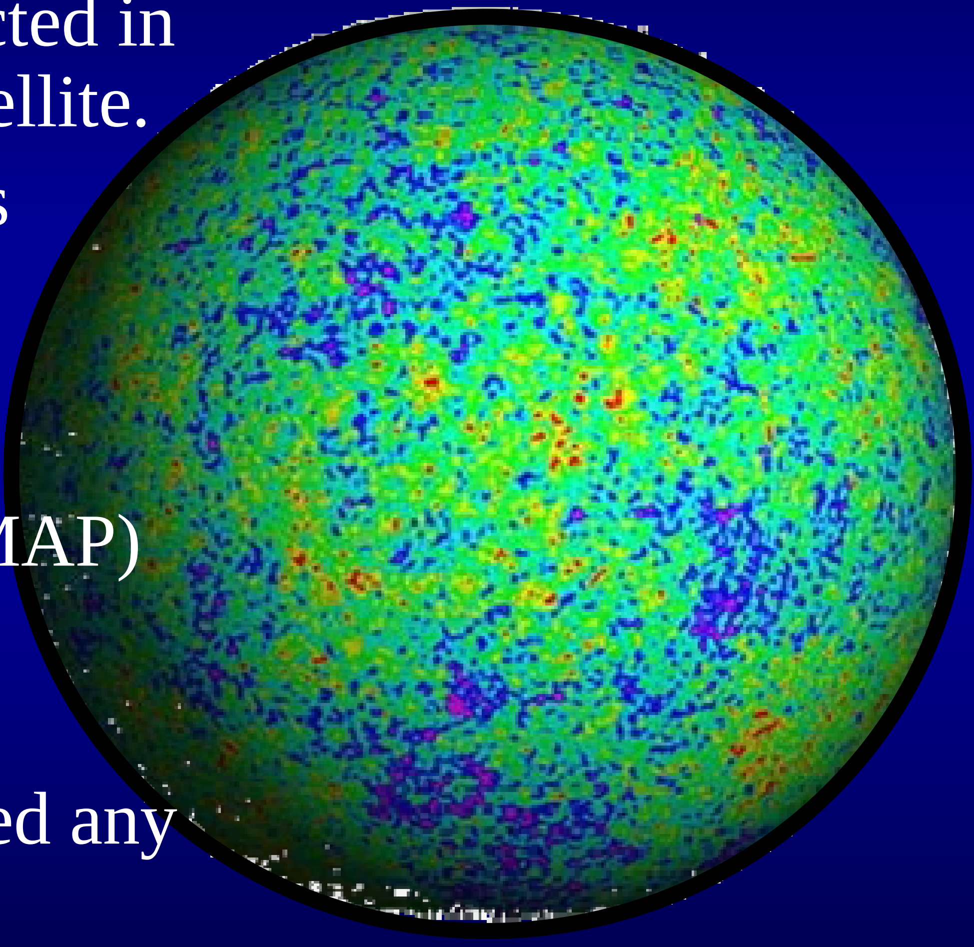
Anisotropies in the CMB

- Recall in 1930’s George Gamow predicts an afterglow from the Big Bang.
- Detected in 1969 by Arno Penzias & Robert Wilson (who won Nobel Prize).
- Formed at reionization at z = 1100.
- In 1960’s Sachs and Wolfe predicted there should be anisotropies due to potential perturbations.



Anisotropies in the CMB

- Anisotropies first detected in 1992 by the COBE satellite.
- On large angular scales $\Delta T/T=10^{-5}$
- Detail images from the Wilkinson Microwave Anisotropy Probe (WMAP) in 2003.
- First year data.
- Third year data expected any day now.



Spherical Harmonic Analysis of the CMB

- Expand fluctuations in CMB temperature field in spherical harmonics on the celestial sphere:

$$\Delta \mathrm{T}(\theta,\phi) = \sum_{\ell m} T_{\ell m}\, Y_{\ell m}(\theta,\phi)$$

Spherical Harmonic Analysis of the CMB

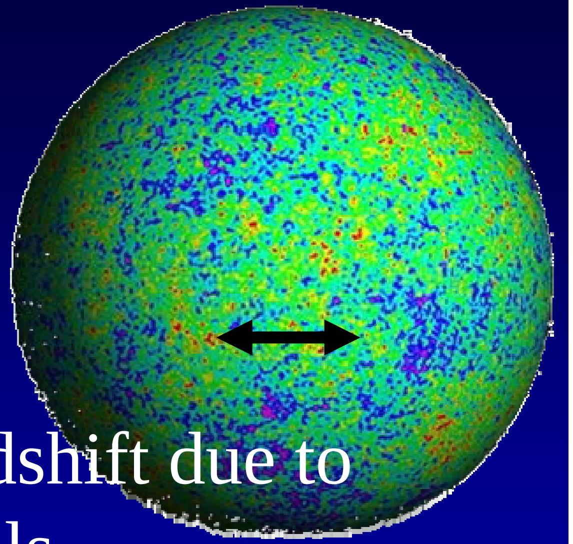
- The squared harmonic modes of the temperature can be averaged to give a power spectrum:

$$\left\langle \left| T_{\ell m} \right|^2 \right\rangle = C_\ell^{TT}$$

where

$$\frac{\ell(\ell+1)C_\ell^{TT}}{2\pi} \approx \left(\frac{\Delta T}{T} \right)^2 (\theta \approx 2\pi / \ell)$$

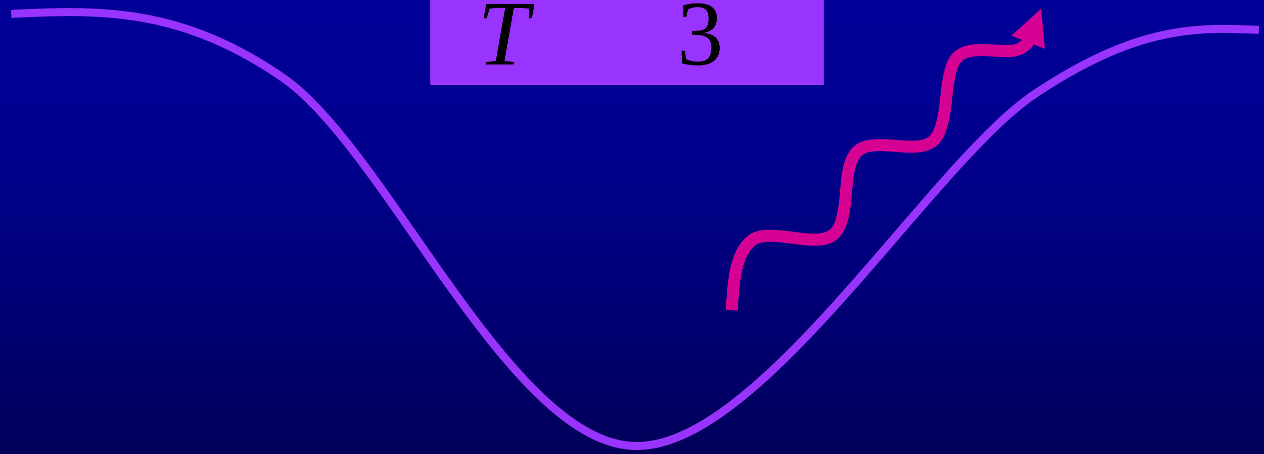
ΔT on super-horizon scales (>1º)



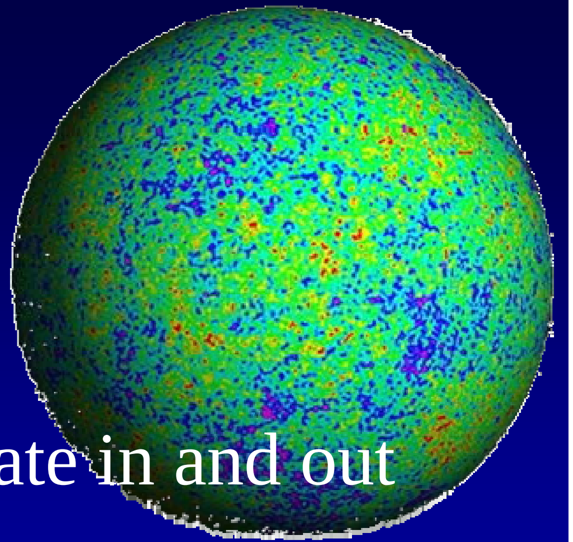
- Sachs-Wolfe Effect: Gravitational redshift due to photons climbing out of potential wells,

$$\frac{\Delta T}{T} = \frac{1}{3}\Phi$$

Φ Newtonian potential

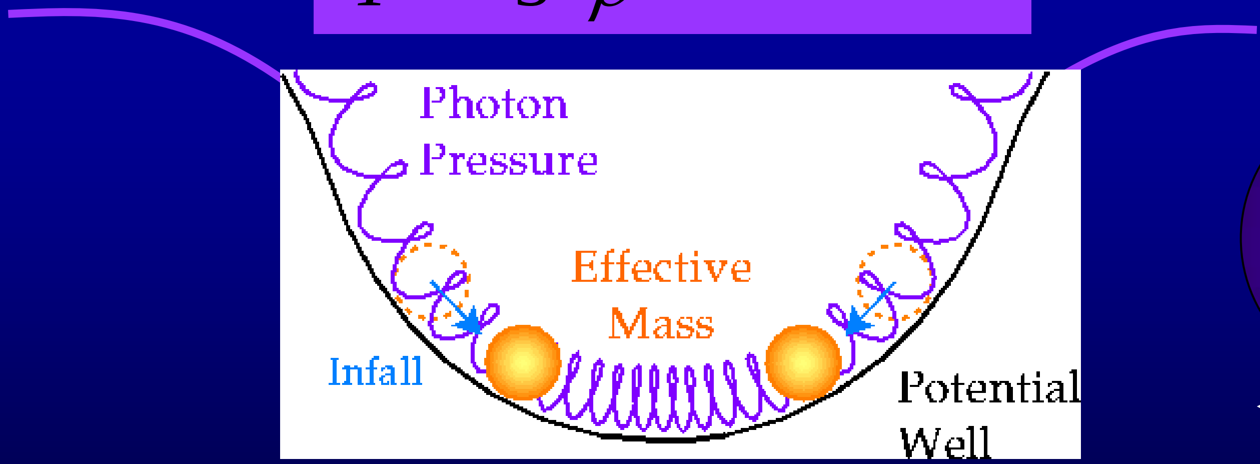


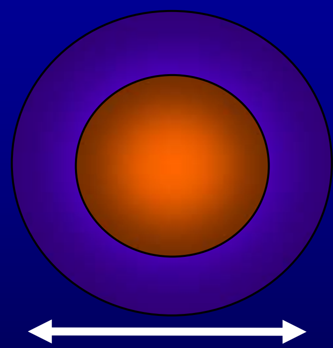
ΔT on sub-horizon scales (<1^{o})



- Acoustic Oscillations: Baryons oscillate in and out of dark matter potential wells,

$$\frac{\Delta T}{T} = \frac{4}{3}\frac{\delta\rho}{\rho} \approx A\cos(k\tau)$$



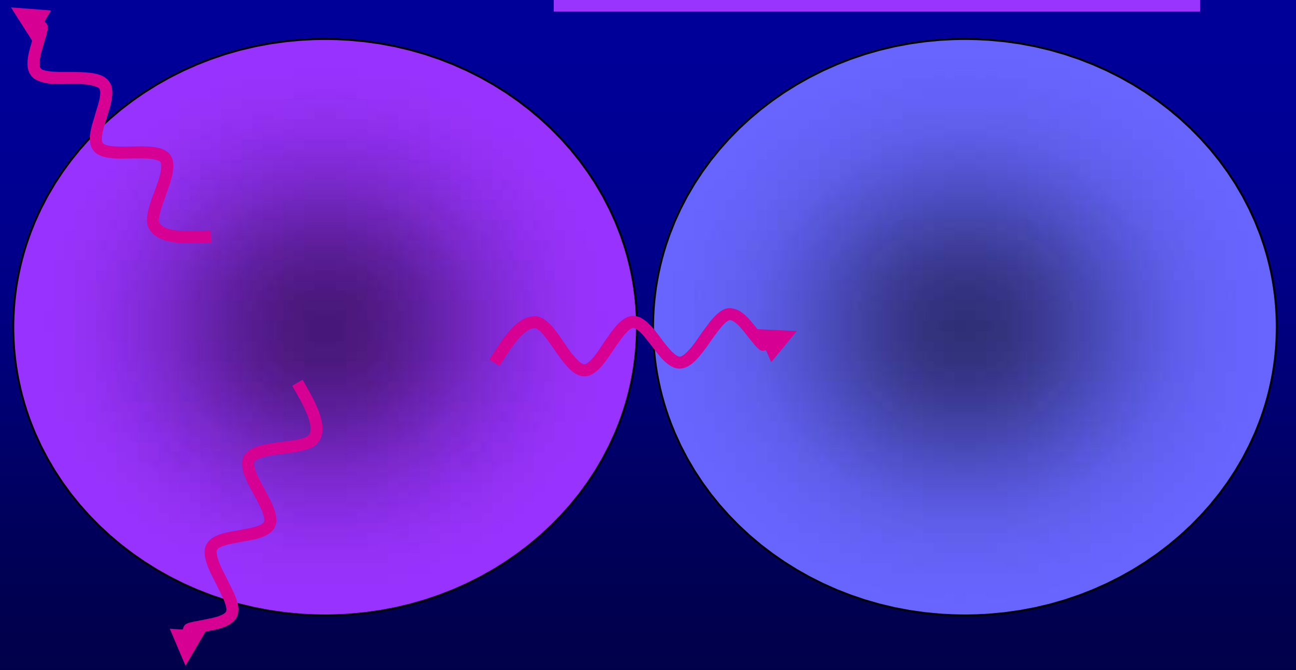


$\lambda << ct$

ΔT on sub-horizon scales (<1°)

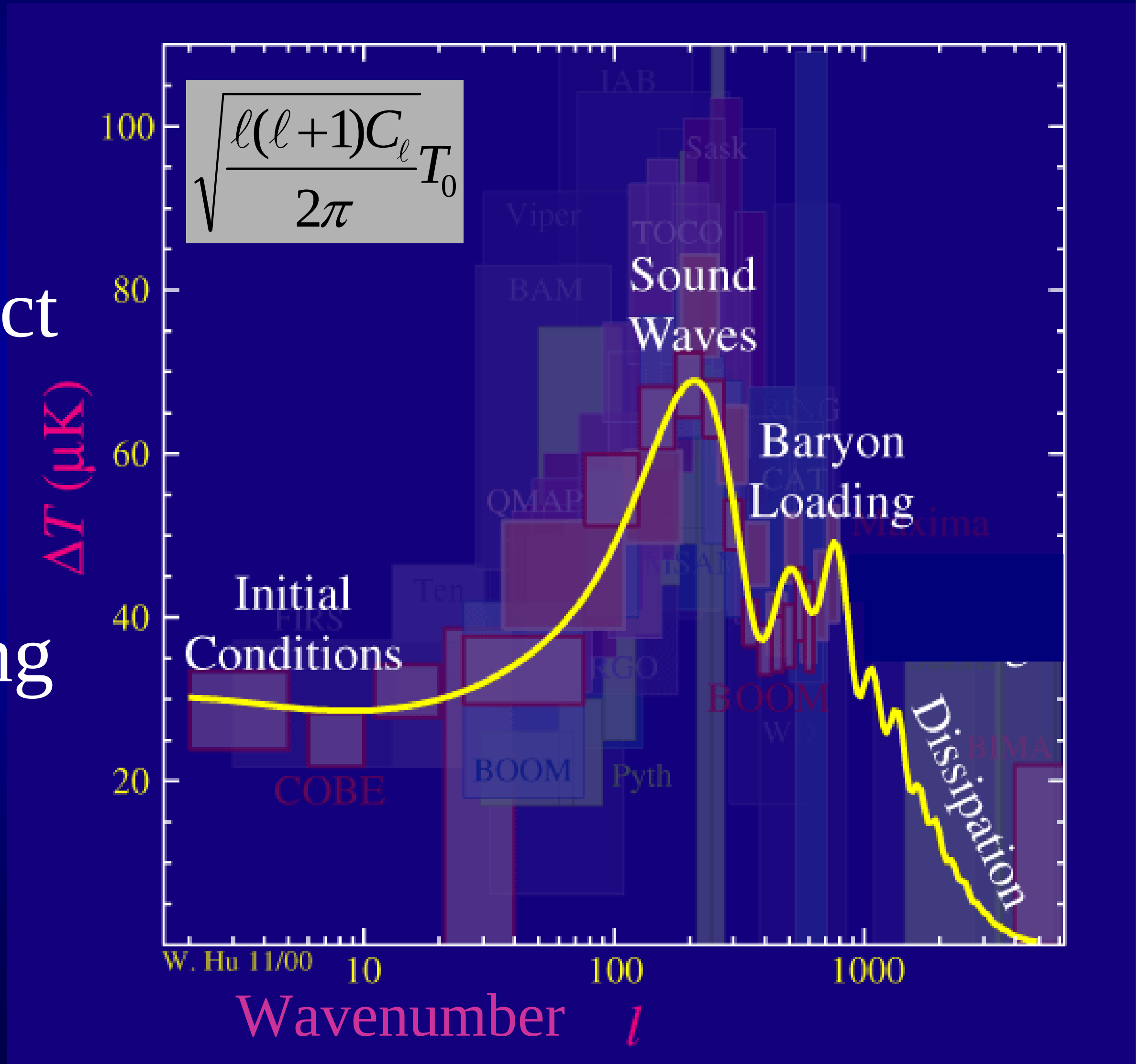
- Photon Diffusion: Photons random walk out of potential wells,

$$\frac{\Delta T}{T} \propto \exp(-\ell^2 \theta_s^2 / 2)$$

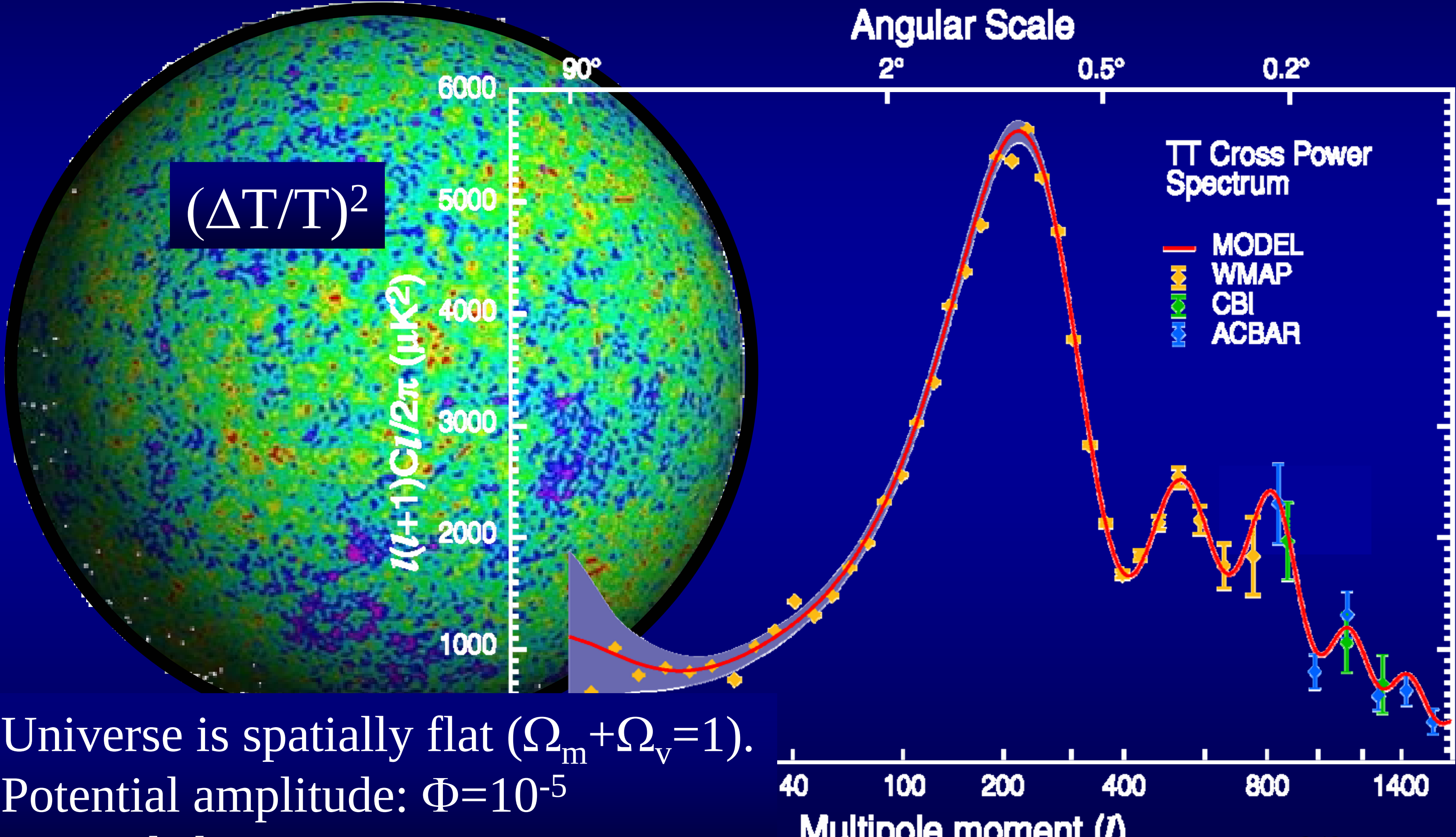


The CMB Temperature Power Spectrum

- Sachs-Wolfe Effect
- Acoustic Peaks
- Diffusion Damping

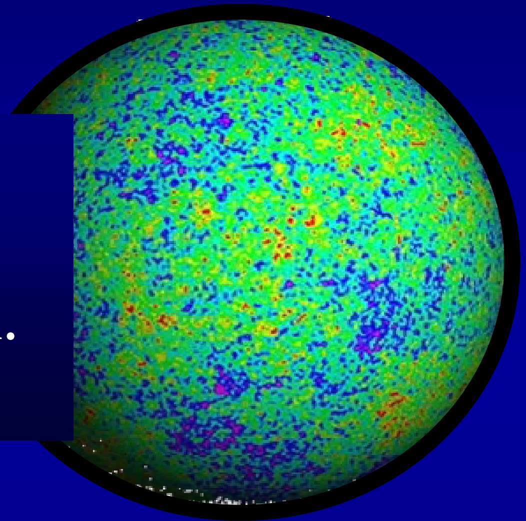


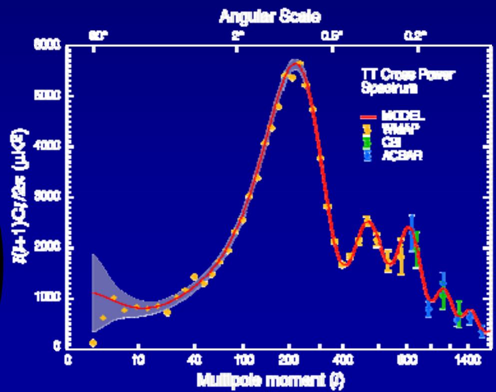
CMB Temperature Power Spectrum seen by WMAP (2003)



- Universe is spatially flat ($\Omega_m+\Omega_v=1$).
- Potential amplitude: $\Phi=10^{-5}$
- Spectral slope n~1

Geometry of the Universe from the CMB Temperature Power Spectrum





- The peak of the CMB power spectrum is given by the horizon size at recombination.
- Hence it is a fixed “ruler”.

Matter only universe:

$$D_H(z) = R_0 \int_z^\infty dr = \frac{2c}{H_0}\left[\Omega_m(1+z)\right]^{-1/2} \approx 181(\Omega_m h^2)^{-1/2} Mpc$$

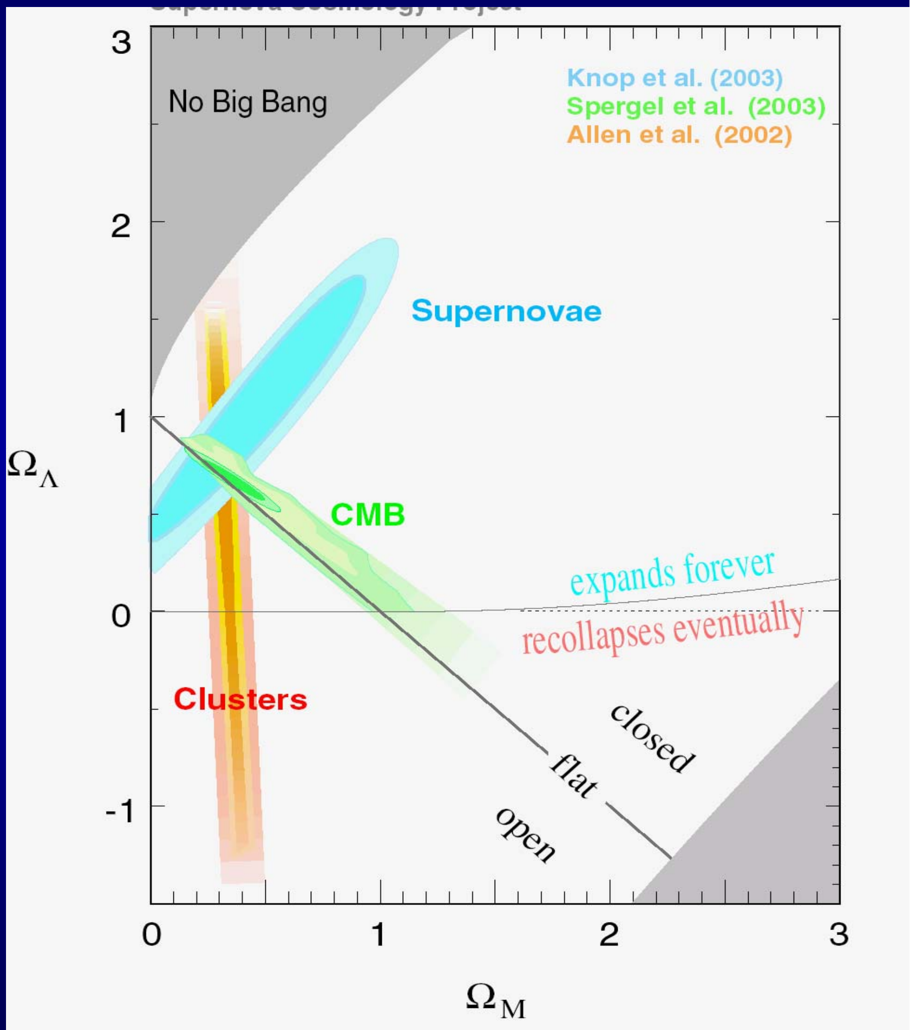
$$\theta_H \approx 1.8\Omega_m^{-1/2} \deg$$

Dark Matter and Vacuum Energy

- The CMB acoustic peak measure spatial curvature of Universe.
- Combine with supernova, galaxy clusters, galaxy clustering.
- Likelihood contours for Ω_m-Ω_V plane.

$$\Omega_V = 0.7, \Omega_m = 0.3$$

- So four independent methods converge on one model.

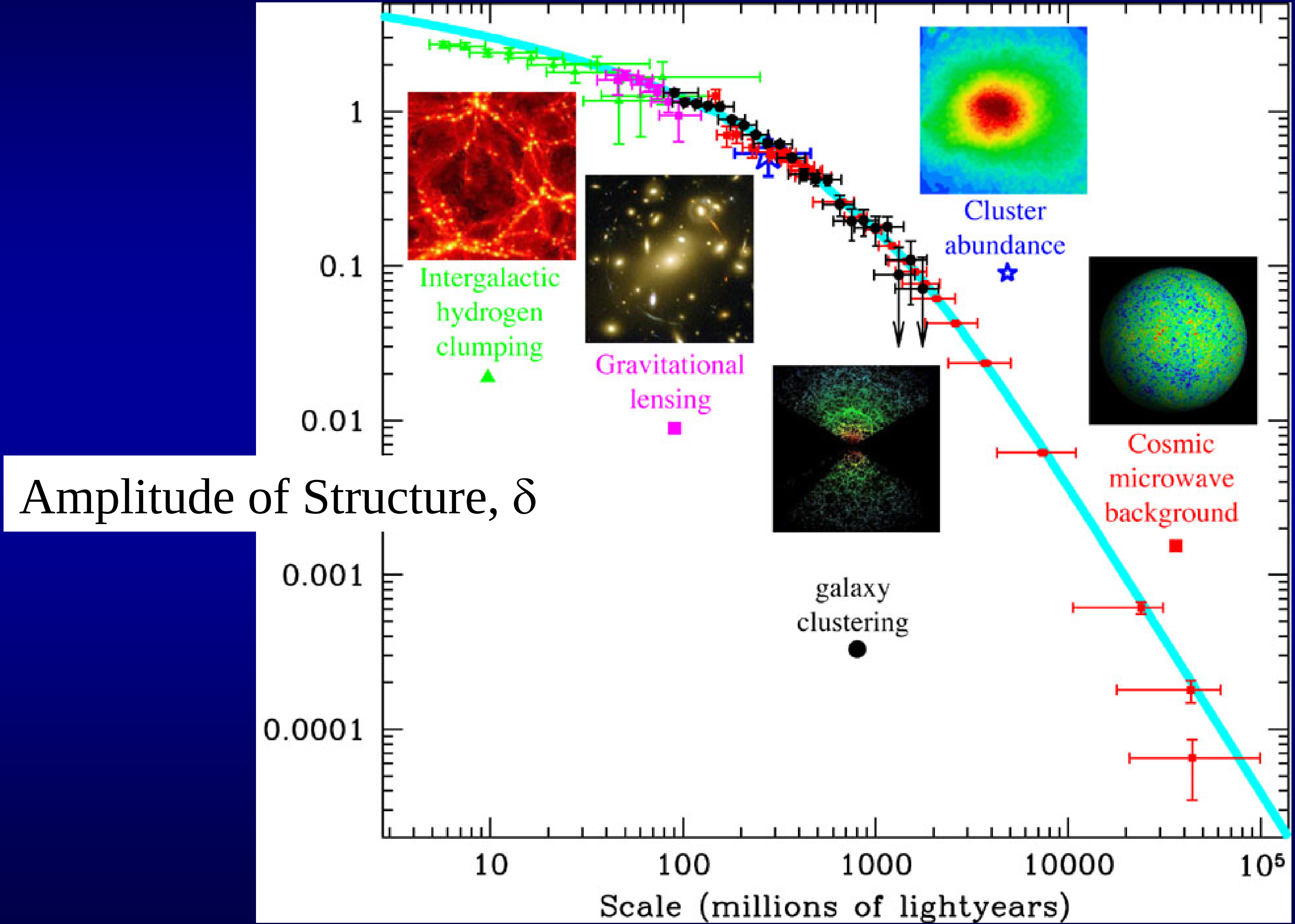


Putting it all together

The “Standard Model” of Cosmology:

- CMB acoustic peaks: $\Omega_m+\Omega_V=1$
- Supernova: $2\Omega_V-\Omega_m=1.1$
- CMB + SN: $\Omega_V=0.7$
- Galaxy clusters and galaxy clustering: $\Omega_m=0.3$
- BBN and CMB: $\Omega_B=0.04$
- CMB Sachs-Wolfe effect: $\Phi=10^{-5}$
- CMB + galaxy clustering: $n\sim1$
- HST key programme and CMB: $h=0.7$
- Age of Universe: 13.7+/- 0.2 Gyrs

Structure in the Universe with CDM

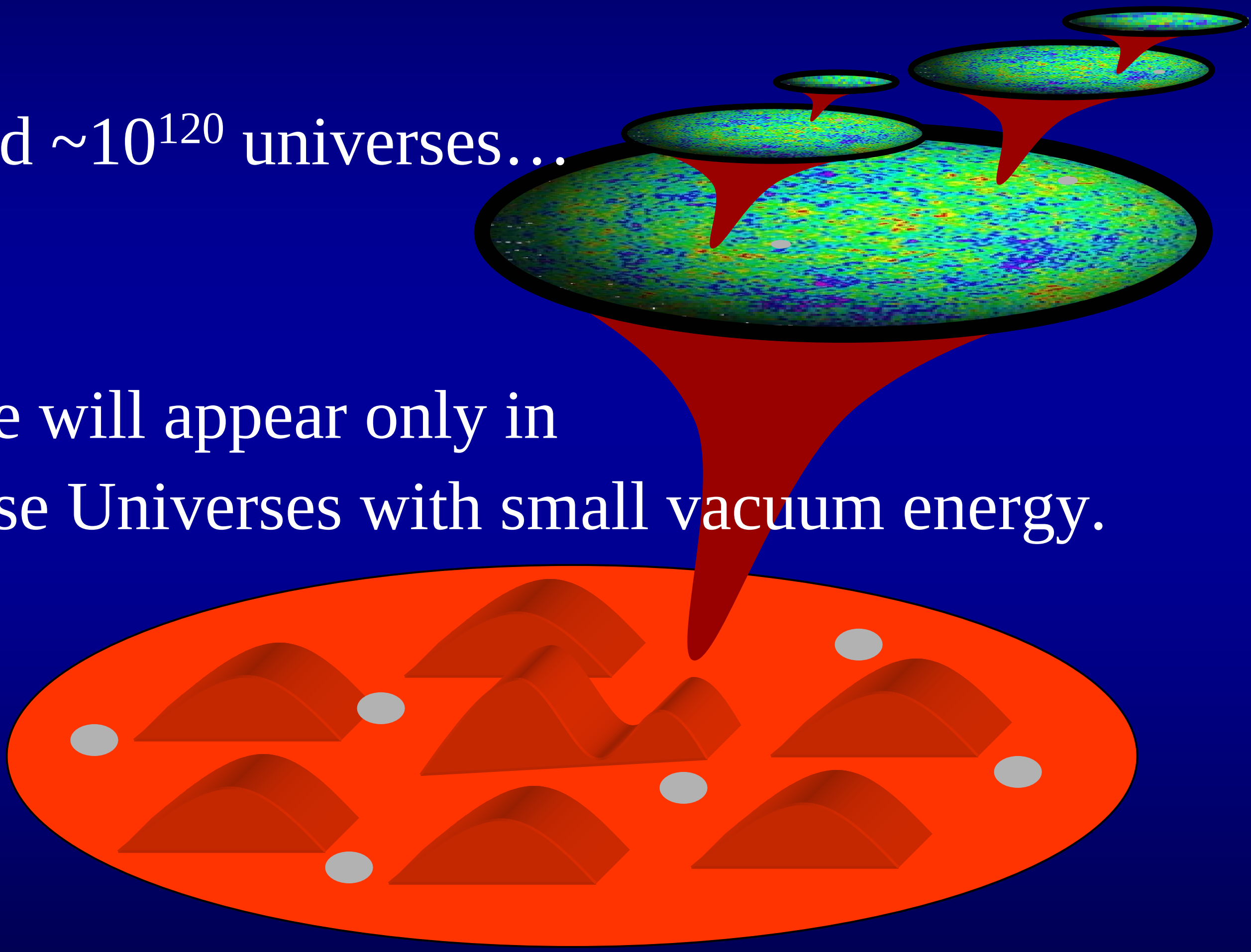


Open Questions and Speculations

- Need to explain this strange Universe.
  - What is vacuum energy (dark energy)?
  - Why is $\rho_V \sim (1\ \text{eV})^4 \sim \rho_{m?}$?
    - New particle physics, change gravity?
  - What is the Cold Dark Matter?
    - CDM – neutralino in LHC?
  - Did inflation happen?
    - Detect gravity wave background?
  - How did galaxies form?
    - Watch them form at high-z?

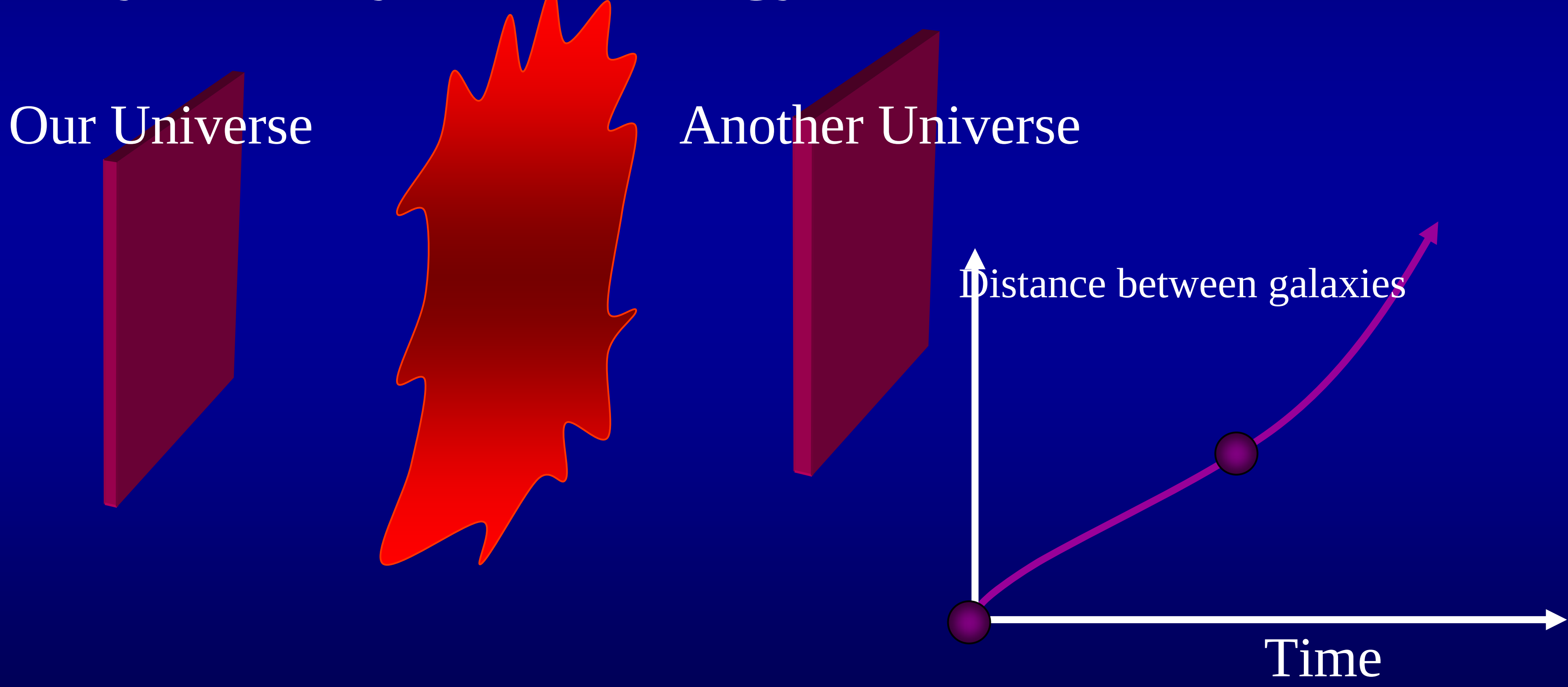
A Multiverse?

- Inflation and superstring theory both predict a multiverse.
- Find ~10^{120} universes…
- Life will appear only in those Universes with small vacuum energy.



Parallel Universe Cosmologies

- Speculative ideas like the Ekpyrotic Universe try to unify Dark Energy and Inflation.




The End